An Open Endowment Economy

(following **very closely** Ch. 2 in *OEM*)

Octubre 2020

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- Consumption smoothing gives rise to equilibrium movements in the *TB* and *CA*.

The framework is called the **intertemporal approach to the current account**.

The Model

Problem of the hh: choose processes $\{c_t, d_t\}_{t=0}^{\infty}$ to maximize:

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_t) \tag{2.1}$$

subject to the sequential budget constraint

$$c_t + (1+r)d_{t-1} = y_t + d_t, \tag{2.2}$$

and to the no-Ponzi-game constraint

$$\lim_{j\to\infty} E_t \frac{d_{t+j}}{(1+r)^j} \leq 0, \tag{2.3}$$

given d_{-1}, where y_t is an exogenous endowment and r is a constant interest rate.

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In the **closed economy** context: permanent income model.

Optimality Conditions

The Lagrangian associated with the household's problem is

$$\mathcal{L}_0 = E_0 \sum_{t=0}^{\infty} \beta^t \{U(c_t) + \lambda_t [d_t + y_t - (1+r)d_{t-1} - c_t]\},$$

where $\beta^t \lambda_t$ is a the Lagrange multiplier associated with b.c. of period t.

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Interpretation: at the margin, the household is indifferent between consuming a unit of good today or saving it and consuming it the next period along with the interest.

We can define the equilibrium here...

A **Rational Expectation Equilibrium** is given by the sequences $\{c_t, d_t\}_{t=0}^{\infty}$ satisfying:

1. the flow budget constraint (2.2),
2. the NPG condition (2.3) with equality, and
3. the Euler Eq. (2.4),

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So, all the statements that we'll make next can be understood as *equilibrium behavior*.

The Intertemporal Resource Constraint

Writing the sequential budget constraint (2.2) for period $t+j$, dividing by $(1+r)^j$, and taking expected values conditional on information available in period t yields:

$$\frac{E_t c_{t+j}}{(1+r)^j} + \frac{E_t d_{t-1+j}}{(1+r)^{j-1}} = \frac{E_t y_{t+j}}{(1+r)^j} + \frac{E_t d_{t+j}}{(1+r)^j}.$$

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Then sum for $j=0$ to $j=J$:

$$\sum_{j=0}^{J} \frac{E_t c_{t+j}}{(1+r)^j} + (1+r) d_{t-1} = \sum_{j=0}^{J} \frac{E_t y_{t+j}}{(1+r)^j} + \frac{E_t d_{t+J}}{(1+r)^J}.$$

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Taking limit for $J \to \infty$ and using the transversality condition (equation (2.3) holding with equality) yields the intertemporal resource constraint

$$(1+r)d_{t-1} = \sum_{j=0}^{\infty} \frac{E_t(y_{t+j} - c_{t+j})}{(1+r)^j}. \tag{2.5}$$

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Interpretation: at every point in time, the economy must be expected to put aside a stream of resources, $\{E_t y_{t+j} - E_t c_{t+j}\}_{j=0}^{\infty}$, large enough in present discounted value to cover the outstanding external debt.

Can An Economy Run A Perpetual Trade Deficit?

Since there is only one good, the trade balance (tb_t) is given by the difference between output and consumption:

$$tb_t = y_t - c_t. \tag{2.6}$$

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The answer to the above question depends on the country's initial debt:

- If the country is a net debtor ($d_{t-1} > 0$), then it must generate an expected trade surplus in at least one period, $E_t tb_{t+j} > 0$ for some j.
- **So:** If the country starts out as a net debtor, then it **cannot** run perpetual trade deficits.

Two Simplifying Assumptions

The following two assumptions are handy because they allow for a closed-form solution of the model:

1. The subjective and market discount rates are equal
$$\beta = \frac{1}{1+r}.$$
2. The period utility function is quadratic
$$U(c) = -\frac{1}{2}(c - \overline{c})^2, \tag{2.7}$$
where $\overline{c}$ is a large enough number so that $c < \overline{c}$ in equilibrium.

