On Wars, Sanctions and Sovereign Default *

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Abstract

This paper explores the role of restrictions on the use of international reserves as economic sanctions. We develop a simple model of the strategic game between a sanctioning (creditor) country and a sanctioned (debtor) country. We characterize how the sanctioning country should impose restrictions optimally, internalizing the geopolitical benefits and the potential losses of a default by the sanctioned country.

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JEL classification: E42, F32, F41.

*First draft, April 2022. The views expressed herein are those of the authors and not necessarily those of the Federal Reserve Bank of Minneapolis or the Federal Reserve System. E-mails: javier.i.bianchi@gmail.com and csosapad@nd.edu.

1 Introduction

Following the invasion of Ukraine, Russia faced a freezing of its international reserves, which amounted to close to 30% of its GDP. While the goal of the sanctions was to hinder the financing of the war, Russia was allowed to continue tapping reserves to make payments on its sovereign bonds. On April 4, 2002, however, the US Treasury blocked these payments, and Russia failed to meet its obligations. A few days later, Russia was declared in default.[1]

In this paper, we explore the role of restrictions on international reserves as economic sanctions and develop a simple model that can account for this set of events. The model has two countries: a debtor country, Russia (the sanctioned country), and a creditor country, the US (the sanctioning country). The sanctioned country can default on its debt and choose external borrowing and international reserves. The sanctioning country can impose restrictions on the use of reserves by the sanctioned country, and its utility is decreasing in the utility of the sanctioned country. We refer to this latter feature as a "geopolitical externality." In this environment, we search for the Stackelberg Nash equilibrium in which the sanctioning country takes into account the strategic response of the sanctioned country when it chooses the restriction on the use of reserves.

We show that soft restrictions on the use of reserves by the sanctioned country are a free lunch for the sanctioning country. They impose some limits on war financing and come at no cost for the sanctioning country. Hard restrictions, however, can impose costs on the sanctioning country by precipitating a default by the sanctioned country. The question is then whether it could be optimal for the home country to ever choose a hard enough restriction that induces a default by its debtor. For a low geopolitical externality, we show that the optimal restriction involves squeezing the resources up to the point at which the sanctioned country is indifferent between repaying and defaulting. For a high geopolitical externality, the optimal restriction becomes a complete freezing of reserves and it induces the foreign country to default. This finding may be surprising because the decision to default by the foreign country is optimal and so restrictions tougher than those that make the foreign country indifferent between repaying and defaulting do not actually hurt the foreign country. Key for our result, however, is that the geopolitical externality is tilted towards the war period and a default reduces the utility in that period.

The theory can account for the sequence of events preceding the sovereign debt crisis in Russia (see Figure 1). Following the invasion, yields on Russian government bonds spiked

[1] See "U.S. stops Russian bond payments, raising risk of default," by Megan Davies and Alexandra Alper, Reuters, April 5, 2022 and "Russia's First Default in a Century Looks All But Inevitable Now", Bloomberg News April 9, 2022. Figure 1 presents the evolution of Russian bond yields and reserves during 2022.

up (see panel [a]), largely as a result of the large scale of sanctions on Russia which lead investors to anticipate a default. However, Russia continued paying the coupons that were coming due using its international reserves (see panel [b]). On April 4, 2022, when the US Treasury blocked payments using reserves, we see another increase in bond yields. Shortly after, Russia missed dollar bond payments and S&P declared Russia in default.

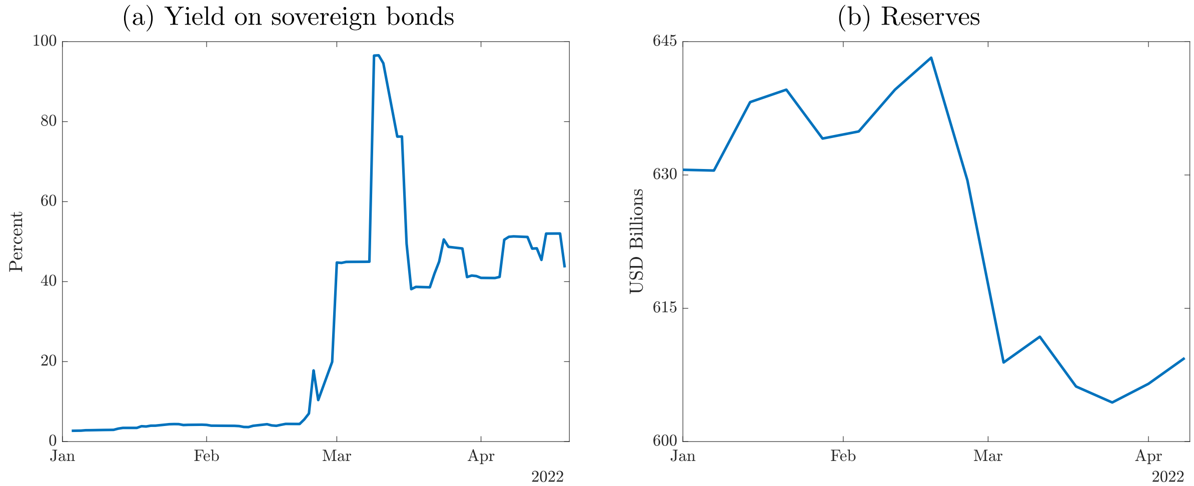

Figure 1: Reserves and government bond yields for Russia in 2022

Literature. Our paper is related to the nascent literature on the economics of sanctions spurred by the Russian-Ukrainian war.[2] Notable examples include Itskhoki and Mukhin (2022) and Lorenzoni and Werning (2022) on the effects of trade sanctions of the Russian ruble exchange rate, Bachmann et al. (2022) and Baqaee et al. (2022) on the effects of a full trade embargo on Europe's economy; and Sturm (2022) who provides a theory of tariffs as economic sanctions from the perspective of terms-of-trade manipulation. Our paper instead develops a model of international financial sanctions, tackling the interaction between a sovereign default in the sanctioned country and the resulting losses for the sanctioning country.

