Reserve Accumulation, Macroeconomic Stabilization, and Sovereign Risk *

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Abstract

In the past three decades, governments in emerging markets have accumulated large amounts of international reserves, especially those with fixed exchange rates. This paper proposes a theory of reserve accumulation that can account for these facts. Using a model of endogenous sovereign default with nominal rigidities, we argue that the interaction between sovereign risk and aggregate demand amplification generates a macroeconomic-stabilization hedging role for international reserves. We show that issuing debt to purchase reserves during good times allows the government to stabilize aggregate demand when sovereign spreads rise and rolling over the debt becomes more expensive. We provide empirical evidence consistent with the model's predictions.

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1 Introduction

The accumulation of international reserves—official public assets that are readily available for use—is one of the most salient features of the international monetary system over the past 30 years. While prevalent across emerging markets, the increase in reserves has been led by countries with fixed exchange rates, which, as we document, increased their reserves-to-GDP ratios from about 10% in the 90s, to 30% in recent years. What accounts for the striking levels of international reserves, and what is the relationship between exchange rate regimes and the accumulation of reserves?

In this paper, we argue that the interaction between sovereign risk and aggregate demand amplification generates a macroeconomic-stabilization hedging role for international reserves. Using a model of endogenous sovereign default with nominal rigidities, we show how accumulating reserves allows the government to reduce the severity of future recessions. In particular, it is optimal for the government to issue debt to accumulate reserves during good times and deploy them during recessions when rolling over the debt becomes more expensive. We establish that this macro-stabilization motive can quantitatively account for the high observed levels of international reserves, a feature of the data that has proven difficult to reconcile with existing models.

The theory provides a new perspective on the link between the exchange rate regime and reserve accumulation. In the celebrated balance of payment crises literature (see e.g., Krugman, 1979; Flood and Garber, 1984), when the government fixes the exchange rate, it loses control of the money supply and thus seigniorage. Therefore, in a situation where the government is pegging and lacks reserves to finance the primary deficits, it has no choice but to abandon the peg.[1] According to this view, reserves are a fiscal necessity to be able to implement a fixed exchange rate regime. Quantitatively, however, the levels of high-powered money seem too modest to account for the levels of reserves in fixed exchange rate economies.[2] In our model, instead, reserves are not necessary to implement a fixed exchange rate regime. Instead, we show they are desirable for macroeconomic stabilization in the presence of sovereign risk.

To understand our argument, consider a negative shock that worsens the borrowing terms faced by a government. Such a shock could come from a decline in income for the government or from foreign lenders' risk premia. The optimal response for the economy is, naturally, a reduction in borrowing and consumption. In the presence of a fixed exchange rate and downward nominal wage rigidity, the reduction in consumption leads to a recession, which further deepens the contraction in consumption. As highlighted by Friedman (1953) and Schmitt-Grohé and Uribe (2016), the lack of exchange rate flexibility prevents the government from using monetary policy

[1] The abandonment of the peg can also be triggered by self-fulfilling expectations (Flood and Garber, 1987; Obstfeld, 1986).

[2] See Obstfeld and Rogoff (1995) for a discussion on how the choice of an exchange rate regime does not seem to be guided in practice by issues of fiscal or technical feasiblity linked to the holdings of international reserves.

to avoid misalignments in real wages and stabilize macroeconomic fluctuations.

We then show that having reserves in these states allows the government to smooth the decline in consumption and mitigate the severity of the recession ex post. From an ex ante point of view, however, the government may also choose to reduce the sovereign debt rather than accumulate reserves. What generates an incentive to accumulate both reserves and debt as a macro-stabilization policy is the fact that in states in which debt becomes more costly to roll over, having reserves allows the government to reduce the slack in the labor market. In a nutshell, having reserves allows the government to avoid rolling over the fraction of debt maturing at high interest rates and frees up resources to stabilize macro fluctuations. We label this channel "macro-stabilization hedging." Moreover, by allowing the government to reduce the severity of recessions, we show that accumulating reserves can also have strong effects on debt sustainability.

In our quantitative analysis, the model features an equilibrium level of reserves of roughly 16% of GDP under a fixed exchange rate. Moreover, under the same calibration, the amount of reserves falls to 7% of GDP when there are no nominal rigidities or when the government follows a fully flexible exchange rate. Across various extensions of the baseline model, including intermediate degrees of exchange rate flexibility, commitment to reserve policies, different forms of nominal rigidities, and an explicit link between devaluations and defaults, we verify that macro-stabilization generates a strong motive for international reserve accumulation. In addition, we show that reserve accumulation has important welfare implications. We find that reserves significantly reduce the costs of maintaining a fixed exchange rate—thus, contributing to making a fixed exchange rate more sustainable. Intuitively, by cushioning the drop in aggregate demand during bad times, reserves make remaining in a fixed exchange rate relatively more attractive. Moreover, a relatively simple rule for reserve accumulation can achieve sizable welfare gains.

We provide empirical evidence that supports the key model predictions. Using a panel of emerging markets, we first show that countries with a lower degree of exchange rate flexibility hold on average more reserves. Moreover, we also establish that a common increase in risk premia can account for the steeper upward trend in reserves for fixed exchange rate economies. We then test the model's implications regarding the link between depreciations and the use of reserves during crises. One prediction of the model is that countries with a lower degree of exchange rate flexibility find it optimal to deploy a larger portion of their reserves in response to adverse shocks. Another prediction is that countries that face a crisis with a higher amount of reserves experience a lower depreciation of their exchange rates. Using the 2008 financial crisis as a case of study, we find that these predictions are strongly supported by the data.

Related literature. The paper contributes to several strands of the literature. In the first place, it relates to a vast literature on international reserves, in particular the one emphasizing the precautionary role of reserves in the context of real models.[3] The decision problem faced by the government in our model is similar to the one in Bianchi, Hatchondo and Martinez (2018), which in turn builds on the canonical sovereign default model in the tradition of Eaton and Gersovitz (1981), Aguiar and Gopinath (2006), and Arellano (2008). We depart from the existing work by incorporating nominal rigidities and show how doing so gives rise to a macroeconomic stabilization hedging role for reserves. A contribution of our paper is to develop a theory that is quantitatively consistent with the observed levels of reserves in emerging markets and to link, both theoretically and empirically, the accumulation of international reserves to the exchange rate regime.

Our paper also belongs to a nascent literature analyzing fiscal and monetary policy in the context of sovereign default models with nominal rigidities. Na, Schmitt-Grohé, Uribe and Yue (2018) first introduced nominal rigidities and showed that the optimal time-consistent exchange rate policy delivers full employment. Moreover, they show that the model can deliver the so-called "twin Ds", that is, episodes in which large devaluations coincide with a sovereign default. Bianchi, Ottonello and Presno (2019) consider a fixed exchange rate regime and analyze the dilemma of choosing between austerity and stimulus, finding that the optimal fiscal policy is consistent with the observed procyclicality. Arellano, Bai and Mihalache (2020) study the interaction between monetary policy conducted through interest rules and sovereign risk. Bianchi and Mondragon (2022) show that an economy under a fixed exchange rate regime is more vulnerable to rollover crises. De Ferra and Romei (2021) develop a model of a monetary union with heterogenous countries and investigate the interactions between incentives to default and the union-wide monetary policy. In contrast to these studies, we allow for the accumulation of international reserves and study how the optimal holdings of reserves differ depending on the monetary policy regime.

Our paper is also related to the literature on aggregate demand externalities under nominal rigidities and constraints on monetary policy (e.g., Schmitt-Grohé and Uribe, 2016; Farhi and Werning 2016, 2017). Our focus on portfolio management shares elements with Farhi and Werning (2016), who show that the government can generically improve welfare by controlling households' portfolios of Arrow-Debreu securities and redirecting demand across states.[4] However, they abstract from the risk of default, which is what gives rise to the macro-stabilizing

[3]See, for example, Aizenman and Lee (2007); Alfaro and Kanczuk (2009); Caballero and Panageas (2008); Jeanne and Ranciere (2011); Hur and Kondo (2016); Arce, Bengui and Bianchi (2019). A different strand of the literature analyzes the role of reserves in implementing exchange rate policies in the context of limits to international arbitrage (e.g., Amador et al., 2020, 2018; Fanelli and Straub, 2021). Another related line studies the role of reserves for lender-of-last-resort support (e.g., Bocola and Lorenzoni, 2018; Céspedes and Chang, 2019) and to avoiding self-fulfilling speculative attacks (e.g., Obstfeld, 1986).

[4]On the other hand, Fanelli (2017) provides an environment in which the government may find it optimal to distort savings but not the composition of the risky foreign asset portfolio.

hedging benefit in our model.[5] Overall, a key difference from the studies in this literature is that we analyze the specific role of international reserves for macro-stabilization and provide a distinct mechanism linked to endogenous default risk.

Our paper is also related to a closed economy literature that studies portfolio management in which the government faces distortionary taxation and is endowed with commitment. As shown by Angeletos (2002) and Buera and Nicolini (2004), trading at different maturities allows the government to alter households' marginal utility and bond prices and, through this channel, improve spanning and complete markets. Our model differs in that the government cannot commit to repay, and fluctuations in bond prices arise because of changes in default probabilities. To the best of our knowledge, our paper is the first to analyze how the presence of nominal rigidities shapes the optimal government portfolio and to uncover a macro-stabilization benefit from carrying larger gross positions.[6]

Finally, our analysis of reserve accumulation rules is related to the literature that studies the design of fiscal and monetary rules when there are tradeoffs between flexibility and commitment.[7] Our paper complements this literature by studying the commitment role of reserves. We show, in particular, that commitment to keeping higher reserve holdings in good states and lower reserve holdings in bad states is beneficial as this increases incentives to repay in the future and improves current bond prices.

Layout. The rest of the article proceeds as follows. Section 2 presents the model. Sections 3 and 4 present the quantitative analysis. Section 5 presents the empirical analysis. Section 6 presents a welfare analysis and Section 7 discusses extensions and sensitivity. Section 8 concludes.

2 Model

We consider a two-sector small open economy model in which the government issues long-term defaultable bonds and invests in risk-free assets. We assume there is a stochastic endowment for tradable goods, while non-tradable goods are produced using labor. There are two currencies that serve as units of account: the domestic currency and the foreign currency. Wages are downwardly rigid in domestic currency, as in Schmitt-Grohé and Uribe (2016), creating the

[5] Auclert and Mitman (2019) consider a model of household default and explore the macroprudential role of bankruptcy legislation.

[6] Other papers in different strands of this literature include Lustig, Sleet and Yeltekin (2008); Arellano and Ramanarayanan (2012); Faraglia, Marcet and Scott (2010); Ottonello and Perez (2019); Alfaro and Kanczuk (2019); Debortoli, Nunes and Yared (2017); and Bocola and Dovis (2019). Particularly relevant is Lustig et al., who consider a fiscal hedging benefit of long-term nominal debt in an environment with nominal rigidities. In their model, however, the government cannot accumulate assets, and there are no monetary policy constraints.

[7] Examples include Athey, Atkeson and Kehoe (2005), Halac and Yared (2014), and Hatchondo, Martinez and Roch (2022).

possibility of involuntary unemployment. We consider several monetary policy regimes that vary in the degree of exchange rate flexibility.

2.1 Households

The small open economy is populated by a measure one of households. Households' preferences are given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(c_t), \tag{1}$$

where c denotes a consumption bundle that we describe below, $\beta \in (0,1)$ is the discount factor, and $\mathbb{E}$ is the expectation operator. We assume that the utility function is given by $U(c) = \frac{c^{1-\gamma}}{1-\gamma}$, where γ is the coefficient of relative risk aversion.

The consumption good c is a composite of tradable (c^T) and non-tradable goods (c^N), with a constant elasticity of substitution (CES):

$$c = [\omega(c^T)^{-\mu} + (1-\omega)(c^N)^{-\mu}]^{-1/\mu}, \tag{2}$$

where $\omega \in (0,1)$ and $\mu > -1$.

In each period, households receive an endowment of tradable goods, y_t^T, which is assumed to follow a stationary first-order Markov process given by

$$\log(y_t^T) = (1-\rho)\mu_y + \rho \log(y_{t-1}^T) + \varepsilon_t,$$

with $|\rho| < 1$ and $\varepsilon \overset{iid}{\sim} N(0, \sigma_\varepsilon^2)$.

Households inelastically supply $\bar{h}$ hours in the labor market. When the downward wage rigidity constraint (to be discussed below) binds, households will be able to work only the number of hours demanded by firms, and we will have $h_t < \bar{h}$. Households also receive lump-sum transfers from the government, T_t (expressed in units of tradables) and profits from the ownership of firms producing non-tradable goods, ϕ_t^N (expressed in domestic currency). As is common in the sovereign debt literature, we assume that households do not have access to external capital markets.

The households' budget constraint expressed in domestic currency is given by

$$P_t^T c_t^T + P_t^N c_t^N = P_t^T y_t^T + \phi_t^N + W_t h_t + P_t^T T_t, \tag{3}$$

where P_t^N and P_t^T denote respectively the price of non-tradables and tradables, and W_t denotes the wage, all expressed in domestic currency. We define the nominal exchange rate e_t as the price of foreign currency in terms of domestic currency (an increase in e thus reflects a depreciation of the domestic currency). Assuming that the law of one price holds for tradables and that the price of tradable goods in foreign currency is constant and normalized to one, we obtain $P_t^T = e_t$.

The households' problem consists of choosing sequences of consumption $\{c_t^N, c_t^T\}_{t=0}^{\infty}$ to maximize (1) given prices $\{P_t^N, P_t^T\}_{t=0}^{\infty}$, labor income $\{W_t h_t\}_{t=0}^{\infty}$, profits $\{\phi_t^N\}_{t=0}^{\infty}$, and government transfers $\{T_t\}_{t=0}^{\infty}$. The optimality condition of this problem yields the standard condition equating the marginal rate of substitution between the two goods to the relative price:

$$\frac{P_t^N}{P_t^T} = \frac{1-\omega}{\omega} \left(\frac{c_t^T}{c_t^N} \right)^{\mu+1}. \tag{4}$$

2.2 Firms

There is a continuum of firms of measure one that produce non-tradable goods using labor. Firms' production function is such that $y^N = F(h)$, where $F(h) = h^\alpha$ and $\alpha \in (0, 1]$. Their profits, expressed in domestic currency, are given by

$$\phi_t^N = \max_h P_t^N F(h) - W_t h_t.$$

The optimal choice of labor, h_t, equates the value of the marginal product of labor to the wage:

$$P_t^N F'(h_t) = W_t. \tag{5}$$

2.3 Government

The government sets monetary policy, chooses issuances of long-term bonds and holdings of risk-free assets (i.e., reserves), and provides lump-sum transfers to households. The government has no commitment and can default on its debt.