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$$c_t = E_t c_{t+1}, \tag{2.8}$$

that is, consumption follows a **random walk**. Households expect to maintain their current level of consumption forever

$$E_t c_{t+j} = c_t$$

for all $j > 0$.

Two useful expressions

Before presenting the closed form solutions to the model we introduce two variables that will be convenient in their characterization and interpretation:

- nonfinancial permanent income, y_t^p
- difference between current income and permanent income, $y_t - y_t^p$

Nonfinancial permanent income, y_t^p

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This equation says that y_t^p is a weighted average of expected future income levels, with weights adding up to one.

The difference between current and permanent income, $y_t - y_t^p$

$$y_t - y_t^p = y_t - y_t^p + \frac{1}{r} y_t^p - \frac{1}{r} y_t^p$$

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$\Rightarrow$

$$y_t - y_t^p = -\sum_{j=1}^{\infty} \frac{E_t \Delta y_{t+j}}{(1+r)^j} \tag{2.27}$$

Interpretation: Permanent income exceeds current income when income is expected to grow in the future.

The Closed-Form Equilibrium Solution

Use the fact that $E_t c_{t+j} = c_t$ to eliminate $E_t c_{t+j}$ from the intertemporal resource constraint (2.5):

$$c_t = y_t^p - r d_{t-1}, \tag{2.11}$$

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My question to you: what happens if I have positive assets?

The Equilibrium Behavior of the Trade Balance

Using the equil. level of consumption given in (2.11), we obtain the closed-form solution for the trade balance, $tb_t = y_t - c_t$:

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$\Rightarrow$ the trade balance responds countercyclically to changes in current income if permanent income increases by more than current income in response to increases in current income. This is an important result: recall evidence on the trade balance being countercyclical.

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To capture this fact in the context of our model, the endowment process must be such that permanent income increases by more than one for one with current income. What type of process satisfies this requirement? We will return to this point soon.

Last class we ended with...

When $\beta(1+r) = 1$ and $U(c) = -1/2(c - \bar{c})^2$, then the model has the following closed-form solution:

$$c_t = y_t^p - rd_{t-1} \tag{2.11}$$

$$tb_t = y_t - y_t^p + rd_{t-1} \tag{2.16}$$

where

$$y_t^p \equiv \frac{r}{1+r} \sum_{j=0}^{\infty} \frac{E_t y_{t+j}}{(1+r)^j} \tag{2.10}$$

and d_{-1} and the stochastic process for y_t are exogenously given.

The Equilibrium Behavior of External Debt

Use (2.11) again to eliminate c_t from the sequential budget constraint (2.2) to get

$$d_t - d_{t-1} = y_t^p - y_t. \tag{2.12}$$

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$$d_t = 1 d_{t-1} + (y_t^p - y_t)$$

The Equilibrium Behavior of the Current Account

The current account is defined as the trade balance minus interest payments on the external debt

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Fundamental balance of payments identity: the CA equals the change in the country's net asset position.

Results so far

When $\beta(1+r)=1$ and $U(c)=-1/2(c-\bar{c})^2$, then the model has the following closed-form solution:

$$c_t = y_t^p - rd_{t-1} \tag{2.11}$$

$$d_t = d_{t-1} + y_t^p - y_t \tag{2.12}$$

$$ca_t = y_t - y_t^p \tag{2.15}$$

$$tb_t = y_t - y_t^p + rd_{t-1} \tag{2.16}$$

where

$$y_t^p \equiv \frac{r}{1+r}\sum_{j=0}^{\infty}\frac{E_t y_{t+j}}{(1+r)^j} \tag{2.10}$$

and d_{-1} and the stochastic process for y_t are exogenously given.

Memo item:

$$ca_t = y_t - y_t^p = -\sum_{j=1}^{\infty}\frac{E_t \Delta y_{t+j}}{(1+r)^j} \tag{2.27}$$

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- By contrast, permanent output shocks, that is movements in y_t that leave $y_t - y_t^p$ unchanged do not produce movements in the current account.

Thus, the following principle emerges:

Finance temporary shocks (by running current account surpluses or deficits with little change in consumption) and *adjust to permanent shocks* (by changing consumption but not the current account).