Our paper also draws on the literature on sovereign debt and international reserves (Alfaro and Kanczuk, 2009; Bianchi, Hatchondo and Martinez, 2018; Bianchi and Sosa-Padilla, 2020), which in turn builds on the workhorse sovereign default model (Eaton and Gersovitz, 1981; Aguiar and Gopinath, 2006; Arellano, 2008). Most of the literature also focuses on the case where investors are either risk neutral or their holdings of sovereign bonds are too small to affect their marginal utility.[3] Our is the first paper to analyze the role of restrictions on reserves as international sanctions and the strategic interaction between a creditor and a

[2] See Van Bergeijk (2021) for a review of the broader literature on sanctions at the intersection of political science and international economics.

[3] Notable exceptions include Park (2014), Arellano, Bai and Lizarazo (2017), and Morelli, Ottonello and Perez (2022).

debtor country in the presence of a geopolitical externality.

2 A Model of Financial Sanctions and Sovereign Default

We present a model of the strategic interactions that result from international sanctions and the possibility of sovereign default. There are two countries, a foreign country, which we think of as Russia, and the home country, which we think about as the US. The economy is also populated by financial intermediaries that discount future payoffs at rate r. Time is discrete, the horizon is infinite and there is no uncertainty.

2.1 Foreign Country

We start the description of the model with the foreign country. The foreign country starts at period 0 with a portfolio of reserves and debt (a_0^*, b_0^*) and receives a constant income y^*. Reserves are one period non-negative, risk-free assets that may be subject to restrictions, given economic sanctions. Debt is long-term with a maturity parameter δ. In particular, a bond issued in period t promises to pay $\kappa(1-\delta)^{j-1}$ units of the tradable good in period $t+j$, for all $j \geq 1$.[4]

The government budget constraint is given by

$$c_t^* + g_t^* + \frac{a_{t+1}^*}{1+r} + \kappa b_t^* = a_t^* + y^* + q(a_{t+1}^*, b_{t+1}^*)[b_{t+1}^* - (1-\delta)b_t^*] \tag{1}$$

where g_t^* is fixed war expenditures and q is the bond price schedule for government bonds. Given the assumption that the war last only one period, $g_t^* = 0$ for all $t > 0$.[5]

Relative to the standard repayment problem of the government in the sovereign debt literature, our model has two extra constraints. First, we have a constraint that restricts the use of reserves:

$$\frac{a_1^*}{1+r} \geq \underline{a}\,, \tag{2}$$

We assume that $\underline{a} \leq a_0^*$ (i.e, the government cannot be forced to increase the amount of reserve holdings). Constraint (2) encompasses the case $\underline{a} = a_0^* - \kappa b_0^*$, which restricts reserves for purposes other than debt repayments, as established at the onset of the Russian-Ukrainian war. The harshest punishment is when $\underline{a} = a_0^*$, which implies that reserves cannot be used at

[4]We normalize the coupon size to $\kappa = (\delta + r)/(1+r)$, which guarantees that a default-free bond with the same maturity and coupon structure trades at a price of $1/(1+r)$.

[5]What will be important for our analysis is $y_0^* - g_0^*$ is lower in the initial period. This could result from lower initial output or higher expenditures.

all and interest payments cannot be repatriated.

Second, there is a constraint on new issuances of bonds,

$$b_1^* \leq b_0^*(1-\delta); \tag{3}$$

that is, the country cannot issue new bonds. To focus on the optimal determination of (2), we take (3) as given (i.e., as part of the existing sanctions imposed by the rest of the world).

If the country defaults, it faces an income cost ϕ^D in the period in which the default takes place, and it cannot borrow. A direct cost from defaulting, either in output or utility, is standard in the sovereign default literature to support positive levels of debt in equilibrium. In the context of the Russian default, the economy was already subject to sanctions, which likely reduced the cost of defaulting. What is crucial for our analysis is that there are non-negative costs from defaulting above and beyond those coming from other sanctions. In addition, another theme widely discussed was that repaying its debt was a way for Russia to signal that its economy was not in dire straits following the war sanctions. The cost ϕ^D captures all these dimensions by introducing a gap between the value of defaulting and repaying.[6]

We also assume there is re-entry to financial markets the period after default. This assumption is without loss of generality, because the economy is stationary the period following default. The budget constraint under default is therefore

$$c_0^* + g^* = y^* - \phi^D.$$

Notice that in the budget constraint above, we already impose that the US would impose the stringent feasible constraint (i.e., $\underline{a} = a_0^*$) in case of a default. We can show that this is without loss of generality, as it is always optimal for the home country to impose the harshest possible punishment under default. The logic is that punishing the foreign country in default has the twofold benefit for the home country of restricting the resources of the foreign country and increase the chances of getting repaid.