Long-term bonds are introduced following Hatchondo and Martinez (2009), Chatterjee and Eyigungor (2012), and Arellano and Ramanarayanan (2012). Specifically, a bond in period t promises to pay $\delta(1-\delta)^{j-1}$ units of the tradable good in period $t + j$, for all $j \geq 1$.[8] Denoting by b_t the initial face value of the debt and i_t the amount of new issuances, we have that

$$b_{t+1} = (1-\delta) b_t + i_t. \tag{6}$$

International reserves are risk-free assets that pay one unit of tradable consumption goods. We let $a_t \geq 0$ denote the government's reserve holdings at the beginning of period t.

[8]Because the price of tradable goods is constant in foreign currency, it is equivalent to assume that foreign debt is denominated in foreign currency ("dollars"). Under a fixed exchange rate regime, it is of course equivalent to have the debt denominated in domestic currency. However, under a flexible exchange rate, assuming that debt is denominated in domestic currency introduces additional portfolio considerations, as this allows the government to inflate away the debt (see Alfaro and Kanczuk, 2019 and Ottonello and Perez, 2019). In Section 7, we consider an extension of our model where a share of total debt is denominated in domestic currency.

We use q and q_a to denote the price of government bonds and reserves in units of tradable goods. As we will see, in equilibrium, the price of government bonds will depend on the government's portfolio decisions and exogenous shocks. Meanwhile, the price of reserves will be constant and determined by the exogenous risk-free rate.

If the government chooses to default, it retains control of its reserves and access to savings but cannot borrow in the default period. A default entails a utility loss $\psi_d(y^T)$, which depends on the realization of the tradable endowment. We think of this utility loss as capturing various default costs related to reputation, sanctions, or misallocation of resources; we do not model these explicitly.[9] We abstract from financial exclusion as an additional source of default penalty, that is, the government can once again borrow from international markets in the period following a default.

Letting $d_t = 1(0)$ if the government defaults (repays), we can write the budget constraint of the government as

$$T_t = \begin{cases} a_t + q_t i_t - q_a a_{t+1} - \delta b_t & \text{if} \quad d_t = 0 \\ a_t - q_a a_{t+1} & \text{if} \quad d_t = 1 \,. \end{cases} \tag{7}$$

2.4 Foreign Lenders

Bonds are priced in a competitive market inhabited by a large number of identical lenders. To capture global factors that are exogenous to domestic fundamentals, we introduce risk premium shocks. These shocks are not critical for the mechanism but enrich the analysis and are in line with a large empirical literature on the role of global shocks driving spreads and credit flows.[10]

Foreign lenders price the payoffs of bonds using the following stochastic discount factor, following Vasicek (1977):

$$m_{t,t+1} = e^{-r-\kappa_t\left(\varepsilon_{t+1}+0.5\kappa_t\sigma_\varepsilon^2\right)} \tag{8}$$

where r represents the international risk-free rate and $\kappa_t \geq 0$ is a stochastic parameter governing the risk premium shock. Notice that (8) implies that a higher value of κ makes bond payoffs more valuable for investors when the small open economy faces a negative shock to its tradable endowment, ε, capturing the positive degree of correlation between the small open economy and the lenders' income process and their limited diversification. To the extent that the government is more likely to default when it faces adverse shocks, this implies that lenders demand a positive premium to be willing to invest in government bonds.

[9] An alternative assumption in the literature specifies an exogenous cost of default in terms of output. Assuming logarithmic utility over the composite consumption and that output losses from default are proportional to the composite consumption in default, the losses from default are identical for the output and utility cost specifications.

[10] See for example Longstaff et al. (2011); Forbes and Warnock (2012); Uribe and Yue (2006), Rey (2015); Johri, Khan and Sosa-Padilla (Forthcoming).

The risk premium shock κ follows a two-state Markov switching regime with values $\kappa_L = 0$ and $\kappa_H > 0$ and transition probabilities π_{LH}, π_{HL}. We assume that $\kappa_L = 0$ so that the stochastic discount factor reduces to $m_{t,t+1} = e^{-r}$, thus eliminating risk premia in that state. The value of κ_H will be calibrated to match increases in spreads during spikes of global risk premia.

The standard asset pricing condition for government bonds is therefore

$$q_t = \mathbb{E}_t \left\{ m_{t,t+1}(1 - \hat{d}_{t+1}) \left[\delta + (1-\delta) q_{t+1}\right] \right\}, \tag{9}$$

where $\hat{d}_{t+1}$ is the equilibrium government default decision in $t+1$.

We can also derive a condition analogous to (9) for the pricing of risk-free assets. In that case, we obtain that the price of reserves must be such that $q_a = e^{-r}$, a result that follows from the log-normal structure of the lenders' discount factor. It thus follows using (9) that if $\kappa = 0$, the expected return on bonds equals the risk-free return on reserves.

2.5 Downward Nominal Wage Rigidity

Following Schmitt-Grohé and Uribe (2016), we assume that nominal wages in domestic currency are downwardly rigid. In particular, we assume that wages cannot fall below $\overline{W}$. That is, market wages must satisfy $W_t \geq \overline{W}$ for all t.[11]

When the nominal wage that clears the labor market is above $\overline{W}$, we have a full employment equilibrium. On the other hand, if the nominal wage that clears the labor market is below $\overline{W}$, employment is determined by firms' labor demand, and we have $h_t < \bar{h}$. These rationing conditions can be summarized by

$$\left(W_t - \overline{W}\right)\left(h_t - \bar{h}\right) = 0. \tag{10}$$

2.6 Competitive Equilibrium

In equilibrium, market clearing for non-tradable goods requires that output is consumed domestically:

$$c_t^N = F(h_t). \tag{11}$$

We can now define a competitive equilibrium for given government policies.

Definition 1 (Equilibrium given policies)**.** Given initial values $\{a_0, b_0\}$, exogenous processes $\{y_t^T, \kappa_t\}_{t=0}^{\infty}$, government policies for exchange rates, transfers, debt and reserves $\{e_t, T_t, b_{t+1}, a_{t+1}\}_{t=0}^{\infty}$, and a default decision $\{d_t\}_{t=0}^{\infty}$, a competitive equilibrium is a sequence of allocations $\{c_t^T, c_t^N, h_t\}_{t=0}^{\infty}$ and prices $\{P_t^N, P_t^T, W_t, q_t\}_{t=0}^{\infty}$ such that

[11] In Schmitt-Grohé and Uribe (2016), the wage floor depends on the previous period's wage. For numerical tractability, we follow Bianchi et al. (2019) and set the wage floor as an exogenous constant value.

(i) Allocations solve households' and firms' problems at given prices;

(ii) Government policies satisfy the government budget constraint (7);

(iii) The bond pricing equation, (9) holds and $P_t^T = e_t$;

(iv) The market for non-tradable goods clears (equation 11); and

(v) The labor market satisfies conditions $W_t \geq \overline{W}$, $h_t \leq \bar{h}$ and (10).

To characterize the optimal government problem, notice that we can combine (4), (5), and (11) to obtain an expression for equilibrium employment as a function of tradable consumption and the exchange rate, which will serve as a key implementability constraint in the government's problem:

$$\mathcal{H}\left(c_t^T, e_t\right) = \min\left\{\left[\frac{1-\omega}{\omega}\left(\frac{\alpha e_t}{\overline{W}}\right)\right]^{\frac{1}{1+\alpha\mu}}\left(c_t^T\right)^{\frac{1+\mu}{1+\alpha\mu}}, \bar{h}\right\}, \tag{12}$$

where c_t^T follows from the tradable resource constraint and is given by [12]

$$c_t^T = y_t^T + a_t - q_a a_{t+1} + (1-d_t)[q_t i_t - \delta b_t].$$

Equation (12) establishes that equilibrium employment is an increasing function of c^T as long as the economy is not at full employment. The underlying mechanism is due to the the aggregate demand effects emerging from the nominal rigidities. A higher level of tradable consumption reflects higher resources in the economy. Higher resources in turn raise domestic aggregate demand. Given a rigid wage, this leads to a higher demand for labor and more employment in equilibrium.

Equation (12) also shows that the government can reduce unemployment by depreciating the exchange rate, in line with the traditional argument by Friedman (1953).[13] The increase in the exchange rate leads to an expenditure switching effect that raises the demand for non-tradable goods. In turn, this increase in the demand for non-tradables leads to an increase in firms' labor demand for given wages. The outcome is that as long as the economy is not at full employment, a depreciation boosts employment.[14]

2.7 Optimal Government Problem

[12]This follows using the households' budget constraint (3), the definition of the firms' profits, the market clearing condition (11), and the government budget constraint (7).

[13]See Schmitt-Grohé and Uribe (2011) for a modern treatment and a quantification.

[14]Another way to see this is to note that $P_t^N = (1-\omega)/\omega\left(c_t^T/c_t^N\right)^{\mu+1} e_t$ and $W_t = P_t^N F'(h_t)$. For a given level of consumption, the government can push up the nominal price of non-tradables and, in turn, raise the market nominal wages above $\overline{W}$.

The government is benevolent and unable to commit to repayment or any other future policies. We consider a Markov equilibrium in which all policies depend on the payoff-relevant states (a, b, s) where $s \equiv \{y^T, \kappa\}$. Every period, the government optimizes, taking as given its future policies. Specifically, the government directly chooses the repayment/default decision, bond issuances, reserves, and transfers, as well as employment, consumption and all prices subject to the competitive equilibrium conditions.

Per the discussion above, a government that has a completely flexible exchange rate regime is able to overcome involuntary unemployment by using the exchange rate as a shock absorber. To motivate a macroeconomic tradeoff, we introduce a cost from exchange rate fluctuations. Specifically, we assume that the utility flow of the government is given by

$$u(c_t^T, c_t^N) - \Phi(e_t) - d_t \psi_d(y_t^T),$$

where $\Phi(\cdot)$ denotes a convex devaluation cost that has a minimum at a target value $\bar{e}$. The setup nests a flexible exchange rate by setting $\Phi(e_t) = 0$ for all e_t and a fixed exchange rate by taking $\Phi(e_t) = \infty$ for all $e_t \neq \bar{e}$. We think about these costs as reflecting reputation considerations, redistribution effects, or currency mismatches, but we abstract from modelling them explicitly.[15] Besides being parsimonious, one advantage of this formulation is that we will be able to empirically exploit the link between the exchange rate flexibility, as characterized by Φ, and the reserve accumulation decisions by the government. We return to this in Section 5.

We now characterize the optimal government problem. At the beginning of each period, the government solves

$$V(a, b, s) = \max_{d \in \{0,1\}} \left\{ (1-d)\, V^R(a, b, s) + d\, V^D(a, s) \right\} \tag{13}$$

where $V^R(a, b, s)$ and $V^D(a, s)$ represent the value functions from repaying and defaulting. Using the implementability constraint (12) as well as the tradable resource constraint, we can express the value of repayment as

$$V^R(a, b, s) = \max_{a', b', c^T, h, e} \left\{ u(c_t^T, F(h)) - \Phi(e) + \beta\, \mathbb{E}_{s'|s} \left[V(a', b', s') \right] \right\} \tag{14}$$

subject to

$$\begin{aligned} c^T + q_a a' + \delta b =& a + y^T + q\left(a', b',\ y^T\right)[b' - (1-\delta) b], \\ h \leq& \mathcal{H}(c^T, e). \end{aligned}$$

[15] Adverse effects from depreciations can arise, for example, from rises in corporate spreads (Céspedes et al., 2004), households' borrowing frictions (Ottonello, 2022, Bianchi and Coulibaly, 2023), or from redistribution between households (De Ferra, Mitman and Romei, 2020). Bianchi and Lorenzoni (2022) explore the implications of this exchange rate friction for capital controls.

Notice that as long as $\delta < 1$, the value function upon repayment depends on the composition of the portfolio (a, b), not just the net position. On the other hand, with one-period debt, $\delta = 1$, the state variable under repayment can be summarized entirely by the net foreign asset position, $a - b$.

The value of default is given by

$$V^D(a, s) = \max_{a', c^T, h, e} \left\{ u(c^T, F(h)) - \Phi(e) - \psi_d\left(y^T\right) + \beta\, \mathbb{E}_{s'|s}\left[V\left(0, a', s'\right)\right] \right\} \tag{15}$$

$$\text{subject to}$$

$$\begin{aligned} c^T + q_a a' &= y^T + a, \\ h &\leq \mathcal{H}(c^T, e). \end{aligned}$$

It is important to notice that in the case in which there are no costs from depreciation, problems (14) and (15) imply that the government would find it optimal to choose a depreciation large enough to ensure full employment.[16]

A Markov perfect equilibrium is then defined as follows.

Definition 2 (Markov perfect equilibrium)**.** A Markov perfect equilibrium is defined by value functions $\{V(a,b,s), V^R(a,b,s), V^D(a,s)\}$, policy functions $\left\{\hat{d}(a,b,s), \hat{a}(a,b,s), \hat{b}(a,b,s),\ \hat{c}^T(a,b,s), \hat{h}(a,b,s), \hat{e}(a,b,s)\right\}$, and a bond price schedule $q(a', b', s)$ such that

(i) Given the bond price schedule, the policy functions and value functions solve problems (13), (14), and (15);

(ii) The bond price schedule satisfies the bond pricing equation

$$q\left(a', b', s\right) = \mathbb{E}_{s'|s}\left\{ m\left(s', s\right)\left[1 - \hat{d}(a', b', s')\right]\left[\delta + (1-\delta) q\left(a'', b'', s'\right)\right] \right\}, \tag{16}$$

where

$$b'' = \hat{b}\left(a', b', s'\right) \text{ and } a'' = \hat{a}\left(a', b', s'\right).$$

2.8 The Macro-Stabilization Role of Reserves

This section presents our key results on how aggregate demand and sovereign risk considerations shape the optimal government portfolio.

Euler equations. We start by presenting the optimality conditions of the government problem (13)-(15). To simplify the expressions, we assume in the rest of this section that there is a linear

[16] This can be seen by noting that if $\Phi = 0$, the exchange rate only shows up in the constraint $h \leq \mathcal{H}(c^T, e)$. See also Na et al. (2018).

production function ($\alpha = 1$), which, per equation (12), gives that employment is a linear function of tradable consumption.