The Income Process

We have established that in order for the present model to account for the observed countercyclicality of the current account and the trade balance, it is required that $y_t - y_t^p$ be countercyclical.

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In turn, the cyclicality of $y_t - y_t^p$ depends on the particular process assumed for the endowment y_t.

For this reason, we now characterize the equilibrium under three representative income processes:

- Two Stationary Processes
  – y_t follows an AR(1) process
  – y_t follows an AR(2) process
- A nonstationary process, Δy_t follows an AR(1) process.

An AR(1) Income Process

Suppose y_t follows the law of motion

$$y_t - \bar{y} = \rho(y_{t-1} - \bar{y}) + \epsilon_t,$$

where ϵ_t is a white noise with mean zero and variance σ_ϵ^2 and $\bar{y}$ is a positive constant. The parameter $\rho \in (-1, 1)$ measures persistence.

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$$E_t y_{t+j} = \bar{y} + \rho^j (y_t - \bar{y}).$$

Thus, if $\rho > 0$, which is the case of greatest empirical interest, the forecast converges monotonically to the unconditional mean of the process, namely $\bar{y}$. The more persistent the process (the higher ρ) is, the slower the speed of convergence of the forecast to the unconditional mean will be.

Permanent Income with AR(1) Endowment

Recall (2.10): $y_t^p \equiv \frac{r}{1+r}\sum_{j=0}^{\infty}\frac{E_t y_{t+j}}{(1+r)^j}$

Use $E_t y_{t+j} - \bar{y} = \rho^j(y_t - \bar{y})$ to eliminate $E_t y_{t+j}$ from (2.10) to obtain

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- if the endowment process is temporary ($\rho \to 0$), then only a small fraction, $r/(1+r)$, of movements in the endowment are incorporated into permanent income.

- if the endowment process is highly persistent ($\rho \to 1$), most of movements in the current endowment are reflected in movements in permanent income.

A key variable is $y_t - y_t^p$, which under the AR(1) process is given by

$$y_t - y_t^p = \frac{1-\rho}{1+r-\rho}(y_t - \bar{y}). \qquad (*)$$

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FAIL !

Consumption Adjustment with AR(1) Income

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- if $\rho = 0$ consumption increases less than one for one with current income. Why? Because current income is higher than permanent income.
- if $\rho \approx 1$, consumption adjusts one-for-one with current output, as current income is equal to permanent income.

External Debt Adjustment with AR(1) Income

By (2.12) and (*)

$$d_t = d_{t-1} + (y_t^p - y_t) = d_{t-1} - \frac{1-\rho}{1+r-\rho}(y_t - \bar{y}) \qquad (2.21)$$

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- if $\rho = 0$ debt decreases almost one-for-one with income. Why? The country saves most of the temporary income increase to be able to consume not only more in the current period but also in all future periods.

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- if $\rho \approx 1$, debt is unchanged. Why? Because income will be also higher in the future and hence there is no need to save.

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- if $\rho = 0$ the change in the current account is close to the change in current income. Why? The country saves most of the temporary income increase to be able to consume not only more in the current period but also in all future periods.

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- if $\rho = 0$, then most of the change in current income is **exported** resulting in a trade balance improvement.
- if $\rho \approx 1$, then none of the change in current income is exported and the trade balance is unchanged.

Summing up

- We've just derived the implications for all the time series of interest: $\{c_t, d_t, tb_t, ca_t\}$

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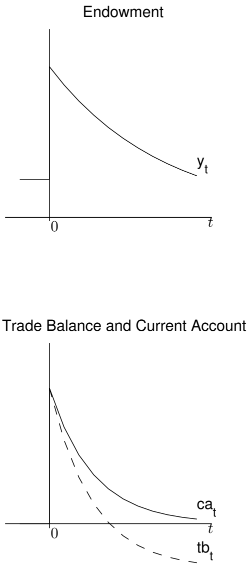
- The next graph displays the impulse response of y_t, c_t, d_t, tb_t, and ca_t to a unit increase in y_t assuming zero initial debt, $d_{t-1} = 0$ for a value of $\rho \in (0, 1)$.