Therefore, the value of defaulting for the foreign country is given by

$$V_D^*(a_0^*) = u(y^* - \phi^D - g^*) + \frac{\beta}{1-\beta} u\left(y^* + \frac{r}{1+r} a_0^*\right), \tag{4}$$

where the utility function satisfies standard properties $u' > 0$ and $u'' < 0$ and Inada conditions.

In deriving the continuation utility in (4), we used that the country starts period 1 with

[6] Notice that the cost does not have to be necessarily an income cost. Under the assumption that utility is log, a proportional income cost is equivalent to a utility loss separable from consumption.

$a_0^*(1+r)$ assets and given that $\beta(1+r)=1$, it consumes the income plus the annuity value for $t \geq 1$.

We now present the value under repayment. Given that $\beta(1+r)=1$, the government would like to equalize consumption over time. Because of the extra war expenditures at time zero, this requires either reducing reserves or issuing debt.[7] However, achieving this constant consumption path may not be feasible in the presence of constraints (2) and (3) We assume that these constraints bind, which can be guaranteed by the following assumption:

Assumption 1 (Binding reserve constraint)**.** *The foreign country's initial gross positions and government spending satisfy*

$$g^* + \kappa b_0^* - a_0^* > (1-\beta)(1-\delta)b_0^*.$$

If the government uses all reserves today and pays the coupons, it is able to consume $y^* + a_0^* - g^* - \kappa b_0^*$. In turn, tomorrow the debt is $(1-\delta)b_0^*$, and this allows for a stationary consumption level of $y^* - (1-\beta)(1-\delta)b_0^*$. In other words, Assumption 1 says that if the government uses all reserves to pay coupon payments and war expenses, this leaves fewer resources for consumption today relative to tomorrow. The implication is then that the government is liquidity constrained.

Notice that since all variables are constant from $t \geq 1$, we have that

$$c_t = y^* + (a_t^* - b_t^*)(1-\beta)$$

for $t \geq 1$. If we use that $a_1 \geq 0$ and that (3) binds even in the absence of a restriction by the US, per Assumption 1, it follows that (2) and (3) also bind when there is a restriction in place. We then have that if the country chooses to repay, the value is

$$V_R^*(a^*, b^*; \underline{a}) = \max_{c^*} \left\{ u(c^*) + \frac{\beta}{1-\beta} u(y^* + (1-\beta)(\underline{a}(1+r) - (1-\delta)b^*)) \right\} \quad (5)$$

$$\text{subject to}$$

$$c^* + g^* + \underline{a} + \kappa b^* = a^* + y^*$$

We have used in writing the continuation value that the country repays for $t \geq 1$, a result that follows because compared with its state in period 0, the government in period 1 has lower debt, lower expenditures and no reserve restrictions. Notice that the constraint set in (5) becomes empty if $g^* > y^* + a^* - \underline{a} - \kappa b^*$ and the government is forced to default.

[7] This implies a $c_t^* = y^* + (1-\beta)(a_0^* - b_0^* - g_0^*)$ and therefore a net foreign asset position for next period of $a_1^* - b_1^* = (1+r)\left[a_0^* - b_0 - (1-\beta)(a_0^* - b_0^* - g_0^*)\right]$.

We assume throughout that the budget set is non-empty. It is straightforward, however, to allow for a punishment that would make the government budget set empty and generate an "involuntary" default.

Default decision. The decision to default at time 0 is as follows

$$d^*(a^*, b^*; \underline{a}) = \begin{cases} 0 & \text{if } V_R^*(a^*, b^*; \underline{a}) \geq V_D^*(a) \\ 1 & \text{if } V_R^*(a^*, b^*; \underline{a}) < V_D^*(a) \end{cases}$$

We make the following parametric assumptions about the default cost ϕ.

Assumption 2 (Default costs)**.** *We assume that*

$$u(y^* - \phi^D - g^*) + \frac{\beta}{1-\beta} u\left(y^* + \frac{r}{1+r} a_0^*\right) < u(y^* + a_0^* - g^* + \kappa b_0^*) + \frac{\beta}{1-\beta} u\left(y^* - (1-\beta)(1-\delta) b_0^*\right)$$

and that

$$u(y^* - \phi^D - g^*) + \frac{\beta}{1-\beta} u\left(y^* + \frac{r}{1+r} a_0^*\right) > u(y^* - g^* - \kappa b_0^*) + \frac{\beta}{1-\beta} u(y^* - (1-\beta)(a_0^*(1+r) - (1-\delta) b_0^*).$$

The first assumption implies that the government finds it optimal to repay when there are no restrictions on the use of reserves other than non-negativity of reserves (i.e., when $\underline{a} = 0$). The second assumption implies that the government finds it optimal to default when the harshest possible restriction is imposed on the use of its reserves (i.e., when $\underline{a} = a_0^*$).

With these assumptions in hand, we can now state the following lemma that establishes a cutoff for the restriction on reserves that induces default.

Lemma 1. *Suppose Assumption 2 holds. Let (a^*, b^*) be the initial financial position, then there exists a restriction on the use of reserves $0 \leq \hat{a} \leq a_0^*$ such that $V^R(a^*, b^*; \underline{a}) \geq V^D(a^*)$ if and only if $\underline{a} \leq \hat{a}$.*

Proof. See Appendix A.2. □

2.2 Home Country

The home country values the utility of the stream of consumption and puts a negative weight on the utility of the foreign country. We refer to this second channel as a "geopolitical externality." The home country's preferences are therefore given by

$$W = \sum_{t=0}^{\infty} \beta^t u(c_t) - \eta u(c_0^*)$$

where $\eta > 0$ measures the intensity with which the home country wishes to punish the foreign country during the war. A higher η can be interpreted as capturing how a reduction in utility for the foreign country can decrease the probability that it would win the war, either because it has fewer resources available or because of the popularity of the political regime declines.