Using the first-order conditions and the envelope conditions, we can obtain the following necessary conditions for optimality:[17]

$$\left(u_T + \mathbb{I}_{h<\bar{h}}\, u_N \frac{c_t^N}{c_t^T}\right)\left[q + \frac{\partial q(a',b',s)}{\partial b'} i\right] = \beta\, \mathbb{E}_{s'|s}\left[\left(u'_T + \mathbb{I}_{h'<\bar{h}}\, u'_N \frac{c_{t+1}^N}{c_{t+1}^T}\right)[\delta + (1-\delta)q'](1-d')\right], \tag{17}$$

$$\left(u_T + \mathbb{I}_{h<\bar{h}}\, u_N \frac{c_t^N}{c_t^T}\right)\left[q_a - \frac{\partial q(a',b',s)}{\partial a'} i\right] \geq \beta\, \mathbb{E}_{s'|s}\left(u'_T + \mathbb{I}_{h'<\bar{h}}\, u'_N \frac{c_{t+1}^N}{c_{t+1}^T}\right) \tag{18}$$

where (18) holds with equality if $a' > 0$. Here we used u_T and u_N to denote the current marginal utility from tradable and non-tradable consumption (and *primed variables* to denote next-period values). In addition, $\mathbb{I}_{h<\bar{h}}$ is an indicator function for unemployment.

Equation (17) is the debt Euler equation for the government. The left-hand side represents the marginal benefits of issuing an additional unit of debt in the current period. When the government borrows one more unit, it raises q units of consumption, but it also lowers the revenue from the inframarginal issuances by $\frac{\partial q(b',a',s)}{\partial b'} i$. Each unit of consumption has a direct marginal utility benefit, u_T, and an indirect marginal utility benefit, $u_N(c_t^T, c_t^N)c_t^N/c_t^T$, operative only when there is unemployment.[18] The latter is a manifestation of an aggregate demand amplification at work. When there is unemployment, the government internalizes that additional borrowing raises aggregate demand, leading to higher employment and non-tradable consumption. The intuition for the expression for the indirect marginal utility benefit is as follows. Under linear production, the price of non-tradables is fixed at $\overline{W}$ in a state with unemployment. Moreover, homothetic preferences imply that households split the extra units of resources between tradables and non-tradables at the same initial proportion. Thus, the extra unit of borrowing by the government raises demand for non-tradables by c_t^N/c_t^T, which has a marginal value of u_N.

Consider now the right-hand side of (17), which represents the marginal utility cost of carrying one more unit of debt. When the government borrows one more unit and repays in the next period, the cost is given by the coupon payments δ plus the cost of carrying $(1-\delta)$ more units of debt at the market price q'. Importantly, the marginal utility costs of repaying those resources have analogous direct and indirect effects. Namely, if the economy is in a situation with unemployment tomorrow, repaying those resources has an additional cost because it raises

[17]For illustration purposes, we assume that the bond price and the value function are differentiable. Clausen and Strub (2017) show that indeed in the canonical default model, the objective function is continuously differentiable at the optimal choices, and a version of the envelope theorem applies. Aguiar et al. (2019) show similar results with shocks to outside options to default. Our computational method does does not rely on differentiability.

[18]Technically, an indirect marginal benefit would also be present in states where the wage rigidity is strictly binding while still having $h = \bar{h}$. Our numerical solution allows for this possibility.

even more unemployment.

Notice that the possibility of default generates an extra cost from borrowing to finance consumption that goes beyond the expected repayment to the creditors. When the government borrows more, it raises the probability of default, and hence the costs associated with it. The default costs do not explicitly appear in (17), because in effect, they cancel out with the resources that the government "saves" by defaulting.[19] However, because investors price the default risk, this implies that the government obtains less revenue from bond issuances, as reflected in the term $\frac{\partial q(a',b',s)}{\partial b'}i$, discussed above.

Equation (18) represents the reserves Euler equation. The left-hand side represents the marginal costs of purchasing one unit of reserves in the current period. Purchasing one unit of reserves requires q_a units of consumption, but because reserves affect the price at which the government is able to issue debt, the overall cost must be netted from the effect that the increase in reserves has on the revenue from the debt issuances, $\frac{\partial q(a',b',s)}{\partial a'}i$. The right-hand side shows the marginal utility benefit of holding one more unit of reserves in all states of nature tomorrow. As discussed above, the marginal value of those resources today and tomorrow depends on the slack in the labor market.

Taking stock, it is worth highlighting three key elements behind the Euler equations. First, the presence of default risk alters the marginal costs of borrowing and acquiring reserves, as both affect future incentives to default and current spreads. Second, the presence of long-term bonds implies that repaying debt is less costly when future bond prices become lower. Third, the value associated with resources in each state depends on the slack in the labor market. This last effect arises because the government internalizes the aggregate demand amplification at work. Crucially, this effect would not be present in the portfolio problem of an individual household.

These three key elements are at the heart of the motive to accumulate reserves, as we will see below. Notice that in a model without default, there would not be scope for reserve accumulation. In that case, only the net foreign asset position matters, and the government cannot improve hedging by issuing debt and accumulating reserves: both debt and reserves have the same return in each state of nature.[20]

[19] Notice that when taking the first-order condition, the changes in the default threshold due to changes in b' cancel out. This is because V^R equals V^D at the the default threshold.

[20] This can be seen more clearly by noting that in the absence of default (which implies a constant bond price) and assuming a strictly interior solution and a non-binding wage rigidity, the Euler equations for reserves and debt become

$$
\begin{aligned}
q u_T(t) &= \beta(1-\delta+q), \mathbb{E}_{s'|s} u_T(t+1) \\
q_a u_T(t) &\leq \beta\, \mathbb{E}_{s'|s} u_T(t+1)
\end{aligned}
$$

Notice that if the return on bonds were higher than the return on reserves, the government would be at a corner solution with zero reserves.

Debt-financed reserves. To further inspect the trade-offs behind the optimal portfolio, let us analyze a financial operation by which the government purchases an additional unit of reserves and finances it by issuing debt. Specifically, starting from a set of initial states (a, b, s), assume that the government sets a certain level of transfers to households, $\overline{T}$ (which in turn determines real allocations for c^T, c^N, and h). We denote by "candidate portfolios" the pairs of (a', b') that are consistent with those transfers:

$$\overline{T} = a - q_a a' + q\left(a', b',\ y^T\right)[b' - (1-\delta)b] - \delta b\,. \tag{19}$$

Assuming that the government follows the optimal policies from tomorrow onward, the expected continuation value is then given by $\mathbb{E}_{s'|s} V(a', b', s')$. Notice that because all the candidate portfolios deliver the same current consumption, given an exchange rate, the difference in utility for the government is entirely due to differences in continuation values. At the optimum candidate portfolio, we must have that the government equates the net marginal benefits of debt-financed reserves to zero. If we totally differentiate $\mathbb{E}_{s'|s} V(a', b', s')$, use the associated envelope conditions from (14)-(15), and totally differentiate (19), we obtain

$$\overbrace{\underbrace{\left(\frac{q + \frac{\partial q(a',b',s)}{\partial b'} i}{q_a - \frac{\partial q(b',a',s)}{\partial a'} i}\right)}_{\text{Reserves bought}} \mathbb{E}_{s'|s}\left[u'_T + \mathbb{I}_{h'<\bar{h}}\, u'_N \frac{c^N_{t+1}}{c^T_{t+1}}\right]}^{\text{Mg utility benefit of issuing debt to buy reserves}} =$$

$$\mathbb{E}_{s'|s}[1-d']\Bigg\{\mathbb{E}_{s'|s,d'=0}\left[\delta + (1-\delta)q'\right]\mathbb{E}_{s'|s,d'=0}\left[u'_T + \mathbb{I}_{h'<\bar{h}}\, u'_N \frac{c^N_{t+1}}{c^T_{t+1}}\right] +$$

$$\underbrace{\mathbb{COV}_{s'|s,d'=0}\left(\delta + (1-\delta)q',\, u'_T + \mathbb{I}_{h'<\bar{h}} \frac{c^N_{t+1}}{c^T_{t+1}}\right)}_{\text{Macro-stabilization hedging}}\Bigg\}. \tag{20}$$

The left-hand side in (20) represents the marginal utility benefits of debt-financed reserves. The marginal benefits are given by the product of the reserves that can be bought by issuing one more unit of debt times the marginal utility benefits of having the reserves available in the next period. The amount of reserves that can be bought is essentially given by the relative prices of bonds and reserves, both of which are adjusted by how the change in positions affects the inframarginal units of debt issued.

The right-hand side in (20) represents the marginal utility costs of debt-financed reserves, which can in turn be decomposed into two terms. The first term is the expected marginal utility costs of repaying the debt in the next period. Notice that if $\kappa = 0$, using (9), this term becomes the product of the gross risk-free rate e^r times the expected marginal utility costs from repayment. When $\kappa > 0$, this term becomes larger, as investors need to be compensated for the credit risk.

The second term in the right-hand side of (20) constitutes a risk adjustment that emerges from the countercyclicality of default risk and depends crucially on the aggregate demand amplification effects. In particular, states in which there is slack in the labor market tend to coincide with states with low q', thus making it less costly for the government to repay the debt precisely when more spending in domestic goods would lower unemployment (that is, the covariance in this term will be negative). In effect, by borrowing and accumulating reserves, the government is transferring resources to states of nature in which resources are valued especially for aggregate demand management. Importantly, the "macro-stabilization hedging" benefits arise only if bonds have long maturity as in Bianchi et al. (2018), but in contrast with the mechanism they highlight, the hedging benefits would arise even if the government were risk neutral.[21]

Illustration. Equation (20) holds at an optimum with strictly positive reserves. While it is not possible to derive explicit analytical solutions for the optimum portfolio, we can numerically illustrate how the key terms in that condition vary for all possible candidate portfolios. In Figure 1, we plot three terms as a function of the amount of reserves purchased for given initial conditions and the target transfer presented in the note of the figure.[22] The first term, depicted with a solid blue line, is "reserves bought," which represents how many reserves the government can purchase by issuing one additional unit of debt while keeping transfers at a constant target. This term starts below one, reflecting a lower price of bonds relative to that of reserves, and it is downward sloping. That is, as the government increases its gross positions, spreads increase, and the government is able to obtain fewer reserves for every additional unit of debt issuances.

The second term, depicted with a dashed red line, represents the normalized marginal costs, which we construct as the right-hand side of equation (20) divided by the expected marginal utility benefit of an additional unit of reserve. The crossing of this line with the blue solid line described above denotes the optimal portfolio, given the initial states. A key component of the marginal costs is given by the macro-stabilization hedging term, which is represented with a dotted black line. This term is negative, indicating that the macro-stabilization hedging effect makes debt-financed reserves less costly. Moreover, it is also upward sloping, indicating that as the government increases the stock of reserves, the macro-stabilization hedging benefit becomes smaller at the margin.

[21] This can be seen by noting that if marginal utility were constant, the covariance would not vanish as long as $\delta < 1$.

[22] The associated level of debt emerges from solving (19). Notice that the figure is constructed for the calibrated version of the model, which we discuss in Section 3, and throughout the paper, debt and reserves are expressed in terms of GDP.

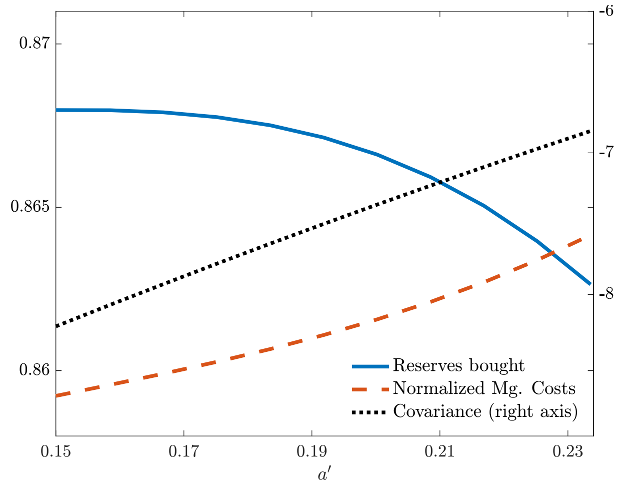

Figure 1: Macro-stabilization benefits of reserves

Note: The figure is computed ensuring that candidate portfolios keep current tradable consumption fixed at 80% of mean tradable output. The figure also assumes initial reserves equal to 17% of GDP, initial debt of 41% of GDP, tradable income is one standard deviation below mean, and no risk premium shock ($\kappa = 0$).

2.9 Discussion

In sum, the key implication of the theory is that a government that faces limited exchange rate flexibility has incentives to accumulate reserves over and beyond those of a government that has a flexible exchange rate. Namely, there is a macro-stabilization hedging benefit from holding reserves when a government is unable to stabilize the economy by using monetary policy.

It is important to highlight that although we have made a particular set of assumptions regarding the production structure, the form of the nominal rigidities, and the monetary policy regime, our results do not hinge on these specific assumptions (as we will show in Section 7). What is key for our mechanism is that output is determined partly by domestic aggregate demand and that drawing on the accumulated reserves can help stabilize aggregate demand.[23]

In the next section, we provide a quantitative analysis of the mechanism and then look at the predictions of the model vis à vis the data.

[23] For example, it should be clear that the fact that under $\Phi = 0$, the government finds it optimal to implement a zero labor wedge allocation is not crucial to our result that fixed exchange rate economies have incentives to accumulate more reserves. If there were markup shocks, for example, the government under flexible exchange rate would depart from a zero labor wedge allocation. Still, the availability of monetary policy would imply that it can insulate the economy from fluctuations in tradable consumption relative better than it could under a fixed exchange rate regime.

3 Quantitative Analysis

3.1 Numerical Solution

As in Hatchondo, Martinez and Sapriza (2010), we solve for the equilibrium by computing the limit of the finite-horizon version of our economy. The recursive government problem is solved using value function iteration. For each state, we solve the optimal portfolio allocation by searching over a grid of debt and reserve levels and then using the best portfolio on that grid as an initial guess in a nonlinear optimization routine. The value functions V^R and V^D and the equilibrium bond price function $q(\hat{a}(\cdot), \hat{b}(\cdot), s)$ are approximated using linear interpolation over y^T and cubic spline interpolation over debt and reserves positions.

3.2 Calibration

A period in the model refers to a year. We split the parameters of the model into two groups, which are listed in Table 1. The first group of parameters takes values that can be set either directly from the data or using typical values from the literature. The second group of parameter values is set by simultaneously matching key moments from the data. As a data reference, we use a panel of emerging economies, which are described in the empirical analysis in Section 5.

Following Bianchi et al. (2018), we assume the following utility cost of default:

$$\psi_d(y^T) = \psi_0 + \psi_1 \log(y^T) .$$

As in Chatterjee and Eyigungor (2012), having two parameters in the cost of default gives us enough flexibility to match the spread dynamics observed in the data.