Response to a positive and persistent endowment shock: AR(1) process



Counterfactual Predictions with AR(1) Endowment

The figure on the previous slide illustrates that a positive endowment shock causes an improvement in the TB and the CA. This is **counterfactual**.

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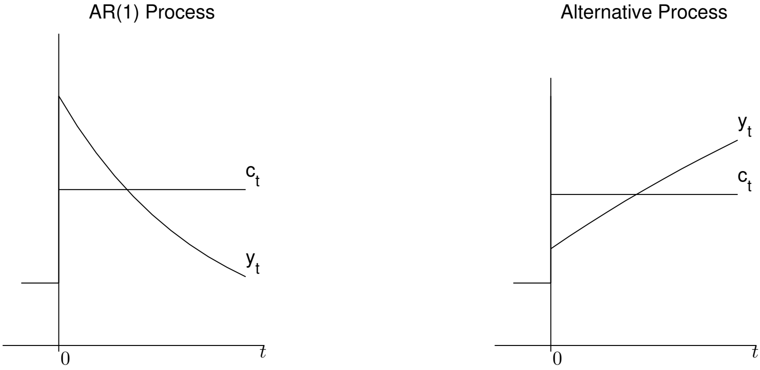

Counterfactual Predictions with AR(1) Endowment

- Under the AR(1) process, an increase in output in the current period is expected to die out over time. So future output is expected to be lower than current output.
- As a result, consumption smoothing households save part of the current increase in output for future consumption.

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- Now suppose we could come up with an output process like the one shown on the right panel of the figure:
- There, an increase in output today creates expectations of even higher output in the future.
- As a result, consumption today increases by more than output, as consumption-smoothing households borrow against future income.
- Here, an output shock would tend to worsen the trade balance and the current account, which is more in line with the data.

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- Here, an output shock would tend to worsen the trade balance and the current account, which is more in line with the data.

- What type of endowment processes can give rise to an upward sloping impulse response for output?

An AR(2) Income Process

$$y_t = \bar{y} + \rho_1(y_{t-1} - \bar{y}) + \rho_2(y_{t-2} - \bar{y}) + \epsilon_t \qquad (2.22)$$

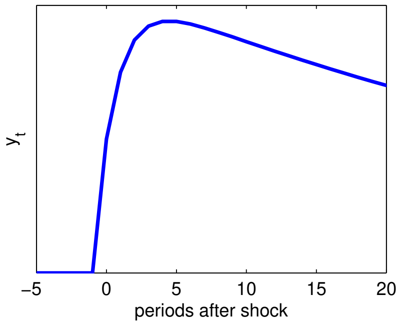

Figure: AR(2) with $\rho_1 = 1.5$ and $\rho_2 = -0.51$.

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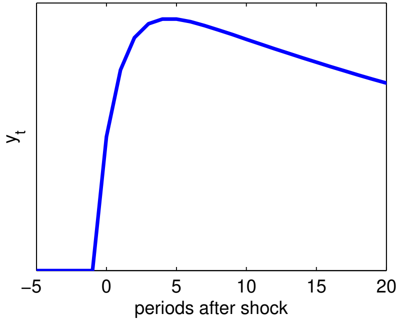


Figure: AR(2) with $\rho_1 = 1.5$ and $\rho_2 = -0.51$.

• impulse response is hump-shaped, that is, the peak output response occurs several periods after the shock occurs.

• current level of output **may** rise by less than permanent income, that is, the change in y_t may be less than the change in y_t^p.

• If so, the trade balance and the current account will deteriorate in response to an increase in output, bringing the model closer to the data.