It is worth highlighting that we are implicitly assuming that the foreign country's utility flow only induces a negative externality on the home country over the initial period. However, as we will see below, the key assumption is that the geopolitical externality is relatively more important during the war period. We also note that we abstract from the primitives of the geopolitical externality as well as the ethical foundations for it. Notice that, from a modelling perspective, our formulation of the geopolitical externality is akin to negative altruism *à la* Becker and Barro (1988).

The home country owns αb_0^* units of the foreign country's debt and other portfolio of assets and liabilities with net position k_0. With a constant income over time, and with a return on the portfolio equal to $1+r$, optimal consumption is then given by

$$c_t = y + (1-\beta)(\alpha b_0^*(1-d^*) + k_0)$$

for all $t \geq 0$.[8]

We can write the home country's welfare as

$$W(\underline{a}; d^*) = \frac{1}{1-\beta} u\left(y + (1-\beta)(\alpha b_0^*(1-d^*) + k_0)\right) - \eta u(c_0^*(\underline{a}, d^*)), \tag{6}$$

where with some abuse of notation we denote by $c_0^*(\underline{a}, d^*)$ the consumption available for the foreign country as a function of $\underline{a}$ and its default decision d^*.

The home country's welfare is therefore determined entirely by the initial default decision

[8] Recall that bonds of the foreign country are also traded by deep-pocketed investors with linear utility that discount future payoffs at rate $1+r$, and thus this pins down the equilibrium bond price. In particular, using that the environment is stationary for $t \geq 1$, we have that that equilibrium bond price is $(1+r)^{-1}$ if $b_{t+1} \leq \frac{\phi}{1-\beta}$ and zero otherwise.

by the foreign country and by c_0^*. The home country can affect these two outcomes by controlling the restrictions on the use of foreign country's reserves. The trade-off is that imposing restrictions reduces the geopolitical externality, but it may trigger a default, which implies fewer resources for the home country. Next, we analyze how this decision is made and solve for the equilibrium.

2.3 Nash Equilibrium

We assume the home country moves first by setting restrictions, followed by the foreign country's decision to repay or not. Our equilibrium concept is a Stackelberg equilibrium, defined below and illustrated in Figure A.1.

Definition 1 (Stackelberg Nash equilibrium)**.** *A Stackelberg Nash equilibrium is a policy for the home country $\underline{\mathbb{A}}$ and a best response for the foreign country $\mathbb{D}^*(\underline{a})$ such that*

$$\underline{\mathbb{A}} = \underset{\underline{a}}{\operatorname{argmax}}\, W(\underline{a}; \mathbb{D}^*(\underline{a})),$$

where

$$\mathbb{D}^*(\underline{a}) = \begin{cases} 0 & \text{if } \; V_R\left(a^*, b^*; \underline{a}\right) \geq V_D\left(a^*\right), \\ 1 & \text{if } \; V_R\left(a^*, b^*; \underline{a}\right) < V_D\left(a^*\right), \end{cases}$$

and V_R, V_D and W were defined in (4), (5), and (6), respectively.

We can solve the equilibrium by backward induction. We distinguish the payoffs when he foreign country defaults and when it repays, an outcome which is a function of $\underline{a}$.

Payoffs under default. Suppose the foreign country defaults, then the values for the foreign and home countries are, respectively,

$$V_D\left(a^*\right) = u(y^* - \phi^D - g^*) + \frac{\beta}{1-\beta} u(y^* + ra^*) \tag{7}$$

and

$$W(\underline{a}, 1) = \frac{1}{1-\beta} u(y^* + (1-\beta)k_0) - \eta u\left(y^* - g^* - \phi^D\right) \tag{8}$$

Payoffs under repayment. When the foreign country repays, the value for the foreign and home country are given by

$$V_R\left(a^*, b^*; \underline{a}\right) = u\left(y^* - g^* + a_0^* - \underline{a} - \kappa b_0^*\right) + \frac{\beta}{1-\beta} u\left(y^* + (1-\beta)\left(\underline{a}(1+r) - (1-\delta) b_0^*\right)\right) \tag{9}$$

and

$$W\left(\underline{a}, 0\right) = \frac{1}{1-\beta} u(y + (1-\beta)(\alpha b_0^* + k_0)) - \eta u\left(y^* - g^* + a_0^* - \underline{a} - \kappa b_0^*\right) \tag{10}$$

Home Country optimal policy. Given the optimal response by the foreign country, the problem solved by the home country is given by

$$\max_{\underline{a}} W\left(\underline{a}; d^*(a^*, b^*; \underline{a})\right)$$

where $d^*(a^*, b^*; \underline{a})$ is the foreign country default decision. How this decision varies with $\underline{a}$ is characterized in Lemma 1.

Let us define the payoff function $\tilde{W}(\underline{a}) = W\left(\underline{a}; d^*(a, b; \underline{a})\right)$. We have that

$$\tilde{W}(\underline{a}) = \begin{cases} \frac{1}{1-\beta} u(y + (1-\beta)(\alpha b_0^* + k_0)) - \eta u\left(y^* - g^* + a_0^* - \underline{a} - \kappa b_0^*\right) & \text{if } \underline{a} \leq \hat{a}. \\ \frac{1}{1-\beta} u(y + (1-\beta) k_0) - \eta u\left(y^* - g^* - \phi^D\right) & \text{if } \underline{a} > \hat{a}. \end{cases} \tag{11}$$

On important observation is that $\tilde{W}(\underline{a})$ is strictly increasing in $\underline{a}$ if $\underline{a} \leq \hat{a}$ while it is independent of $\underline{a}$ if $\underline{a} > \hat{a}$. In addition, $\tilde{W}(\underline{a})$ features in general a discontinuity at $\underline{a} = \hat{a}$.