The parameter values that govern the tradable endowment process are chosen to mimic the average behavior of logged and linearly detrended tradable GDP. Estimating an AR(1) for each country in the sample and taking the average yields $\sigma_\varepsilon = 0.045$ and $\rho = 0.84$. We set $\mu_y = -\frac{1}{2}\sigma_\varepsilon^2$ so that mean tradable income is normalized to 1.

The values of the risk-free interest rate and the domestic discount factor are set to $r = 0.04$ and $\beta = 0.90$, which are standard in quantitative sovereign default studies. We set $\delta = 0.2845$. With this value and the targeted level of sovereign spread, the Macaulay duration of debt is three years, which is roughly the average duration of public debt in our panel of emerging economies.

Following Bianchi et al. (2018), we use the average EMBI+ spread to parameterize the shock process to lenders' risk aversion. We assume that a period with high lenders' risk aversion is one in which the global EMBI+ excluding countries in default is one standard deviation above the median over the sample period. With this procedure, we obtain three episodes of a high risk premium every 20 years with an average duration equal to 1.25 years for each episode, which

Table 1: Parameter values.

Parameter	Description	Value
r	Risk-free rate	0.04
α	Labor share in the non-tradable sector	0.75
β	Domestic discount factor	0.90
π_{LH}	Prob. of transitioning to high risk premium	0.15
π_{HL}	Prob. of transitioning to low risk premium	0.80
σ_ε	Std. dev. of innovation to $log(y^T)$	0.045
ρ	Autocorrelation of $\log(y^T)$	0.84
μ_y	Mean of $\log(y^T)$	$-\frac{1}{2}\sigma_\varepsilon^2$
δ	Coupon decaying rate	0.2845
$1/(1+\mu)$	Elasticity of subs. between T-NT	0.44
γ	Coefficient of relative risk aversion	2.27
$\bar{h}$	Time endowment	1
	Parameters set by simulation	
ω	Share of tradables	0.40
ψ_0	Default cost parameter	3.76
ψ_1	Default cost parameter	22.20
κ_H	Pricing kernel parameter	15
$\overline{W}$	Lower bound on wages	0.98

implies $\pi_{LH} = 0.15$ and $\pi_{HL} = 0.8$. On average, the global EMBI+ was 200 basis points higher in those episodes than in normal periods.

The households' endowment of hours to work ($\bar{h}$) is normalized to 1. The labor share in the production of non-tradable goods is set to $\alpha = 0.75$, the estimate from Uribe (1997) for Argentina. We set $1/(1+\mu) = 0.44$, which is the elasticity of substitution between tradables and non-tradables estimated by Gonzalez-Rozada et al. (2003) and Akinci (2011). The coefficient of relative risk aversion is set to $\gamma = 1+\mu$. We do this for two reasons. First, the implied value (2.273) is close to 2, a value commonly used in the literature; and second, this is a convenient parameterization because it implies that the dynamics of the "tradable block" (debt, reserves, and consumption of tradables) are independent of the "non-tradable block" in the absence of rigidities.[24]

Targeted moments. The remaining parameters to calibrate are the weight of tradables in the utility function (ω), the default cost parameters (ψ_0 and ψ_1), the risk premium parameter (κ_H), the lower bound on wages ($\overline{W}$), and the cost of exchange rate fluctuations (Φ). Given a cost Φ, to be described below, we target the following five moments from the data: (i) a share of tradable output to total output of 41%, (ii) a mean debt-to-GDP ratio of 44%, (iii) a mean

[24]This feature follows from the fact that the CRRA utility function and CES aggregator imply that the cross-partial derivative u_{TN} equala zero when the intertemporal elasticity of substitution equals the intratemporal elasticity across goods.

sovereign spread of 2.9%, (iv) an increase of 200 basis points in the spread during high risk premium periods, and (v) an increase of 2 percentage points in the cyclical unemployment rate in a one year window around a default event.[25] Regarding the cost of exchange rate fluctuations, we assume a quadratic function $\frac{\Phi}{2}(e-\bar{e})^2$, where $\bar{e}$ is normalized to one. Our two benchmark results contrast $\Phi = 0$ and $\Phi = \infty$, but we will consider intermediate cases as well to match the exchange rate variability.

4 Results of the Quantitative Analysis

4.1 Simulation Moments

We start by showing that the model is able to account for salient features of business cycles in our panel of emerging economies. Table 2 reports moments from the data and from the model simulations. We present results for both the model under a fixed exchange rate ($\Phi = \infty$) and under a perfectly flexible exchange rate ($\Phi = 0$), labelled respectively "fixed" and "flexible." Because the optimal exchange rate policy prescribes full employment in all states, the latter can also be interpreted as an economy with flexible wages.

The top panel of Table 2 shows that our model closely matches the targeted moments. The middle panel shows that the model also performs well in accounting for several non-targeted moments of the data. In particular, the model features consumption that is procyclical and slightly more volatile than output, and sovereign spreads that are volatile and countercyclical.

The bottom panel of Table 2 provides the model predictions for reserves. We present the average level of reserves, expressed both in terms of output and as a fraction of debt, and the correlation of reserves with spreads. All these moments are non-targeted. Remarkably, one can see that the fixed exchange rate economy generates a mean reserves-to-output ratio of 16%, in line with the average of our panel of emerging markets. In contrast, the flexible exchange rate economy generates holdings of international reserves that are much lower, averaging 7% of GDP. This large gap between the two underscores the crucial role of the macroeconomic stabilization role for reserves in accounting for the level of reserves observed in the data.

It is important to point out that while the economy under nominal rigidities features higher

[25] To compute the sovereign spread that is implicit in a bond price, we first compute the yield i_b, defined as the return an investor would earn if he were to hold the bond to maturity (forever) and no default is declared. This yield satisfies

$$q_t = \sum_{j=1}^{\infty} \delta(1-\delta)^{j-1} e^{-j i_b}.$$

The sovereign spread is then computed as the difference between the yield i_b and the risk-free rate r. Debt levels in the simulations are calculated as the present value of future payment obligations discounted at the risk-free rate—that is, $\frac{\delta}{1-(1-\delta)e^{-r}} b_t$.

Table 2: Key statistics – model and data.

	Data	Model	
		Fixed	Flexible
Targeted			
Mean debt (b/y)	44	44	45
Mean r_s	2.9	2.9	2.9
Δr_s w/ risk-prem. shock	2.0	2.0	2.0
Δ UR around crises	2.0	2.0	0.0
Mean y^T/y	41	41	40
Non-targeted			
$\sigma(c)/\sigma(y)$	1.1	1.0	1.1
$\sigma(r_s)$ (in %)	1.7	3.1	2.8
$\rho(r_s, y)$	-0.3	-0.7	-0.8
$\rho(c, y)$	0.8	1.0	1.0
Mean reserves (a/y)	16	16	7
Mean reserves/debt (a/b)	35	37	17
$\rho(a/y, r_s)$	-0.2	-0.4	-0.6

Note: Moments in the model are computed for the average of pre-default simulation samples (except for the change in the unemployment rate around default crises). We simulate the model for 1,000 samples of 300 periods each. We then take the last 35 observations of each sample in which the last default was observed at least 25 periods before the beginning of the sample.

volatility and lower employment, this is *not* the reason why the government accumulates more reserves. In fact, if we assume separable preferences for tradables and non-tradables (as we did in the calibration), a higher volatility of non-tradable output is completely irrelevant to the decision to accumulate reserves under flexible wages. Key for our mechanism is that the government *internalizes* that through its portfolio choice, it can alter the level and the volatility of employment.

4.2 Macro-Stabilization Hedging Benefits

In this section, we inspect the key channel at work in the model and show how reserves contribute to improving macroeconomic stabilization. To do so, we examine how the entire set of candidate portfolios chosen at time t, as constructed in (19), affects the distribution of unemployment rates in $t+1$. That is, we fix an initial state (a, b, s) and consider all the possible portfolios from which the government could pick, subject to delivering a given amount of transfers to households. The results of this exercise for the fixed exchange rate economy are illustrated in Figure 2. The left panel shows the mean and the volatility of the $t+1$ unemployment rate for a range of values of reserves. The right panel shows the entire distributions of the $t+1$ unemployment rate for two possible values of reserve accumulation (0% and 2% of GDP).[26]

(a) Mean & std. dev. of t+1 unemployment (b) Distribution of t+1 unemployment

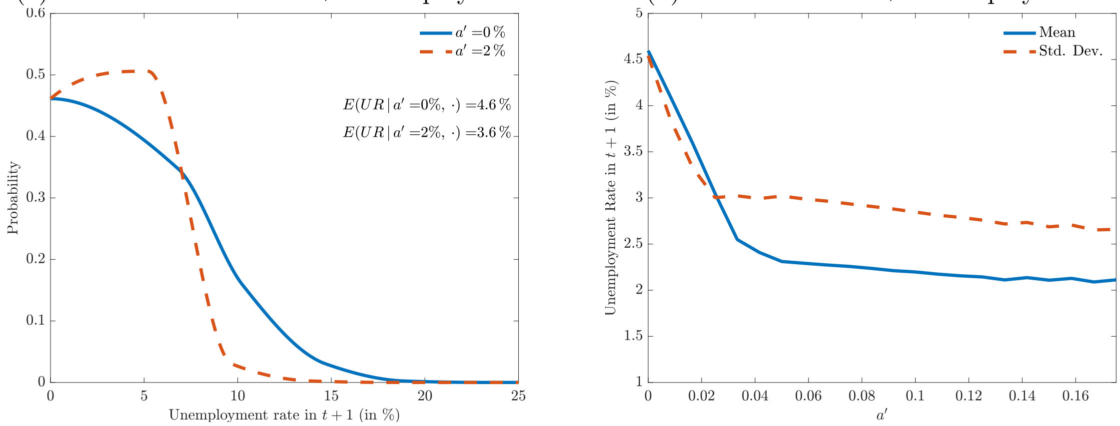

Figure 2: Unemployment tomorrow as a function of reserves

Note: The figure assumes an initial portfolio of $a = 8\%$ and $b = 36\%$ of mean GDP, an initial tradable income that is 10% below mean, and $\kappa = \kappa_L = 0$.

The key result from Figure 2 is that a portfolio with larger gross asset and debt positions helps reduce future unemployment. Panel (a) shows a lower mean and volatility of unemployment with higher reserves; Panel (b) shows how with higher reserves, the distribution of unemployment places a lower probability mass in states with higher unemployment levels. Notice that states with high unemployment in the distribution are associated with low tradable income shocks or adverse risk-premium shocks. The figure therefore shows that having more reserves available in those states allows for a significant reduction of the slack in the labor market.

4.3 Portfolios and Spreads

A central element in the government's optimal portfolio choice is how spreads respond to the portfolio composition. As highlighted in equation (17), the more the spread increases in response to a debt-financed reserves operation, the smaller the amount of reserves that can be purchased is, and hence the lower the net marginal benefits from accumulating reserves are.

Default sets. To understand how portfolios affect spreads, it is useful to start by considering for which states the government finds it optimal to default or repay. In Figure 3, we fix the initial value of income y^T at one standard deviation below its mean and $\kappa = 0$ (no risk premia case), then analyze the default decision as a function of (a, b). We label "default set" the combinations of (a, b) such that $V^D(a, b, s) > V^R(a, b, s)$. For the fixed exchange rate economy, this set is the area with vertical stripes in Figure 3. For comparison, we also show the default set under

[26] For convenience, all figures in the paper express debt and reserves as percentage of mean GDP.

a flexible exchange rate (for the same initial values of the exogenous state variables), which is illustrated by the shaded area in the figure. As one can see, the default set under a flexible exchange rate is contained by the default set under a fixed exchange rate.[27]

An important observation that emerges from Figure 3 is that default sets are decreasing in reserves. That is, for a given level of debt, higher reserves increase the incentives to repay. This result reflects the fact that while both the value of repayment and the value of default are increasing in reserves, the former is even more so (see Figure D.1 for an illustration). To understand the logic behind this result, it is useful to go back to the expression for the marginal value of reserve holdings which, using the envelope condition, is given by $u_T + \mathbb{I}_{h<\bar{h}} u_N \frac{c_t^N}{c_t^T}$. In a state where the government is facing steep borrowing terms, it follows that tradable resources are relatively more scarce under repayment than under default. This, in turn, makes one more unit of reserves more valuable under repayment, particularly when there is slack in the labor market and the reserves can help mitigate the recession.

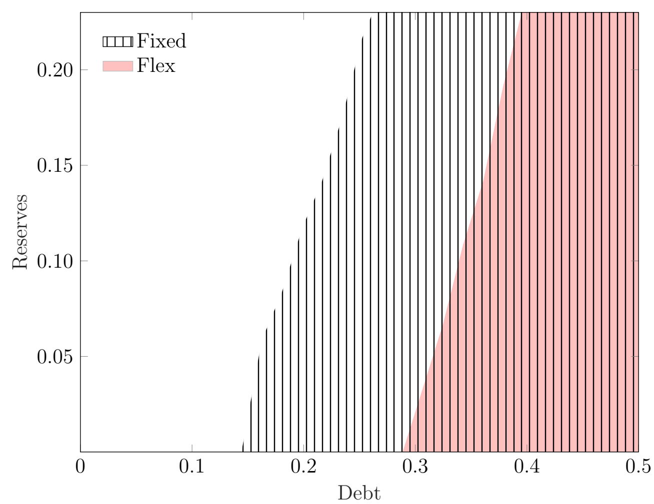

Figure 3: Default sets

Note: The area with dark vertical stripes is the default set for the economy under a fixed exchange rate regime, and the area with a pink shading is the default set for the economy under a flexible exchange rate regime.

Spread-debt menus. The previous results on the default set naturally translate into the equilibrium spread schedule that the government faces. The left panel in Figure 4 shows the menu of spreads and next-period debt level combinations from which the sovereign can choose, keeping the level of a' fixed at the mean value observed in the simulations and initial values for all state variables set at their means. In line with the results on the default sets, the spread is increasing in the end-of-period debt, and the spread is higher under fixed than under flex for any given level of debt. The right panel of Figure 4 shows how much the spreads-debt menu

[27]See Bianchi and Mondragon (2022) for a formal characterization of how a fixed exchange rate affects the incentives to default.