Restrictions on ρ_1 and ρ_2 to ensure stationarity

• income process should be mean reverting (stationary), i.e., $E_t y_{t+j}$ exists for all $j \geq 0$ and

$$\lim_{j \to \infty} E_t y_{t+j} = \bar{y} \quad \forall t \geq 0$$

Let $Y_t = \begin{bmatrix} y_t - \bar{y} \\ y_{t-1} - \bar{y} \end{bmatrix}$ Then write (2.22) as:

$$Y_t = R\, Y_{t-1} + \begin{bmatrix} \epsilon_t \\ 0 \end{bmatrix} \qquad \text{with } R = \begin{bmatrix} \rho_1 & \rho_2 \\ 1 & 0 \end{bmatrix}$$

This implies that

$$E_t Y_{t+j} = R^j Y_t \tag{2.23}$$

Thus $\lim_{j \to \infty} E_t y_{t+j} = \bar{y}$ holds for any initial conditions y_t, y_{t-1}, if and only if both eigenvalues of **R** lie inside the unit circle.

The eigenvalues of any 2x2 matrix **M** lie inside the unit circle if and only if

$$|det(M)| < 1$$
$$|tr(M)| < 1 + det(M)$$

In our application we have,

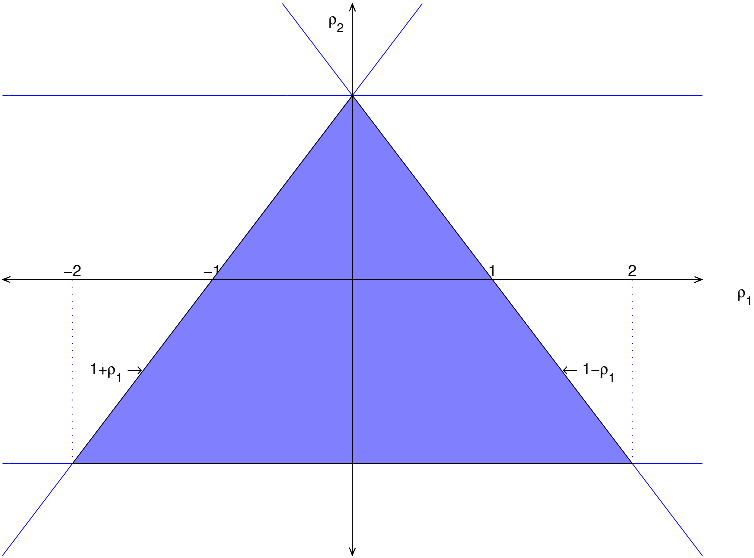
$$det(R) = -\rho_2 \quad \text{and} \quad tr(R) = \rho_1$$

It follows that the AR(2) process is mean reverting iff

$$\rho_2 < 1 - \rho_1$$
$$\rho_2 < 1 + \rho_1$$
$$\rho_2 > -1$$

Take a look at the graph on the next slide. The set of allowable pairs (ρ_1, ρ_2) are those inside the triangle.

Restrictions on ρ_1 and ρ_2 to ensure stationarity



Restrictions on ρ_1 and ρ_2 to ensure countercyclical impact response of trade balance

Recall that the key variable determining the response of the trade balance and the current account is: $y_t - y_t^p$, the difference between current income and permanent income.

Restrictions on ρ_1 and ρ_2 to ensure countercyclical impact response of trade balance

Recall that the key variable determining the response of the trade balance and the current account is: $y_t - y_t^p$, the difference between current income and permanent income.

In particular, we want to know for which pairs (ρ_1, ρ_2) current income increases by less than permanent income in response to a positive income shock in the AR(2) case.

Permanent Income with AR(2) Endowment

Recall the definition of permanent income, y_t^p, given in (2.10):

$$y_t^p - \bar{y} = \frac{r}{1+r} \sum_{j=0}^{\infty} (1+r)^{-j} E_t(y_{t+j} - \bar{y})$$

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$$\begin{aligned}
E_t Y_{t+j} &= \mathbf{R}^j Y_t \\
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\sum_{j=0}^{\infty} (1+r)^{-j} E_t Y_{t+j} &= \sum_{j=0}^{\infty} (\mathbf{R}/(1+r))^j Y_t \\
\frac{r}{1+r} \sum_{j=0}^{\infty} (1+r)^{-j} E_t Y_{t+j} &= \frac{r}{1+r} [\mathbf{I} - \mathbf{R}/(1+r)]^{-1} Y_t
\end{aligned}$$