It therefore follows that the solution for the home policy satisfies $\underline{a} \geq \hat{a}$. In particular, the solution features either $\underline{a} = \hat{a}$ or any $\underline{a} > \hat{a}$. In other words, conditional on inducing repayment in equilibrium, the home country squeezes the foreign country's resources up to the point at which it becomes indifferent between repaying and defaulting. When the geopolitical externality η is low, the outcome is that the home country induces repayment. However, if η is large, the optimal policy for the home country induces a default by the foreign country. We summarize this result in the next proposition.

Proposition 1. *There exists a threshold* $\hat{\eta} = \frac{u(y+(1-\beta)(\alpha b_0^* + k_0)) - u(y + (1-\beta) k_0)}{(1-\beta)\left[u\left(y^* - g^* + a_0^* - \hat{a} - \kappa b_0^*\right)\right] - u(y^* - g^* - \phi^D)} > 0$ *such that if* $\eta \leq \hat{\eta}$, *the home country chooses* $\underline{a} = \hat{a}$ *and the foreign country repays, and if* $\eta > \hat{\eta}$, *the home country sets* $\underline{a} > \hat{a}$ *and the foreign country defaults.*

Proof. See Appendix A.3 □

Discussion. The result of Proposition 1 underscores how the model can account for the evolution of the economic sanctions on Russia during the war. Following the invasion, the West prevented Russia from using its international reserves for purposes other than debt repayments. As the war escalated and the catastrophe continued, the US Treasury decided to completely freeze Russian reserves, and the default followed.

From a theoretical standpoint, the finding that a creditor country may find it optimal to push the debtor country into default is perhaps surprising. The result is indeed subtle because the decision to default by the foreign country is optimal, which means that restrictions larger than $\hat{a}$ do not actually hurt the foreign country. The crucial feature in the model is that the home country experiences more disutility from the current utility flow of the foreign country relative to the future utility flows, an assumption that is reasonable to capture that the externality is tilted towards the war period—here in particular, we assume for simplicity that the geopolitical externality applies only to the initial war period. In fact, we show in Appendix A.4 that if the home country were to face a constant geopolitical externality over time, it would never be optimal to trigger a default. Instead, the home country would choose $\eta = \hat{\eta}$. That is, the home country would select the harshest penalty such that the foreign government would repay. Underlying this result is that the decision to default is strategic for the foreign country. That is, if the foreign country defaults it is because it finds optimal to do so. Therefore, if the home country's geopolitical externality is proportional to the foreign country's lifetime utility, the home country would always be better off choosing $\eta = \hat{\eta}$. The idea is that at $\eta = \hat{\eta}$, the value for the foreign country is given by by V^D. Punishments larger than $\hat{\eta}$ would not reduce the value for the debtor country, but it would impose losses in the creditor country.[9]

2.4 A Simple Linear Example

We now provide a simple example to obtain further insights into the threshold value of the geopolitical externality, $\hat{\eta}$, at which point the sanctioning country finds it optimal to face the losses from the sanctioned country defaulting. In particular, suppose the restriction is such that the foreign country can only use reserves to pay coupon payments, then we obtain $\Delta \equiv c_0^{*R} - c_0^{*D} = \phi^D$, the difference in the sanctioned country's consumption under repayment under default. Assume also that the utility function for both countries is linear. In this case,

[9] Another assumption worth discussing is that the home country has a continuous technology to punish the foreign country, as given by $\underline{a}$. One could also think, however, about related punishments that are more discrete in nature. An interesting case is when the home country does not allow the foreign country to use banking services to be able to deliver the debt payments. In this case, we can show that the home country may find it optimal to induce default even if there is a constant geopolitical externality over time.

the sanctioning country chooses to default if $b_0^* > \phi^D$. In addition, the condition that makes the sanctioning country indifferent between inducing default or not is given by

$$\hat{\eta}\Delta = \alpha b_0^*$$

where the left-hand side represents the benefits from the reduction in the sanctioned country's consumption and the right-hand side are the losses from the default. Using $\Delta = \phi^D$, we then obtain

$$\hat{\eta} = \frac{\alpha b_0^*}{\phi^D} > \alpha\,, \tag{12}$$

where the inequality follows from the default decision under linear utility. Notice that this means that if $\alpha = 1$, we must have $\eta > 1$ to justify that the sanctioning country absorbs the default losses. The intuition is that if $\alpha = 1$, what the sanctioned country saves from defaulting is a loss for the sanctioning country. Because the default decision is optimal for the sanctioned country, this implies that the debt is higher than the penalty suffered from default. Thus, justifying a penalty that induces default implies that it is more valuable for the sanctioning country to deplete consumption of the sanctioned country by one unit than to have one more unit of consumption for itself.

2.5 A Simple Calibration

We now conduct a simple calibration exercise. The goal is to gauge the quantitative effects of the geopolitical externality and argue that under plausible parameters, the model can account for the sequence of events around the Russian default. We parameterize the Foreign country using Russian data and the home country using US data. Table A.1 reports the parameter values we use.