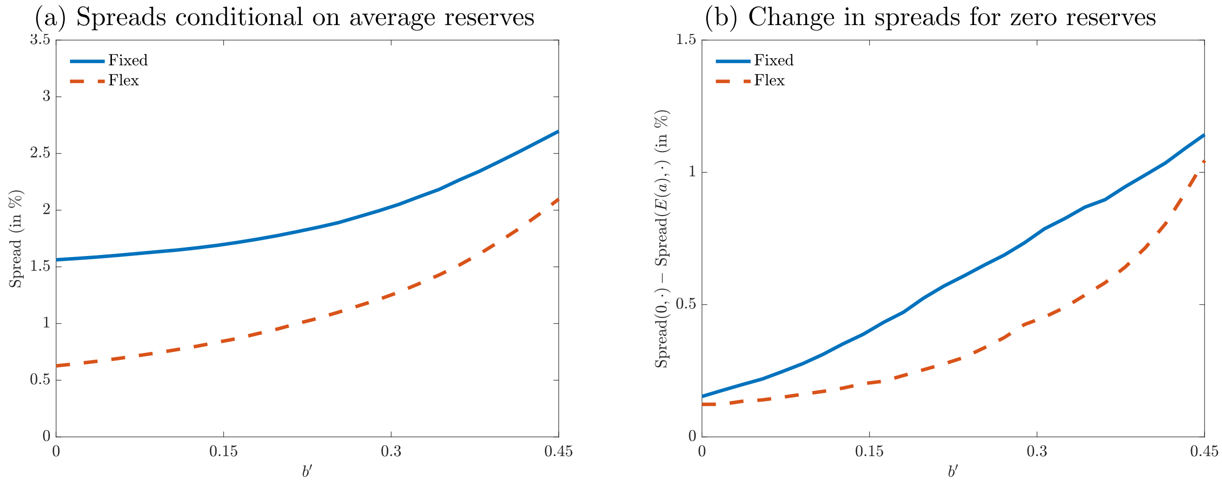

Figure 4: Spread schedules, reserves, and nominal rigidities.

Note: The left panel shows the equilibrium spread-debt menu under "Fixed" and "Flex," conditional on reserves being at the mean observed in the benchmark simulations. The right panel shows how much the spread will increase if the government were to choose zero reserves instead of the average value. Initial states correspond to average income, debt, and reserves, and no risk premium shocks.

would worsen if the government were to accumulate zero reserves instead of the average level. As the figure shows, the increase in spreads is especially significant under fixed exchange rate. This reflects that having more reserves under a fixed exchange rate helps stabilize the economy and makes repayment more likely.

Debt-financed reserves and spreads. The previous results highlight that when the government chooses to hold higher reserves, incentives to default in the future are reduced and spreads are lowered. On the other hand, repaying debt rather than accumulating reserves could lead in principle to even lower incentives to default and lower spreads.

To further investigate this issue, we now examine how default decisions at $t+1$ change when we vary both debt and reserves at time t following the candidate portfolios constructed in (19). We define an income threshold $\tilde{y}$ as the value of next-period income at which the government is indifferent between repaying and defaulting.[28] Figure 5 plots this income threshold for candidate values of a' (and the two possible values that the risk premium shock can take) while keeping government transfers constant. For $y^T < \tilde{y}$, the government defaults (and repays otherwise). We can see in the figure that as the government increases the gross positions, it becomes more likely that it will default (in the sense that under more realizations of the income shock, the government defaults). Under a fixed exchange rate, however, the income threshold responds more modestly to the increase in gross positions. Moreover, when risk aversion for investors is high next period

[28]That is, the value $\tilde{y}$ such that $V^R(a', b', \tilde{y}, \kappa') = V^D(a', \tilde{y}, \kappa')$. We verify numerically that this threshold is unique.

($\kappa' = \kappa^H$), the government may become *less likely* to default for higher levels of gross positions.[29]

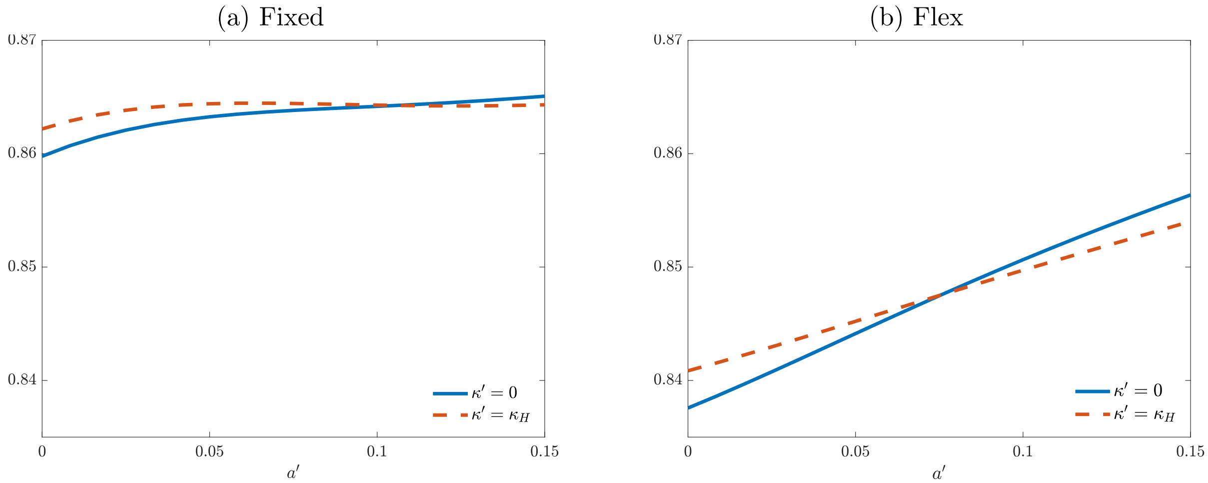

Figure 5: Income-default thresholds.

Note: the left panel shows the income default thresholds for a fixed exchange rate regime, and the right panel shows them for the flexible regime. In both cases, we present the thresholds for $\kappa' = 0$ and $\kappa' = \kappa_H$.

5 Empirical Analysis and Model Validation

In this section, we present empirical evidence on international reserves, sovereign spreads, and exchange rate flexibility that is consistent with the model. Further, we validate the mechanisms of the model by examining several testable implications of the model and contrasting them with their counterparts in the data.

We use data from 1990 to 2015 for a set of 22 emerging economies, that are commonly used in the literature.[30] Table A.1 presents summary statistics of debt, sovereign spreads, and reserves in our sample of countries. On average, the countries in our panel have a debt-to-GDP ratio of 44% and a reserves-to-GDP ratio of 16%. In addition, they face significant default risk. The average mean spread is 2.9%, and the standard deviation is 1.7%. Appendix A.1 has the full details about the dataset and variable definitions.

[29]In Figure D.3, we present an example of initial conditions under which issuing debt to buy reserves does not actually raise sovereign spreads in a fixed exchange rate. The logic is that when the government is very likely to be borrowing constrained, an increase in debt and reserves may lead the government to default in fewer states. This highlights that a government could, in some instances, raise debt at effectively no cost, provided that it accumulates reserves. There is, of course, a limit. At some point, once debt-financed reserves become large enough, the spreads would move against the government.

[30]The 22 countries in our panel are Argentina, Brazil, Chile, China, Colombia, the Czech Republic, Egypt, Hungary, India, Indonesia, Korea, Malaysia, Mexico, Morocco, Pakistan, Peru, Poland, Romania, Russia, South Africa, Thailand, and Turkey.

5.1 Reserve Buffers: Fixed versus Flex

A central prediction of the theory is that economies with fixed exchange rate regimes should accumulate more reserves than economies with flexible exchange rate regimes do. To examine whether this prediction holds in the data, we use the IMF classification of exchange rate regimes and sort countries into two categories, "fixed" and "flex," representing countries with fixed and flexible exchange rate regimes, respectively.

Figure 6 shows that while both sets of countries have experienced an increase in reserve holdings, the surge is most notable for countries with fixed exchange rates.

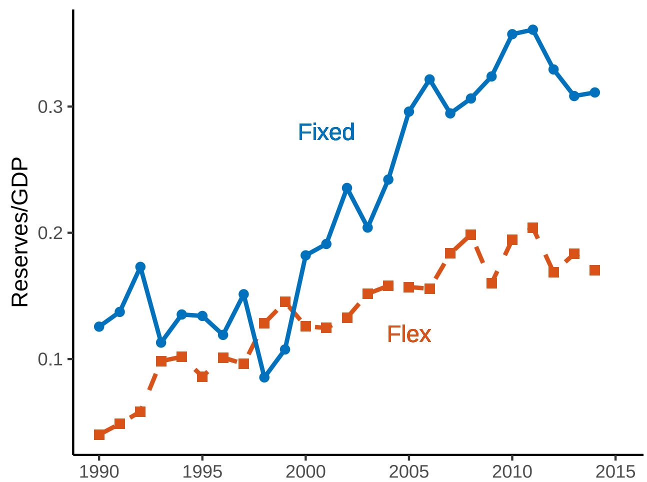

Figure 6: Reserve accumulation and exchange rate regime

5.2 Exchange Rate Variability and Reserve Accumulation

In order to further document the role of exchange rate flexibility in shaping the accumulation of international reserves, we follow the empirical literature (e.g., Tenreyro, 2007) and use a more continuous variable to capture the degree of flexibility of a country's exchange rate. In particular, we define the exchange rate variability (ERV) for country i as the standard deviation of the log first-difference of the exchange rate of country i's currency (against the US dollar). The standard deviation is computed using (centered) rolling windows of three years.

Averages. We begin by documenting that the inverse relationship between ERV and reserve accumulation is apparent when we impose a minimal structure on the data. In particular, Figure 7 presents a scatter plot of the time-series average of reserves-to-GDP ratio against the time-series average of ERV for all countries. (The figure also includes a model counterpart to be explained below.)

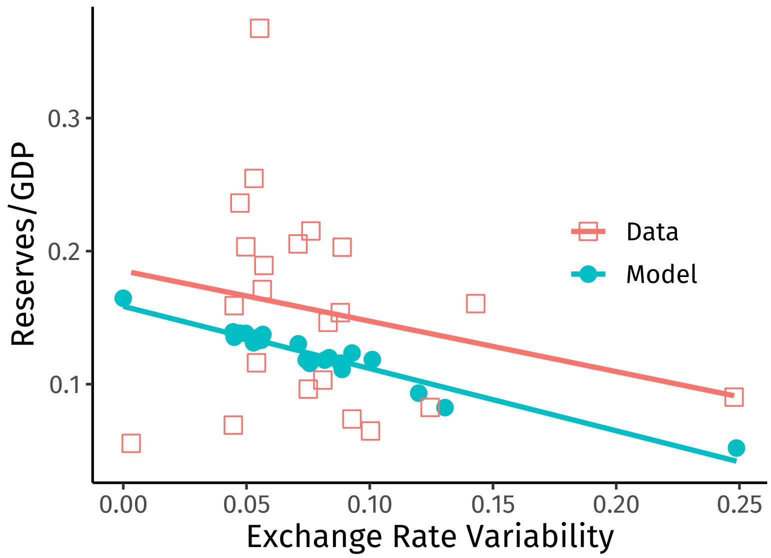

Figure 7: Model vs. Data: Reserves and ERV

Panel data regressions. The simple correlation between ERV and reserve accumulation is mildly negative in our sample (-.13). Since the association between exchange rate variability and reserve accumulation might be driven by other confounding factors, we use a regression framework and control for global and country specific factors. Specifically, we estimate

$$Res_{it} = \beta_{\text{ERV}}\,\text{ERV}_{it-1} + \phi'\,\mathbf{X}_{it-1} + \epsilon_{it}, \tag{21}$$

where Res_{it} is the (logarithm of) of the reserves-to-GDP ratio for country i in year t; ERV_{it-1} is our measure of exchange rate variability for country i in year $t-1$; and $\mathbf{X}_{it-1}$ is a vector of commonly used control variables in reserves regressions: the debt-to-GDP ratio, the country spread, the cyclical component of GDP, and the world interest rate. Finally, ϵ_{it} is a random error term.[31]

Table 3 shows that, other things equal, countries with less exchange rate variability tend to accumulate more reserves, and this finding is robust to various controls and specifications. The magnitudes are also economically significant. In specification (3), for example, a decrease of 10 percentage points in country i's exchange rate variability is associated with a 10% increase in reserve holdings.[32]

[31] In all the specifications of equation (21) that we estimate, we follow the standard procedure of lagging the explanatory variables one period, to control for endogeneity. As is common in studies of emerging economies, we exclude crisis years. Crisis years are defined following Catão and Mano (2017).

[32] The results in all specifications of Table 3 hold when including country fixed effects in equation (21). See Table A.2 in Appendix A.

Table 3: Data panel regressions

	Dependent variable: log(*Reserves*/*y*)				
	(1)	(2)	(3)	(4)	(5)
ERV	-1.015^{***}	-0.991^{***}	-0.999^{***}	-0.692^{***}	-0.531^{**}
	(0.311)	(0.312)	(0.295)	(0.258)	(0.231)
$\log(Debt/y)$		-0.040	0.034	0.087	0.076
		(0.051)	(0.051)	(0.053)	(0.050)
$\hat{y}$			1.178^{***}	0.277	0.604
			(0.429)	(0.504)	(0.472)
$\log(Spread)$				-0.231^{***}	-0.223^{***}
				(0.038)	(0.036)
r^{world}					-0.144^{***}
					(0.023)
(Intercept)	-1.942^{***}	-1.983^{***}	-1.885^{***}	-1.694^{***}	-1.589^{***}
	(0.041)	(0.068)	(0.064)	(0.070)	(0.068)
Number of countries	22	22	22	22	22
Observations	427	412	396	385	385
R^2	0.030	0.032	0.049	0.119	0.208
F Statistic	13.232^{***}	6.751^{***}	6.740^{***}	12.830^{***}	19.935^{***}

Note: All explanatory variables are lagged one period. The cyclical component of GDP (y) is denoted by $\hat{y}$. Robust standard errors are reported in parentheses. *p<0.1; **p<0.05; ***p<0.01.

Model regressions. In order to validate the workings of our model, we run comparable panel regressions using model simulated data. In our model, the key variable determining the exchange rate variability in the sample is Φ and so we run multiple simulations of the model for a range of values of Φ. Specifically, we consider 22 possible values for Φ to match the ERV in the 22 countries in our sample and recalibrate each economy to match the same targeted moments. For each economy, we construct 200 samples consisting of 35 periods each (and set Φ to match the average ERV in the simulated panel).[33]

Figure 7 presents a scatter plot in blue of the simulated data from the model, with average ERV in the x-axis and average reserves in the y-axis. The figure clearly shows that that the negative relationship found in the data holds as well in our simulated panel.