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E_t Y_{t+j} &= \mathbf{R}^j Y_t \\
(1+r)^{-j} E_t Y_{t+j} &= (\mathbf{R}/(1+r))^j Y_t \\
\sum_{j=0}^{\infty}(1+r)^{-j} E_t Y_{t+j} &= \sum_{j=0}^{\infty}(\mathbf{R}/(1+r))^j Y_t \\
\frac{r}{1+r}\sum_{j=0}^{\infty}(1+r)^{-j} E_t Y_{t+j} &= \frac{r}{1+r}[\mathbf{I} - \mathbf{R}/(1+r)]^{-1} Y_t \\
&= \frac{r}{(1+r)(1+r-\rho_1)-\rho_2}\begin{bmatrix} 1+r & \rho_2 \\ 1 & 1+r-\rho_1 \end{bmatrix} Y_t
\end{aligned}$$

From here we obtain

$$y_t^p - \bar{y} = \frac{r}{(1+r)(1+r-\rho_1)-\rho_2}\left[(1+r)(y_t - \bar{y}) + \rho_2(y_{t-1} - \bar{y})\right] \quad (2.24)$$

$\Rightarrow$ The impact response of y_t^p to $\uparrow y_t$ is always positive. (Can you show this?)

The key variable $y_t - y_t^p$ with AR(2) Endowment

From (2.24) it follows that

$$y_t - y_t^p = \gamma_0(y_t - \bar{y}) + \gamma_1(y_{t-1} - \bar{y})$$

with

$$\gamma_0 = \frac{(1-\rho_1-\rho_2) + r(1-\rho_1)}{(1-\rho_1-\rho_2) + r(1-\rho_1) + r(1+r)} \quad \text{and} \quad \gamma_1 = \ldots$$

The model therefore predicts a counterycyclical response on impact of $y_t - y_t^p$ iff

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$$\rho_2 > (1+r)(1-\rho_1) \qquad (**)$$

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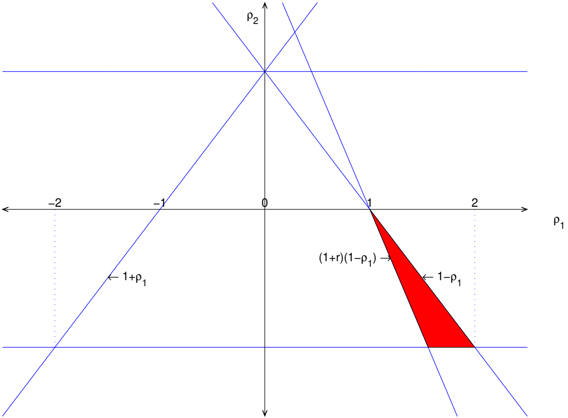
The graph on the next slide shows there exist combinations of (ρ_1, ρ_2) that satisfy this criterion and the stationarity requirements. Thus in principle the model with an AR(2) endowment process can predict a countercyclical response of the trade balance and the current account. It is then an empirical question whether output conforms to those restrictions. (See Exercise 2.8)

Restrictions on ρ_1 and ρ_2 to ensure countercyclical impact response of trade balance with AR(2) income



We need $\rho_1 > 1$. Why? To have future income to be higher than current income. And we need $\rho_2 < 0$ for stationarity, but not too negative to avoid that the initial increase in output is followed by a future decline.

Summary of adjustment with AR(2) income

In response to a positive income shock in period t:
- y_t^p increases
- By (2.11): $c_t = y_t^p - rd_{t-1}$, hence c_t procyclical on impact.

and provided $\rho_2 > (1+r)(1-\rho_1)$
- $y_t^p - y_t$ increases,
- By (2.12): $d_t = d_{t-1} + y_t^p - y_t$, hence d_t increases.
- By (2.15): $ca_t = y_t - y_t^p$, hence ca_t countercyclical
- By (2.16): $tb_t = y_t - y_t^p + rd_{t-1}$, hence tb_t countercyclical

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We have shown that it is possible for the model to predict a counter cyclical trade balance adjustment even under a stationary income process. Thus non-stationarity **is not a necessary assumption** for a countercyclical adjustment. Next we show that a non-stationary income process can give rise to a countercyclical trade balance adjustment as well.