We assume log utility for both countries. We normalize the income in the home country to unity ($y = 1$) and set the income in the Foreign country to match the GDP of Russia relative to the US as observed in the data ($y^* = y/14$). The world interest rate is $r = 0.01$ and $\beta = 1/(1+r)$. The initial financial position of the Foreign country is given by $a_0^* = 0.3y^*$ and $b_0^* = 0.2y^*$, the ratios observed in Russian data. We set the coupon decay rate δ so that the debt duration is 6.8 years, this amounts to $\delta = 0.14$.[10] The net foreign assets of the home country are $k_0 = -0.6y$, which is the magnitude for the US net foreign assets as of 2020, as reported in Atkeson et al., 2022. We set $\alpha = 0.5$, capturing that roughly 50% of Russian bonds are held by foreign investors (see March 15's New York Times article). Given all these

[10]Bai et al. (2017) find that the debt duration for Russia is 6.83 years (using data for the period January 1993 – June 2009). The risk-free Macaulay duration, given our coupon structure, is given by $(1 + r)/(\delta + r)$.

parameters, g^* is set to $0.28y^*$ (which is the lowest value consistent with Assumption 1) and $\phi^D = 0.13y^*$ which guarantees that $\hat{a} = a_0^* - \kappa b_0^*$ (i.e. restricting reserves for any purposes other than debt coupon payments). Finally, we explore several values of the geopolitical externality.

Results. We start by illustrating the workings of the model. In panel (a) of Figure 2 we present the value for the foreign country both under repayment and under default, V_R and V_D. As implied by the Lemma 1, the value under repayment and default become equal at the threshold $\hat{a}$. In Panel (b) we present the value for the home country, $\tilde{W}(\underline{a})$ for two different values of η ($\eta = 0.03$ and $\eta = 0.05$). As shown in (11), the value for the home country is discontinuous at $\hat{a}$. Below $\hat{a}$ it is strictly increasing in the strength of the sanctions. Above $\hat{a}$, the value is independent of $\underline{a}$. The figure illustrates how for the low value of η, it is in the home country's best interest to choose the highest possible constraint without triggering a default in the foreign country. On the other hand, for the high value of η, the home country finds it optimal to introduce a restriction that is larger than $\hat{a}$ and that triggers a default and, therefore, losses for the home country.

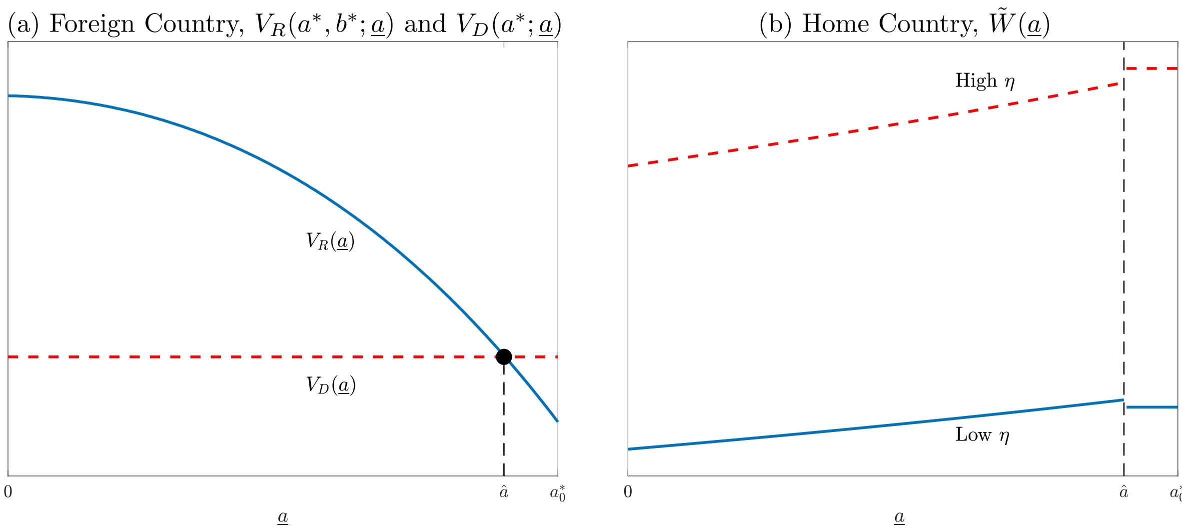

Figure 2: Value functions for Home and Foreign Countries

Note: The left panel shows the value for the Foreign country as function of the sanctions ($\underline{a}$) for both default and repayment. The right panel shows the value for the home country for two levels of the geopolitical externality, low (blue solid lines) and high (red dashed lines).

Based on these calibrated parameters, we obtain that the critical value for the intensity of the externality is $\hat{\eta} = 0.04$. This implies that the home country is willing to cut the utility from its own consumption stream by 0.04 units to reduce the foreign country's current utility by 1 unit. In terms of consumption equivalence, we can express it as follows: starting form a situation in which the foreign country repays, we compute the decrease in permanent consumption that the home country is willing to suffer to reduce the utility of the foreign country to the level under default. The value we obtain is 0.01%. The key takeaway is that it is optimal for the home country to induce default in the foreign country for a plausible range of values of the geopolitical externality.