We then estimate the equivalent panel regressions (equation (21), specifications 1–4) using our simulated data. Table 4 presents the mean and interquartile range of all coefficients in specifications 1–4. This table shows that our main coefficient of interest (β^{ERV}) is, on average,

[33] To be consistent with the sampling used in the simulations of the benchmark model, we choose 35 periods. Results are almost identical if we use panels of 26 periods (years), as in our dataset.

negative in all specifications.[34] That is to say, we find a negative association between reserve accumulation and exchange rate variability, when we follow the same procedure in the model and in the data.

Table 4: Panel regressions using simulated data

	Dependent variable: $\log(Reserves/y)$			
	(1)	(2)	(3)	(4)
ERV	-2.34	-1.37	-1.49	-0.71
	[-2.82, -1.79]	[-1.94, -0.84]	[-2.06, -0.95]	[-1.32, -0.1]
$\log(Debt/y)$		-1.05	-1.10	-0.70
		[-1.27, -0.82]	[-1.32, -0.85]	[-0.86, -0.51]
$\hat{y}$			-1.04	-4.74
			[-1.67, -0.52]	[-5.82, -3.73]
$\log(Spread)$				-0.62
				[-0.74, -0.5]
(Intercept)	-1.98	-2.99	-3.01	-5.35
	[-2.09, -1.85]	[-3.26, -2.66]	[-3.28, -2.68]	[-6, -4.67]

Note: All explanatory variables are lagged one period. The the cyclical component of GDP (y) is denoted by $\hat{y}$. We estimate 200 panel regressions, each of them consisting of 22 model-economies with 35 periods each. We then report the means of the estimated coefficients. The interquartile range is reported in square brackets.

5.3 Labor Market Rigidity

So far the empirical analysis has focused on the exchange rate flexibility as a key determinant on the level of reserve accumulation. However, the model also predicts that countries with a higher degree of labor market rigidities accumulate more reserves. To test this empirical prediction in the data, we use a measure of wage rigidity as proposed by Schmitt-Grohé and Uribe (2016) and Matschke and Nie (2022). The measure is constructed by computing the decline in wages conditional on episodes with an observed increase in the unemployment rate.[35] Besides its simplicity and the fact that it can be constructed with readily available data, an advantage of this measure is that it lends itself to a straightforward international comparison.[36] Figure 8 shows that countries with more severe wage rigidity accumulate more reserves, as is consistent

[34] In Appendix A, we present the kernel densities for β^{ERV} under the four different specifications.

[35] We use in particular the estimates from Matschke and Nie (2022). Their procedure identifies all spells with an increase in cyclical unemployment and then averages the decline in wages across all spells. We thank Johannes Matschke and Jun Nie for sharing their data with us.

[36] There is an active literature using micro data to measure wage rigidity in specific countries. While these papers offer a more granular analysis, estimates are for a very limited set of countries. Another interesting test that we leave for future research would be to relate a measure of volatility of output gaps and reserve accumulation.

with the model.[37]

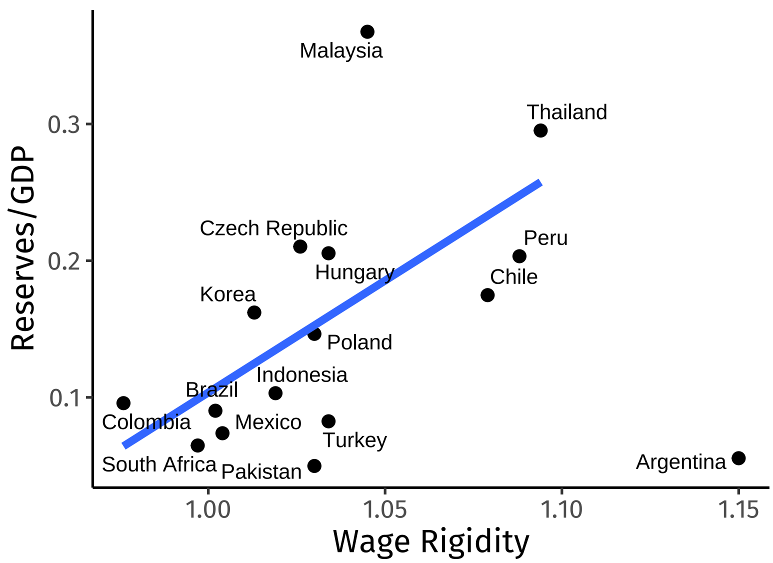

Figure 8: Wage Rigidity and Reserves.

5.4 Reserves and Devaluations

One of the central predictions of our model is that: all else equal, countries with a lower degree of exchange rate flexibility find it optimal to deploy a larger portion of the reserves in response to adverse shocks. In addition, countries that face a crisis with higher holdings of reserves should experience a lower depreciation of the exchange rate, all else equal.

A suitable laboratory to test these two predictions of the model is the Global Financial Crisis. Around September 2008, there was a large increase in sovereign spreads across all emerging economies. Meanwhile, there was a decline in the accumulation of international reserves and a large dispersion in exchange rate depreciation across countries.

In Figure 9, we present empirical evidence that confirms that the two theoretical predictions indeed hold in the data. Panel (a) shows that countries that deployed more reserves during the 2008 Global Financial Crisis were associated with a lower depreciation in the exchange rate.[38] Panel (c) shows that countries that faced the crisis with a higher amount of reserves experienced a lower depreciation of the exchange rate.

Panels (b) and (d) of Figure 9 provide model simulations to zoom in the theoretical predictions and compare them with the data. In panel (b), we consider an economy that starts with a

[37] The regression coefficient is significant when excluding Argentina, which is a clear outlier. We also find that the regression coefficient is more positive for countries below the median ERV, although it is not statistically significant (partly attributable to the modest sample size).

[38] Evidence on the relationship between depreciations and the the level of reserves is also provided by Obstfeld et al. (2009).

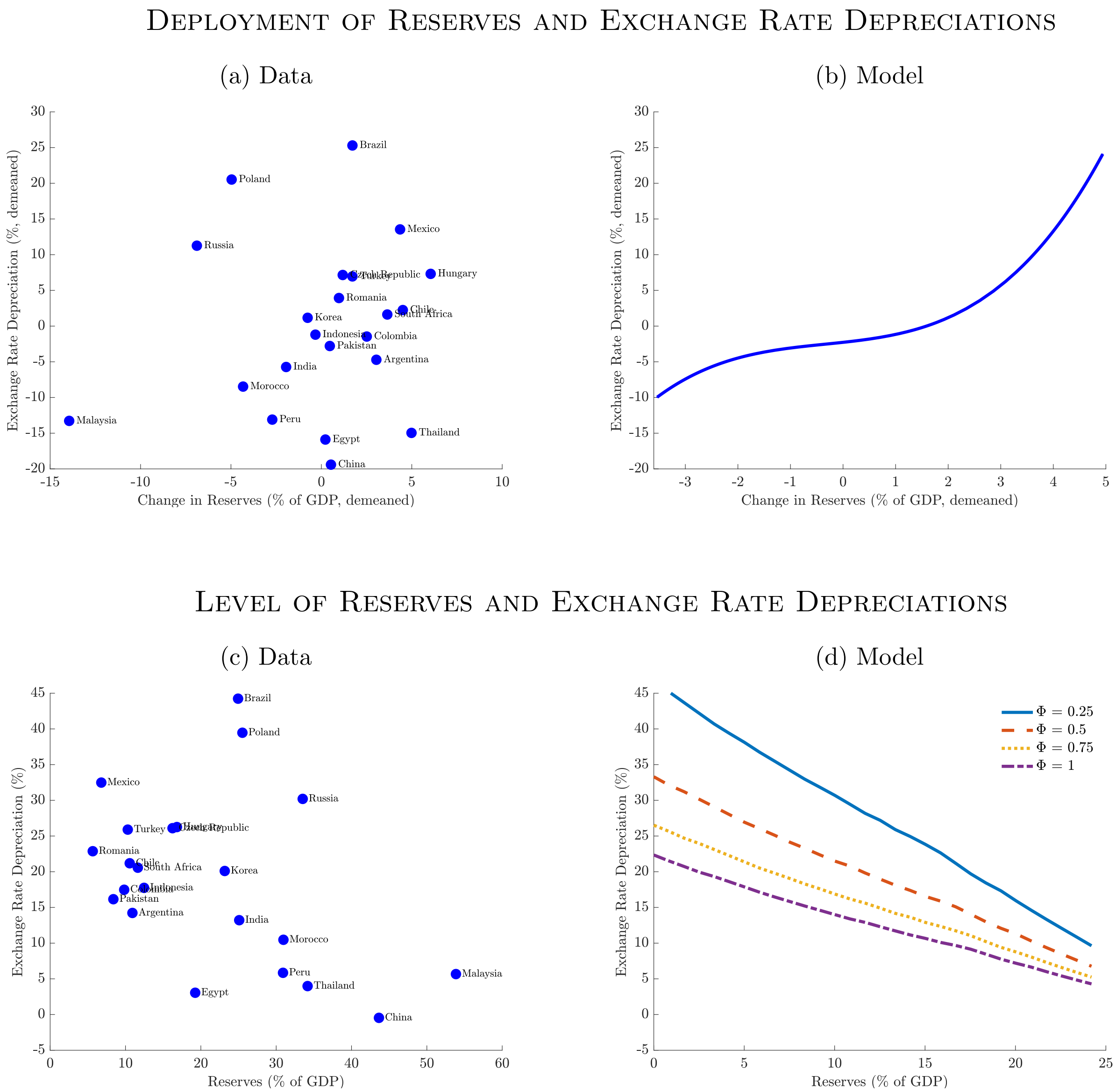

Figure 9: Depreciation rates and reserves

Note: Panel (b) shows the model responses for different values of Φ . Panel (d) shows the exchange rate depreciation as a function of the initial level of reserves for different values of Φ.

portfolio (a, b) around the mean and is hit by a negative income shock that increases spreads by 300 bps for an economy with a fixed exchange rate. We then vary the costs from depreciating the exchange rate in the current period, as determined by Φ, and trace out the reduction in reserves. The figure shows that as Φ is lowered (moving in the north-east direction), there is a higher depreciation rate and a lower decline in reserves.[39] Panel (d) conducts a similar exercise, but it varies the initial level of reserves, for given Φ. Specifically, starting from the average debt level, we hit the economy with a negative income shock and trace the exchange depreciation

[39]Note that plots (a) and (b) are demeaned, so that a positive change in reserves implies that reserves fall *less* than the average. By the same token, a negative value on the vertical axis means that the exchange rate depreciated by *less* than the average.

as a function of the initial level of reserves. As the figure shows, the higher is the initial level of reserves, the lower is the exchange rate depreciation. This negative relationship holds for different values of Φ.

The evidence we present suggests that the predictions of our model regarding the link between devaluations and reserves during crises are consistent with the data. However, it is important to discuss whether there are other potential explanations that can also fit the same pieces of evidence. Consider for example the literature on balance of payment crises à la Krugman (1979). In these models, a negative fiscal shock predicts a permanent rise in the depreciation rate to raise seigniorage and thus balance the intertemporal budget constraint of the monetary authority. However, the evidence discussed indicates a rise in the *level* of the exchange rate upon the shock.[40] Another important line of work considers the government portfolio in monetary models. In Ottonello and Perez (2019), for example, the government faces a portfolio of non-defaultable domestic and foreign currency bonds in the context of an endowment economy. In their model, if there is a higher cost from depreciation (or inflation), higher gross positions, holding fixed the net foreign asset positions, would predict a higher depreciation, a feature that would be difficult to reconcile with the evidence presented in panel (a). The logic is that in Ottonello and Perez (2019), the gains from depreciations emerge from diluting the domestic currency debt, and these gains are larger with higher gross positions. The takeaway is that the macro-stabilization role of reserves that we highlight appears to be crucial in accounting for the evolution of reserves and exchange rates during episodes of financial turbulence.

5.5 Accounting for the Trend in Reserves

The data presented in Figure 6 also shows a clear trend in reserve accumulation over the past decades. While our model is stationary, we can extend our analysis to allow for a sequence of shocks that can speak to this upward trend. In particular, we explore the effects of an increase in risk premia, a feature that is motivated by the widespread view that following the east Asian crisis, there was a realization that capital flows could suddenly dry up for emerging economies (Feldstein, 1999). We will argue that that this specific development in international markets can account for the increase in reserves among fixed exchange rate countries and at the same time can deliver the less pronounced upward trend among countries with flexible exchange rates, as seen in the data.

[40] Moreover, this literature focuses on the net foreign asset position and does not have predictions for gross positions.

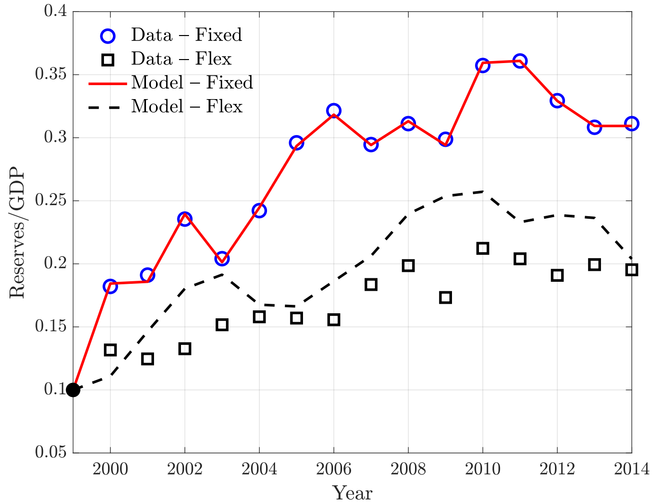

Figure 10: The upward trend in reserves: Model and data.

Note: The lines are for model-generated data and the markers without lines are for the averages from our panel data. The solid red (dashed black) line is for the model under fixed (flexible) exchange rates. The blue circular (black squared) markers denote the average reserves holdings for countries under a fixed (flexible) exchange rate regime.

We proceed in three steps. First, we initialize both model economies (fixed and flexible) with income and debt at their means and with the initial level reserves at 10% of GDP for the year 1999. Second, we look for a sequence of shocks to the beliefs about the values of κ_H such that the model under the fixed regime replicates exactly its data counterpart. The exercise assumes that $\kappa_t = \kappa_L$ in the simulations—and so the risk premium shocks are not realized on the equilibrium path.[41] Third, we then feed this sequence of "risk premium shocks" into the flexible exchange rate economy.