A Nonstationary Income Process

$$\Delta y_t \equiv y_t - y_{t-1} \tag{2.25}$$

$$\Delta y_t = \rho \Delta y_{t-1} + \epsilon_t \tag{2.26}$$

The key variable $y_t - y_t^p$ with Nonstationary Income

In general (i.e., not using any particular stochastic for the income process)

$$y_t - y_t^p = -\sum_{j=1}^{\infty} \frac{E_t \Delta y_{t+j}}{(1+r)^j} \tag{2.27}$$

When income follows (2.26), then

$$E_t \Delta y_{t+j} = \rho^j \Delta y_t$$

and hence (2.27) can be expressed as

$$y_t - y_t^p = -\frac{\rho}{1+r-\rho} \Delta y_t \tag{***}$$

This equation says that permanent income increases by more than current income if $\rho > 0$, that is, then the growth rate shock is persistent. In this case, the level of output is expected to rise in the future.

Current Account Adjustment with Nonstationary Income

By (2.16) and (***)

$$tb_t = y_t - y_t^p + rd_{t-1} = -\frac{\rho}{1+r-\rho}\Delta y_t + rd_{t-1}$$

and by (2.15) and (***)

$$ca_t = y_t - y_t^p = -\frac{\rho}{1+r-\rho}\Delta y_t$$

Hence, as long as, $\boldsymbol{\rho > 0}$, the economy with nonstationary income predicts a countercyclical impact response of the trade balance and of the current account.
Why? Agents increase consumption more than income in anticipation of future income increases.

IRFs under Non-Stationary y_t

Excess Consumption Volatility with Nonstationary Income

Can the model account for the empirical regularity that consumption changes are more volatile than output changes, as documented in Chapter 1.
Let

$$\Delta c_t \equiv c_t - c_{t-1}$$

and $\sigma_{\Delta c}$ and $\sigma_{\Delta y}$ denote the standard deviation of consumption and output changes, respectively.
Use

$$ca_t = y_t - c_t - rd_{t-1}$$

Take differences

$$ca_t - ca_{t-1} = \Delta y_t - \Delta c_t - r(d_{t-1} - d_{t-2}).$$

Noting that $d_{t-1} - d_{t-2} = -ca_{t-1}$ and solving for Δc_t, we obtain:

$$\begin{aligned}
\Delta c_t &= \Delta y_t - ca_t + (1+r)ca_{t-1} \\
&= \Delta y_t + \frac{\rho}{1+r-\rho}\Delta y_t - \frac{\rho(1+r)}{1+r-\rho}\Delta y_{t-1} \\
&= \frac{1+r}{1+r-\rho}\Delta y_t - \frac{\rho(1+r)}{1+r-\rho}\Delta y_{t-1} \\
&= \frac{1+r}{1+r-\rho}\ \epsilon_t. \qquad (2.29)
\end{aligned}$$

The change in consumption is a white noise. Why? By the Euler equation (2.8).

By (2.29)

$$\sigma_{\Delta c} = \frac{1+r}{1+r-\rho}\sigma_\epsilon.$$

and by equation (2.26)

$$\sigma_{\Delta y}\sqrt{1-\rho^2} = \sigma_\epsilon$$

so that

$$\frac{\sigma_{\Delta c}}{\sigma_{\Delta y}} = \left[\frac{1+r}{1+r-\rho}\right]\sqrt{1-\rho^2} \tag{2.30}$$

When $\rho = 0$, consumption and output changes are equally volatile. When $\rho > 0$, consumption changes can become more volatile than output changes. To see this: RHS of (2.30) is increasing in ρ at $\rho = 0$. Since consumption and output changes are equally volatile at $\rho = 0$, it follows that there are values of ρ in the interval $(0, 1)$ for which the volatility of consumption changes is higher than that of output changes.

This property ceases to hold as Δy_t becomes highly persistent. This is because as $\rho \to 1$, the variance of Δy_t becomes infinitely large as changes in income become a random walk, whereas, as expression (2.29) shows, Δc_t follows an i.i.d. process with finite variance for all values of $\rho \in [0, 1]$.