3 Conclusion

We present a simple model to think about the implications of restrictions on the use of international reserves as economic sanctions, a measure recently adopted to punish Russia following the invasion of Ukraine. We find that soft restrictions come at no cost for the sanctioning country—they restrict resources available to the sanctioned country without negative consequences for the sanctioning country. However, a complete freezing of reserves can trigger a default by the sanctioned country and generate losses for the sanctioning country. Even though the decision to default is an optimal response by the sanctioned country, we show that a complete freezing of reserves may be optimal when there are geopolitical externalities during the war period. An interesting avenue for future research is to extend the framework to consider international trade or other sanctions.

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A Appendix

A.1 Additional Tables and Figures

Table A.1: Parameter values

Income in H	y	1
Income in F	$y*$	$y/14$
World interest rate	r	0.01
Discount factor	β	$1/(1+r)$
Initial Reserves in F	a_0^*	$0.3\,y^*$
Initial Debt in F	b_0^*	$0.2\,y^*$
Coupon decay rate	δ	0.138
Other net-foreign-assets in H	k_0	$-0.6\,y$
H's exposure to F's debt	α	0.50
Default cost	ϕ^D	$0.13\,y^*$
War spending	g^*	$0.28\,y^*$
Geopolitical externality	$\{\eta^{Low}, \eta^{High}\}$	$\{0.03,\ 0.05\}$

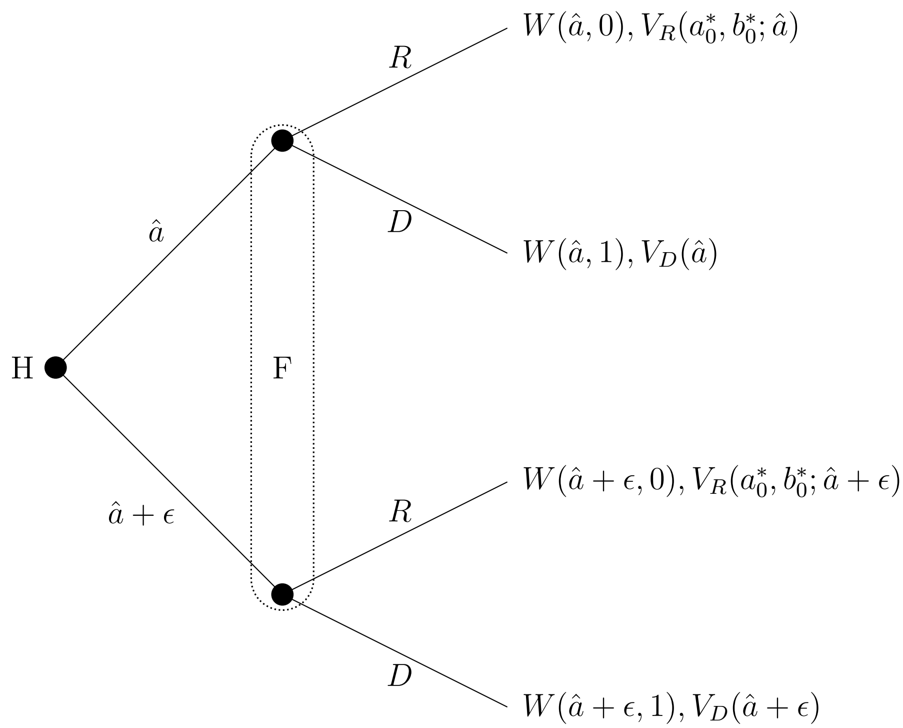

Figure A.1: Extensive-form representation of the game between the Home and Foreign countries.

A.2 Proof of Lemma 1

Proof. By Assumption 2, $V^R(a^*, b^*; a^*) < V^D(a^*)$ and $V^R(a^*, b^*; 0) > V^D(a^*)$. The result then follows by the fact that $V^R(a^*, b^*; \underline{a})$ is continuous and strictly decreasing in $\underline{a}$ while V^D is independent of $\underline{a}$. □

A.3 Proof of Proposition 1

We first prove the following lemma.

Lemma A.2. *Consider $\underline{a} = \hat{a}$ and let $c^*_{0,D}$ and $c^*_{0,R}(\underline{a})$ be the consumption policies under default and repayment in period 0 when the home policy is $\underline{a}$. Then, we have that consumption evaluated a the home policy $\hat{a}$ is higher under repayment: $c^*_{0,D} < c^*_{0,R}(\hat{a})$.*

Proof. By Lemma 1, we know that $V_R(a^*, b^*, \hat{a}) = V_D(a^*)$. It is straightforward that the continuation value is higher under default, which implies that $c^*_{0,D} < c^*_{0,R}(\hat{a})$. □

We now proceed to prove the proposition.

Proof. We first argue that $\hat{\eta} = \frac{u(y+(1-\beta)(\alpha b^*_0+k_0))-u(y+(1-\beta)k_0)}{(1-\beta)\left[u\left(y^*-g^*+a^*_0-\hat{a}-\kappa b^*_0\right)\right]-u(y^*-g^*-\phi^D)}$ is positive, a result that is immediate from Lemma A.2 and the budget constraints under repayment under default. Next, let us define Γ as the difference in value between setting $\hat{a}$ and setting a strictly higher restriction $\hat{a} + \epsilon$

$$\Gamma(\tilde{a}; \eta) = \hat{W}(\tilde{a}; \eta) - \hat{W}(\tilde{a} + \epsilon; \eta) \tag{13}$$