Figure 10 shows the resulting time series for the reserves-to-GDP ratio for the fixed and flexible economies. The figure shows that the risk premium shock delivers the exact the path of reserves to GDP, as calibrated. Moreover, the same sequence of shocks under the flexible regime delivers an increase in reserves that is much less pronounced that under fixed exchange rate (and in line with the path observed in the data). The lesson we obtain is that a common increase in risk premia can explain why fixed exchange rate economies have accumulated more reserves than flexible exchange rate economies in the last 20 years.

6 Welfare Experiments

In this section, we conduct several welfare experiments. We first ask what the welfare gains of accumulating reserves are and to what extent they mitigate the costs of running a fixed exchange

[41]For simplicity, we also keep both income and debt constant at their means and assume that the shocks to the value of κ_H are unanticipated throughout the entire transition.

rate. Then, we examine the desirability from committing ex ante to a reserve accumulation policy. Finally, we study the performance of simple policy rules.

6.1 Welfare Gains from Reserves

We compute the welfare effects of switching from a baseline regime to an alternative regime as

$$\mathbb{W}(a,b,s) = 100\times\left[\left(\frac{(1-\gamma)(1-\beta)V_{baseline}(a,b,s)+1}{(1-\gamma)(1-\beta)V_{alternative}(a,b,s)+1}\right)^{1/(1-\gamma)}-1\right], \tag{22}$$

where $V_{baseline}$ and $V_{alternative}$ denote the respective value functions.

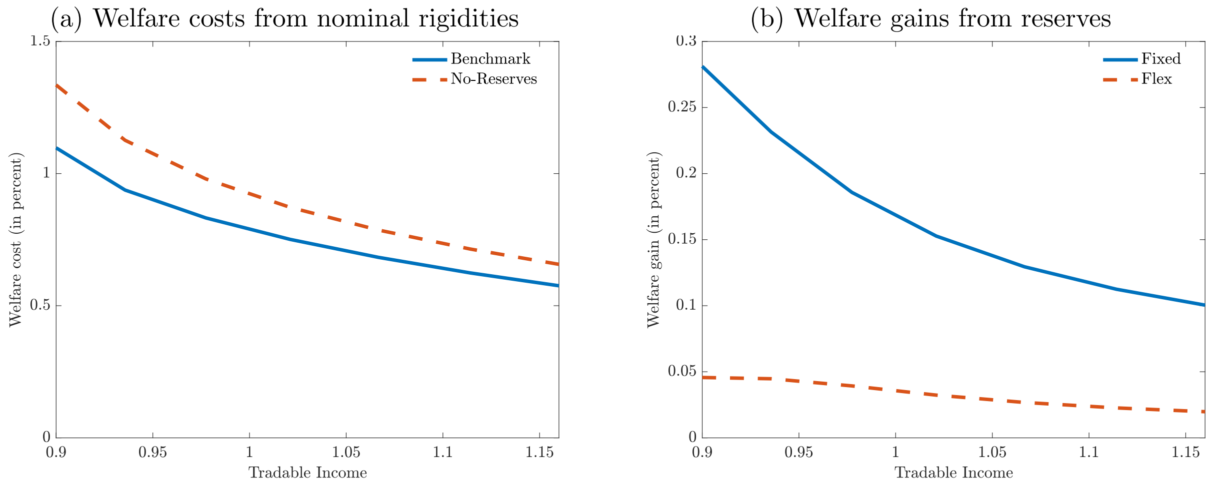

Figure 11: Welfare

Note: The left panel shows the costs of nominal rigidities under fixed exchange rate for our benchmark economy and compares it with an economy where $a_{t+1} = 0$ for all t. The right panel shows the welfare gains from moving from a regime with $a_{t+1} = 0$ for all t to our benchmark economy. The figure is presented for fixed and flexible exchange rate regimes ($\Phi = \infty$ and $\Phi = 0$). Both panels assume zero initial debt and reserves.

Figure 11 shows that nominal rigidities induce large welfare costs under a fixed exchange rate regime, but these costs can be significantly attenuated by optimally managing international reserves. In the left panel, we can see that the welfare costs from nominal rigidities are higher when tradable endowment is low, a result that follows from the fact that those are the states with high unemployment.[42] Moreover, these costs are larger in an alternative regime in which we assume that the government does not have access to reserves.

On the right panel of Figure 11, we delve deeper into the gains from accumulating reserves and study how they depend on the exchange rate regime. As the figure shows, the welfare gains are substantially larger in a fixed exchange rate economy. In line with the macroeconomic stabilization role we highlighted, managing reserves under a fixed exchange rate allows the government

[42] Welfare costs are also larger at higher levels of debt. At the mean value of debt, we find an average welfare cost of 2.8% of permanent consumption in the benchmark regime and 3.0% in the "no-reserves" regime.

to avoid deeper contractions in economic activity and hence deliver larger welfare gains.[43]

6.2 Is Reserve Accumulation Ex Ante Optimal?

In the previous section, we found that under the sequentially optimal policy, reserve accumulation delivers significant gains under a fixed exchange rate. However, there is a question about whether a commitment to a different reserve accumulation policy at time zero would deliver larger gains and, if so, what that policy would look like.

The scope for committing in advance to a reserve accumulation policy emerges from the debt-dilution problem associated with long-term defaultable bonds. That is, when pricing bonds, investors anticipate the evolution of bond prices in the future. To achieve a higher bond price today, the government would like to commit to policies that reduce the probability of default in the future, but ex post, the reward for such policies accrues to the investors that hold the legacy bonds. To shed light on whether the government would want to commit ex ante to higher or lower reserves, we consider a planner's problem in which the government can commit at time t to a certain reserve accumulation policy at $t+1$ (and switch to the Markov perfect equilibrium after that). Following this structure, we can then analyze how the policy the government promises for $t+1$ compares with the one without commitment, depending on the initial conditions at time t and the shocks at $t+1$.

The fact that bond prices are generally increasing in reserves, as we have shown in Section 4.3, suggests that the government should perhaps commit to increasing reserves in the future. However, there is another effect, which is that a commitment to higher reserve accumulation in the future also tends to lead to larger bond issuances in the future.[44] To the extent that bond prices are relatively more sensitive to bond issuances than to reserves, a commitment to higher reserve accumulation may worsen bond prices today. The tension between these two forces can be illustrated by considering the optimality condition for the promise of reserve accumulation in the next period $a_{t+2}(s_{t+1})$, derived in Appendix B.1:

$$u_T(c^T_{t+1}, c^N_{t+1}) \left(q_a - \frac{\partial q\left(a_{t+2}, b_{t+2}, s_{t+1}\right)}{\partial a} i_{t+1} \right) + \beta \mathbb{E}_{t+1} \frac{\partial V\left(a_{t+2}, b_{t+2}, s_{t+2}\right)}{\partial a} = (1-\delta) u_T(c^T_t, c^N_t) \left(\frac{\partial q\left(a'', b'', s\right)}{\partial a''(s')} + \frac{\partial q\left(a'', b'', s\right)}{\partial b''} \frac{\partial b''}{\partial a''} \right) i_t \quad (23)$$

for all s_{t+1} in which the government repays. Notice that in a Markov perfect equilibrium, the

[43]In fact, under some parameterizations, we find that in a flexible exchange rate, reserve accumulation may lead to negative welfare gains. The reason is that having the ability to accumulate reserves may end up leading to worse borrowing terms for the government because accumulating reserves exacerbates the time inconsistency problem associated with debt dilution. See Hatchondo, Martinez and Sosa-Padilla (2016) for a study on debt dilution and sovereign default risk.

[44]Note that here we are considering a commitment only to reserve policy and not to bond issuances.

expression on the left-hand side is equal to zero. In this case, however, the government may choose a policy a'' that is not ex-post optimal at $t+1$ but improves bond prices in period t, as reflected on the right-hand side of (23).

Relative to the discretionary solution, our numerical inspection, described in more detail in the appendix, shows that the government tends to find it optimal to commit to higher reserve accumulation for high income states tomorrow and to commit to lower reserve accumulation for low income states tomorrow. For example, for an initial portfolio equal to the mean in the simulations and an income level one-standard deviation below the mean, the government finds it optimal to commit to increasing the reserve accumulation by 0.8% of GDP in high income states tomorrow and to decreasing it by 1.6% of GDP in low income states tomorrow. This difference reflects that borrowing responds less to a higher stock of reserves in high income times than in low income times, as shown in Figure B.2.[45]

6.3 A Simple Reserve Accumulation Rule

Our previous results highlight the welfare benefits of a state-contingent optimal reserve accumulation policy (and the potential gains from commitment). An important related practical discussion in policy circles is what constitutes an "adequate" amount of reserves. In fact, the IMF often recommends that countries hold a certain amount of reserves as a fraction of imports or some debt measure. Perhaps the most well-known example is the Greenspan-Guidotti rule, which prescribes that a country hold reserves equal to 100% of its short-term liabilities. To help guide these discussions, we now explore the design of simple rules.

Toward an operational policy, we explore the performance of simple reserve accumulation rules that we assume to be linear in the initial debt, initial reserves, tradable income, and the sovereign spread (while the government continues to optimize over debt, consumption, and the default or repayment decision). To compute the best rule, we first compute the welfare gains of moving from the economy without reserves to the economy in which this rule is followed for every possible initial state. Then, we maximize the increase in welfare, starting from the ergodic distribution under no reserve accumulation. We find that average welfare gains amount to 0.07% of permanent consumption, starting from zero initial debt and reserves. These estimated rules imply a reserve accumulation policy that is increasing in the initial level of reserves and in

[45] We have also explored non-state-contingent commitments where the government increases/decreases by a fixed amount $\tilde{\varepsilon}$ the level of reserves relative to that of the discretion case. In that case, we find that the government commits to lower reserves when it starts at time t from low income states and to higher reserves when it starts from high income states.

tradable output and decreasing in spreads and in the initial level of debt.[46]

On the other hand, we find that a rule that prescribes reserves equal to the liabilities maturing within a year features welfare losses. This result illustrates the importance of some degree state contingency for the effectiveness of reserve rules.[47] It is also interesting to note that the preferred simple rule from our analysis prescribes that the government should have more reserves than the ones needed to fulfill the short-term debt obligations. The idea is that when borrowing conditions become adverse, the government must have (on average) liquidity not only to repay existing debt but also to have additional resources for macro-stabilization.

7 Extensions and Sensitivity

In this section, we present several extensions to the baseline model to show how our main theoretical and quantitative results can be generalized.

7.1 Defaults and Devaluations

In practice, when a government operating under a fixed exchange rate defaults on its sovereign debt, it also often abandons the currency peg. Indeed, as shown by Reinhart (2002) and Na et al. (2018), devaluations and default tend to happen together, a phenomenon dubbed "Twin Ds". Our baseline model does not speak to this phenomenon, as we assume the government sticks to the exchange rate regime. We will argue, however, that our key results continue to hold once we incorporate this realistic feature.

We consider the problem of a government that starts with a fixed exchange regime and has the option of abandoning it. Once the peg is abandoned, we assume for simplicity that the country remains in a flexible exchange rate in all future dates. A government that has the option to devalue solves

$$V^o(a, b, s) = \max \left\{ V_{stay}(a, b, s), V_{flex}(a, b, s) - \Lambda^i \right\}, \tag{24}$$

where Λ^i takes the value Λ^R if the government chooses to repay and Λ^D if the government chooses to default. We assume that $\Lambda^R > \Lambda^D$. The idea is to capture that both default and abandonment of a currency peg carry a reputational cost for the government. Once the government chooses

[46] Moreover, we also find that spreads represent the most important coefficient of the rule, followed by debt. Following the rule without reacting to spreads implies an average welfare loss of -0.91% of permanent consumption, while following a rule that does not react to debt implies a loss of -0.31% of permanent consumption. Appendix B.2 has the detailed description of the equilibrium when the government follows this rule and compares the simulated moments with those of the benchmark model.

[47] Arguably, a rule in the vein of Greenspan-Guidotti may be more useful in environments with multiple equilibria, a topic that we leave for future research. See Obstfeld (1986) for a notable example of a model of self-fulfilling speculative attacks, which are triggered by the expectation that the central bank would follow an expansionary monetary policy upon the exit of a peg.

to default, this erodes the reputational level of the government and reduces the costs from abandoning the commitment to a currency peg.[48] In Appendix C.1, we present the full details of this extension of the model.

To parameterize this version of the model, we set the cost of exiting the peg under a default, Λ^D, to zero, and calibrate Λ^R so that the mean duration of a peg is 28.3 years, the average reported in Husain et al. (2005). We keep all other parameters at our benchmark calibration for fixed exchange rates.

Table 5: Twin Ds

Duration of the peg	28.3 yrs
Prob (Default)	2.2%
Prob (Default | Devaluation)	47.1%
Prob (Devaluation | Default)	100%
Prob (Devaluation | Repay)	5.0%
Mean Reserves/GDP	12.2%

Note: The reserves/GDP ratio is computed conditioning on the economy being under the fixed exchange regime. All conditional probabilities use a window $\{t, t+1\}$ around the event (devaluation, default or repayment) at date t.

Table 5 reports the key results. As the table shows, this extended model can account for the Twin Ds. At the same time, it predicts a substantial amount of reserves close to that in our baseline model.[49] The intuition for why the model predicts a similar amount of reserves as that in the baseline is that the key role of reserves is not to stabilize the economy *while in default*. Instead, the key role of reserves is to allow the government to stabilize macro fluctuations while facing increases in sovereign spreads *while in repayment*. Therefore, it follows that allowing for a devaluation while in default does not significantly alter the marginal value of reserve holdings.

7.2 Alternative Nominal Rigidities

Our baseline model features downward wage rigidities. We now examine how the basic results change when the source of nominal rigidities is prices rather than wages and when this rigidity is symmetric. In addition, we also allow households to supply labor elastically. Preferences are

[48] This feature is consistent with the theoretical analysis in Chari and Kehoe (2003).