Plot of $\sigma_{\Delta c}/\sigma_{\Delta y} = \left[\frac{1+r}{1+r-\rho}\right]\sqrt{1-\rho^2}$

Flashback to stationary AR(1)

What is σ_c / σ_y in this model?

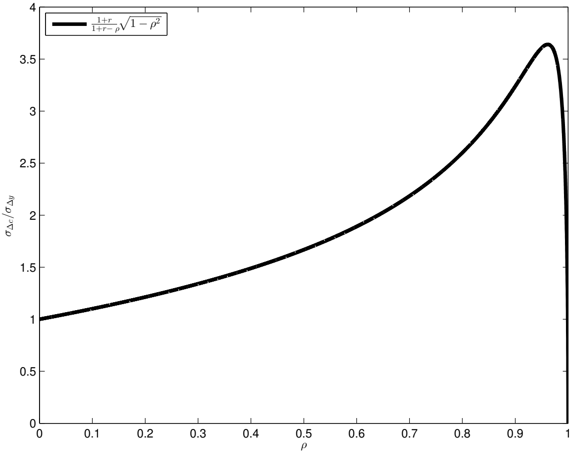
Flashback to stationary AR(1)

What is σ_c / σ_y in this model?

Eq. (2.18) tells us that:

$$c_t = fracr1 + r - \rho - rd_{t-1}$$

Excess Volatility of Consumption Changes and the Persistence of Output Changes



Note. This figure plots equation (2.30) as a function of ρ for an interest rate of 4 percent ($r = 0.04$).

Testing the Intertemporal Approach to the Current Account

- Hall (1978) initiates a large empirical literature testing the random walk hypothesis for consumption implied by the permanent income hypothesis.
- Campbell (1987) tests the predictions of the permanent income model for savings.
- Nason and Rogers (2006) test the predictions regarding the current account.

In the present model savings and the current accounts are equal (because we are abstracting from investment).

Combining (2.15) and (2.27) yields

$$ca_t = y_t - y_t^p = -\sum_{j=1}^{\infty}(1+r)^{-j}E_t\Delta y_{t+j} \qquad (2.28)$$

This equation says that a country runs a current account deficit (i.e., borrows from the rest of the world), when the present discounted value of future income changes is positive. And the country runs a current account surplus (i.e., lends to the rest of the world), when current income exceeds permanent income. It also says that current account data should forecast income changes.

Testable Restrictions of the Intertemporal Approach to the Current Account

The testable restriction we focus on is:

$$ca_t = -\sum_{j=1}^{\infty} \frac{E_t \Delta y_{t+j}}{(1+r)^j} \qquad (2.28\ \text{R})$$

We have observations on the LHS of this equality because we have current account data. But we do not have data on the RHS. How can we get an estimate of the RHS? The idea is to use a vector autoregression (VAR) model in output changes (Δy_t) and the current account (ca_t) to construct an estimate of the RHS given a calibrated value of r. Given the estimate of the RHS, we can test the hypothesis that the LHS is equal to the RHS.

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Punchline:

- Nason and Rogers (2006) find evidence rejecting the hypothesis that the "data" follows the permanent-income endowment model (in its open-economy variant).

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Punchline:

- Nason and Rogers (2006) find evidence rejecting the hypothesis that the "data" follows the permanent-income endowment model (in its open-economy variant).
- This is regardless of stationary vs non-stationary shocks
- We need to enrich the model: its sources of fluctuations and propagation mechanisms.

Conclusions

- propagation mechanism invoked by canonical intertemporal model of the current account does not provide a satisfactory account of the observed current account dynamics.
- for AR(1) output specifications that model predicts the current account to be procyclical, whereas it is countercyclical in the data.
- AR(2) or nonstationary specifications for the income process can in principle imply a countercyclical adjustment of the current account. Yet, empirical tests rejects the basic mechanism of the intertemporal model of the current account namely that current accounts are equal to future expected income changes.
- to bring observed and predicted behavior of the current account closer together, in the coming weeks we will enrich both the model's sources of fluctuations and its propagation mechanism.