We have that $\frac{\partial \Gamma}{\partial \eta} = \eta[u(c^*_{0,D}) - u(c^*_{0,R}(\hat{a}))] < 0$, where the inequality follows from Lemma A.2. The result that $\underline{a} = \hat{a}$ if $\eta \leq \hat{\eta}$ and that $\underline{a} > \hat{a}$ if $\eta > \hat{\eta}$ follows then immediately. □

A.4 Extension with current and future geopolitical externality

In the baseline model, we assume that the geopolitical externality emerges only from the utility flow of the foreign country during the war period. Consider instead the following welfare for the home country:

$$W = \sum_{t=0}^{\infty} \beta^t \left[u(c_t) - \eta_t u(c^*_t)\right]$$

where $\eta_t > 0$ measures the intensity with which the home country "dislikes" the foreign country's consumption at time t. Let us assume that $\eta_0 \geq \bar{\eta} = \eta_t$ for all $t \geq 1$. The idea is capture that it is during the war period that the home country particularly dislikes consumption by the foreign country. The case analyzed in the baseline model is $\bar{\eta} = 0$.

With this assumption, we can then write the value for the home country of inducing default and repayment as

$$W(\underline{a}, 1) = \frac{1}{1-\beta} u(y^* + (1-\beta)k_0) - \eta_0 u\left(y^* - g^* - \phi^D\right) - \frac{\beta}{1-\beta}\bar{\eta} u(y^* + \frac{r}{1+r}a^*)$$

$$W(\underline{a}, 0) = \frac{1}{1-\beta} u(y + (1-\beta)(\alpha b_0^* + k_0)) - \eta_0 u\left(y^* - g^* + a_0^* - \underline{a} - \kappa b_0^*\right) - \frac{\beta}{1-\beta}\bar{\eta} u(y^* + (1-\beta)\left(\underline{a}(1+r) - (1-\delta)b_0^*\right) \quad (14)$$

Similar to (11), we can define $\tilde{W}(\underline{a}) = W\left(\underline{a}; d^*(a, b; \underline{a})\right)$ as

$$\tilde{W}(\underline{a}) = \begin{cases} \frac{1}{1-\beta} u(y+(1-\beta)(\alpha b_0^*+k_0)) - \eta_0 u\left(y^* - g^* + a_0^* - \underline{a} - \kappa b_0^*\right) - \frac{\beta}{1-\beta}\bar{\eta} u\left(y^* - g^* + a_0^* - \underline{a} - \kappa b_0^*\right) & \text{if } \underline{a} \leq \hat{a}. \\ \frac{1}{1-\beta} u(y+(1-\beta)k_0) - \eta_0 u\left(y^* - g^* - \phi^D\right) - \frac{\beta}{1-\beta}\bar{\eta} u(y^* + \frac{r}{1+r}a^*) & \text{if } \underline{a} > \hat{a}. \end{cases} \quad (15)$$

We can then establish the following proposition that underscores the key assumption that $\eta_0 > \eta_t$ for $t \geq 1$ for the prediction that the creditor country may find it optimal to induce a default by the debtor country.

Proposition A.2. *Assume that $\eta_t = \bar{\eta}$ for all t. Then, the optimal policy is $\underline{a} = \hat{a}$ and the foreign country always repays.*

Proof. Using that $\eta_0 = \bar{\eta}$, we have that

$$\begin{aligned} \tilde{W}(\hat{a}) &= \frac{1}{1-\beta} u(y+(1-\beta)(\alpha b_0^*+k_0)) - \bar{\eta}\left[u\left(y^* - g^* + a_0^* - \underline{a} - \kappa b_0^*\right) + \frac{\beta}{1-\beta} u\left(y^* - g^* + a_0^* - \underline{a} - \kappa b_0^*\right)\right] \\ &= \frac{1}{1-\beta} u(y+(1-\beta)(\alpha b_0^*+k_0)) - \bar{\eta} V(\underline{a}, 0) \end{aligned}$$

where the equality uses (7). Consider now $\tilde{W}(\hat{a})$ for $\underline{a} > \hat{a}$. We have that

$$\tilde{W}(\underline{a}) = \frac{1}{1-\beta} u(y+(1-\beta)k_0) - \bar{\eta} V(\underline{a}, 1)$$

where the equality uses (9). Using that $V(\underline{a}, 1) > V(\underline{a}, 0)$ for $\underline{a} > \hat{a}$ and that $u(y+(1-\beta)(\alpha b_0^*+k_0)) > u(y+(1-\beta)k_0)$ leads to the result. □

Finally, we have that if $\eta_0 > \bar{\eta}$, we recover the same result as in the baseline model with $\bar{\eta} = 0$

Proposition A.3. *There exists a threshold $\hat{\eta}_0$ such that if $\eta_0 \leq \hat{\eta}_0$, the home country chooses $\underline{a} = \hat{a}$ and the foreign country repays, and if $\eta_0 > \hat{\eta}_0$, the home country sets $\underline{a} > \hat{a}$ and the foreign country defaults.*

Proof. The proof follows the same steps as Proposition 1 with the difference that now the threshold is given by $\hat{\eta}_0 = \frac{u(y+(1-\beta)(\alpha b_0^*+k_0))-u(y+(1-\beta)k_0)}{(1-\beta)\left[u\left(y^*-g^*+a_0^*-\hat{a}-\kappa b_0^*\right)\right]-u(y^*-g^*-\phi^D)-\frac{\beta}{1-\beta}\bar{\eta}\left[u(y^*+(1-\beta)\left(\underline{a}(1+r)-(1-\delta)b_0^*\right)-u(y^*+\frac{r}{1+r}a^*)\right]}$

$\square$