[49] In particular, the probability of a devaluation conditional on default is 20 times larger than the probability of a devaluation conditional on repayment. Given $\Lambda^D = 0$, the government abandons the peg with probability 1 when it defaults (given our parameterization). However, we have verified that devaluations are much more common under default when there is a less extreme asymmetry between the costs of abandoning a peg under repayment and default.

now given by

$$U(c_t, h_t) = \frac{c_t^{1-\gamma}-1}{1-\gamma} + \varphi \frac{\left(\bar{h}-h_t\right)^{1-1/\phi}-1}{1-1/\phi},$$

We assume that nominal prices of non-tradables cannot fall below $\underline{P}^N$ and cannot exceed $\overline{P}^N$, with $\overline{P}^N > \underline{P}^N$. The flexible price allocation is nested in this framework as we increase $\overline{P}^N$ and decrease $\underline{P}^N$ sufficiently. We adopt the standard rationing assumption. When the price that clears the goods market is inconsistent with the bounds, we assume that firms are off their optimal supply schedule and employ labor to satisfy the household demand for non-tradables. Meanwhile, households are always on their optimal labor supply, implying that the wage must be such that $(W_t/P_t^N)U_N(c_t^T, c_t^N) = U_h$. When the price that clears the goods market is within the bounds, firms are on their optimal supply schedule, and so we arrive at $F'(h_t)U_N(c_t^T, c_t^N) = U_h$, which corresponds to the efficient allocation. As shown formally in Appendix C.2, in this framework, we arrive at an expression for equilibrium employment that echoes the one obtained under wage rigidities, given by equation (12).[50]

Our quantitative results show that the government portfolio under a fixed exchange rate is very similar under sticky prices and sticky wages. We set a Frisch elasticity of labor supply equal to 1, in the range of the empirical estimates, and keep all parameters at their baseline values for the fixed exchange rate, except for $\overline{W}$ (which we set equal to zero) and for $\bar{h}$ (which we renormalize to 3 in order to have an efficient allocation that simultaneously features $h = 1$ and has leisure totalling two-thirds of the time).[51] To gauge the relative importance of price versus wage rigidity and downward versus symmetric rigidity, we proceed in two steps. First, setting $\underline{P}^N$ to target a 2 percentage point drop in employment around defaults, in line with the baseline calibration, we find that the average level of reserve holdings is 15% of GDP, only 1% point lower than the level under wage rigidity.[52] We then set $\overline{P}^N$ so that the frequency with which the downward and upward price rigidity bind is the same. In this case, we find that reserves decrease by 3 percentage points of GDP relative to the baseline with wage rigidity. While reserves remain substantially higher than under a flexible exchange rate, it is interesting to note that a more symmetric rigidity tends to decrease somewhat the desire to hold reserves. The logic is that in a situation in which the upward price rigidity binds, the economy faces overheating, and reserves

[50]The expression is detailed in eq. (C.10) in the appendix. Notice that under $\alpha = 1$, we have $P_t^N = W_t$, and for a given c^T, we obtain an identical level of employment under sticky wages and sticky prices.

[51]We parameterize φ to deliver $h = 1$ in the efficient allocation. Given our baseline assumption on separable preferences between tradables and non-tradables in the calibration, this is achieved by setting $\varphi = (1-\omega)\alpha(\bar{h}-1)^{-1/\phi}$. Then, since the Frisch elasticity can be written as $\left(\frac{\phi(\bar{h}-h)}{h}\right)$, the value of ϕ that delivers a unitary Frisch elasticity is 0.5. See Appendix C.2 for more details.

[52]Notice that under flexible exchange rates, the government portfolio remains identical to our baseline model.

become less valuable.[53,54]

7.3 Inflation Targeting

In this section, we explore a variant to our model, in which the government follows an inflation targeting regime. The goal is to study a regime under which the government has a flexible exchange rate, with $\Phi = 0$, but is unable to completely implement full employment because of other monetary policy objectives.

We assume the government targets the CPI price level.[55] In this regime, the government has the ability to use monetary policy to stabilize macroeconomic fluctuations, but only to some extent. To see this, consider a shock worsening borrowing conditions today. If the government were to keep the exchange rate constant, there would be deflation as the price of non-tradables would fall. By partially depreciating the exchange rate, the government can reduce unemployment while still delivering the targeted inflation.

Table C.2 in the appendix presents the simulation moments for the model under inflation targeting and compares them with the moments from the baseline version.[56] As the third column shows, the government still accumulates about 12% of GDP in reserves under an inflation targeting regime. The key finding is that the macroeconomic stabilization role for international reserves remains strong under an inflation targeting regime.

7.4 Other Extensions/Sensitivity

Reserves during default. The portfolio of reserves and debt matters for two reasons. One is that it transfers resources across repayment states, from states with high bond prices to states with low bond prices. Second, it transfers resources to default states (since the government keeps the reserves and can adjust them freely). We argue that the second channel is quantitatively

[53] The welfare gains from reserves that we analyzed in section 6.1 are quantitatively very similar in this version of the model with sticky prices.

[54] Another formulation of the nominal rigidity that we do not explore here would link the bound on wages (or prices) to the previous market value (e.g., $W(s_t) \geq \tilde{\gamma} W(s_{t-1})$, as in Schmitt-Grohé and Uribe, 2016). We expect that compared with our formulation, a formulation like this would deliver more amplification but less persistence. There would be more amplification because in good times, there is a rise in market wages, which in turn leads to a stricter rigidity in the following period. On the other hand, we expect less persistence because after a negative shock, one would have a decline in wages over time, to the extent that $\tilde{\gamma} < 1$. These two elements are likely to push reserves in opposite directions and so, in principle, it is ambiguous whether the desire to hold reserves increases or decreases.

[55] That is, given a target $\hat{P}$, the government needs to set an exchange rate policy consistent with $\hat{P} = \left(\omega^{\frac{1}{1+\mu}} \left(P^T\right)^{\frac{\mu}{1+\mu}} + (1-\omega)^{\frac{1}{1+\mu}} \left(P^N\right)^{\frac{\mu}{1+\mu}}\right)^{\frac{1+\mu}{\mu}}$. It is worth highlighting that in this environment, a better target for monetary policy is non-tradable inflation. For various reasons, which we do not model explicitly in the paper, governments in practice target CPI inflation.

[56] We parameterize $\hat{P}$ so that it is equal to the mean value observed in the simulations of the flexible-wage economy. All other parameters are kept at their benchmark calibration values.

negligible. In particular, we modified our baseline model, imposing the restriction that when the government defaults it has to choose reserves such that $q_a a' = a$.[57] Keeping our benchmark calibration (for both fixed and flexible exchange rates), we find that mean reserves to GDP are 16.2% under the fixed regime and 7.6% under the flexible one. These ratios are very close to the ones in Table 2 from our baseline model.

Currency mix. Our model assumes that both debt and reserves are denominated in foreign currency. Under a fixed exchange rate, this is of course without loss of generality. However, under a flexible exchange rate, if debt is in domestic currency, a portfolio of local currency debt and foreign reserves can achieve additional spanning due to currency movements, even absent any default considerations (see Ottonello and Perez, 2019).

A relevant question is, therefore, how the desire to accumulate international reserves would change in our model under a flexible exchange rate once we introduce this empirically relevant feature. To explore this question, we now assume that a share of the debt is in local currency (which we assume to be fixed and calibrate to match the average in the data). We consider a version of our model with an intermediate value for Φ. As mentioned above, under $\Phi = \infty$, the denomination does not matter, while under $\Phi = 0$, the government would have the incentive to inflate away entirely the local currency portion of the debt.

The description of the government problem is presented in Appendix C.4. The new dimension that emerges in the government problem is that now the government realizes that by devaluing the currency today, it can reduce the real debt payments to investors. Moreover, ex ante, the government realizes that candidate portfolios with higher gross positions allow it to obtain ex post additional hedging benefits. While this suggests that the scope for reserves may increase, there is an "incentive effect" pushing in the opposite direction. In fact, the devaluation is ex post costly, and investors charge a higher interest rate that compensates for the future devaluation. Therefore, this effect suggests that the government may want to hold fewer gross positions than it does in our benchmark case.[58]

In our quantitative analysis, we calibrate the share of debt in local currency to 25%, which is the mean reported in Ottonello and Perez (2019). We take as a benchmark the case in which the depreciation cost Φ is set to match the average ERV in our dataset. In that economy, we

[57]That is the government has to roll over the reserves, including interests. In a related vein, Bianchi and Sosa-Padilla (2022) explore restrictions on the use of reserves as an optimal sanction imposed by international creditors to a debtor country in the context of geopolitical considerations.

[58]Ottonello and Perez (2019) analyze in detail the tradeoff between insurance and incentives in the portfolio problem. They find that the government does not accumulate reserves but instead issues debt in both local and foreign currency. Alfaro and Kanczuk (2019) analyze a different but related portfolio problem featuring debt in units of non-tradables and reserves in units of tradables. They do obtain that the government accumulates reserves in that setup. However, a key difference with our currency mix setup is that, because debt is real in their model, the government cannot dilute it by depreciating the currency.

find that the average level of reserves is 12% of GDP when the share of debt in local currency is zero. Once we allow for a currency mix in the debt denomination, we find a reserve ratio of 13.4% of GDP. Therefore, this suggests that the currency denomination of the debt does not significantly alter the reserve holdings. Moreover, notice that reserve holdings are still lower than in our model with fixed exchange rates (16%). So, while higher gross positions under a currency peg do not provide hedging through movements in the exchange rate, the fact that there is a stronger macro-stabilization motive calls for higher reserves.[59]

Elasticity parameters. In Figure C.1, we report a sensitivity exercise with respect to changes in the coefficient of relative risk aversion (γ) and in the intratemporal elasticity of substitution between tradables and non-tradables ($1/(1+\mu)$). All other parameters are left unchanged at their values in the benchmark calibration. As the figure shows, all the parameter combinations deliver substantial reserve holdings ranging between 11% and 19% of GDP for a fixed exchange rate. As expected, the higher is the degree of risk aversion, the more reserves the government wants to accumulate. The elasticity of substitution between tradables and non-tradables has a relatively modest and mixed impact on the reserve holdings.

To conclude, the results suggest that the macro-stabilization channel that we uncover is crucial for explaining the observed levels of reserves and the differences between fixed and flexible exchange rates.[60]

8 Conclusions

We provide a theory of reserve accumulation based on the interaction between macroeconomic stabilization and sovereign risk. We show that governments with limited exchange rate flexibility find it optimal to issue debt and accumulate reserves during good times and deploy them during bad times to reduce the severity of recessions. The quantitative findings suggest that the theory can account for the observed levels of reserves in emerging markets and the differences between fixed and flexible exchange rate economies that we document in the paper.

There are many interesting avenues for future research. One possibility we have not considered in this paper is the possibility of rollover crises. As Bianchi and Mondragon (2022) show, a fixed exchange rate regime is more vulnerable to defaults triggered by rollover crises. This suggests

[59] We have also considered how the predictions change when we shut down nominal rigidities. In that case,we find that that a lower degree of exchange rate flexibility leads to slightly higher reserve accumulation but the levels of reserves are much lower than in the data (e.g., under a fixed exchange rate, the government accumulates reserves that are 7% of GDP). Based on this, we conclude that nominal rigidities are crucial to account for both the level of reserves in the data and the differences between fixed and flexible exchange rates.

[60] There is of course space for further refinements of the quantitative analysis. For example, one could introduce a choice of debt maturity, in addition to reserves.

that in our model, the government portfolio should be more sensitive to the possibility of a self-fulfilling rollover crisis under a fixed exchange rate. However, while reserves do provide more liquidity to the government, reducing the debt may be a more effective way to reduce this vulnerability. This question is the subject of our current research.

Data Availability Statement

The data and programs underlying this article are available in Zenodo, at https://zenodo.org/record/7972083.

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Online Appendix to "Reserve Accumulation, Macroeconomic Stabilization, and Sovereign Risk"

A Empirical Appendix

A.1 Data Sources and Variable Definitions

Whenever possible, we take the data from the online appendix of Catão and Mano (2017). We also follow them in terms of variable definitions for debt, reserves, spreads, world interest rate, crisis years, and exchange rate regime classification. In this appendix, we provide a brief description of these variables:[61]

- **Debt:** we focus on "Total Government Debt." The sources are IMF's World Economic Outlook (International Monetary Fund, 2022b) and the World Bank's Global Development Finance databases (now superseded by the International Debt Statistics, The World Bank, 2022b).

- **Foreign exchange reserves:** these are as reported in the IMF's International Financial Statistics (International Monetary Fund, 2022a).

- **Spreads:** the main reference for emerging market spreads is the EMBI spreads (available at The World Bank, 2022a).

- **World interest rate:** the real world interest rate is computed as a GDP-weighted average of the (short-term) money market interest in all G7 countries plus Australia, deflated by CPI inflation. Money market rates, CPI and inflation, and US dollar GDP data for all countries are from the IMF's International Financial Statistics.

- **Exchange rate regime:** this is a dummy variable taking a value of 1 for countries deemed to be under a "Fix" regime and 0 otherwise. This dummy was constructed according to the IMF classification (categories "1" and "2").

- **Crisis years:** these are defined as years in which a given country experienced a "credit event." These events are defined as all the years between the initial default and full (or near full) settlement of arrears as per the definition used by Standard and Poor's.

The variables that were not readily available from Catão and Mano (2017) were obtained (or constructed) as follows:

[61] See Catão and Mano's 2017 data appendix for further details.

- **Exchange rate:** we use the official nominal exchange rate between a country i's currency and the USD from the World Bank's World Development Indicators (WDI, The World Bank, 2022c).

- **Tradable GDP:** we construct tradable GDP as the sum of agriculture value added and industry value added. Both series are expressed in constant local currency units and taken from the WDI.

Table A.1: Summary statistics

	Mean	Median
Debt/GDP	.44	.40
Reserves/GDP	.16	.16
Spread (in %)	2.9	2.0
SD(Spread) (in %)	1.7	1.4
Corr(Reserves/GDP, Spread)	-.24	-.24

A.2 Regressions

In this section, we present further robustness to the motivating empirical evidence discussed in section 5. First, Table A.2 shows that the main results reported in Table 3 hold when controlling for country fixed effects (and with clustered standard errors).

Table A.2: Panel regressions with country fixed effects

	Dependent variable: log($Reserves/y$)				
	(1)	(2)	(3)	(4)	(5)
ERV	-0.582^{*}	-0.575^{**}	-0.556^{**}	-0.502^{**}	-0.293^{*}
	(0.330)	(0.281)	(0.224)	(0.218)	(0.152)
log($Debt/y$)		0.287	0.503^{*}	0.554^{*}	0.468^{**}
		(0.230)	(0.260)	(0.285)	(0.210)
$\hat{y}$			1.316	1.145	1.216
			(1.181)	(1.227)	(0.768)
log($Spread$)				-0.062	-0.044
				(0.072)	(0.056)
r^{world}					-0.152^{***}
					(0.028)
Number of countries	22	22	22	22	22
Observations	425	411	392	382	382
R^2	0.017	0.049	0.102	0.112	0.324
F Statistic	6.781^{***}	10.007^{***}	13.957^{***}	11.259^{***}	34.086^{***}

Note: All explanatory variables are lagged one period to control for endogeneity. The cyclical component of GDP (y) is denoted by $\hat{y}$. All specifications include country fixed effects. Robust standard errors (clustered at the country level) are reported in parentheses. *p<0.1; **p<0.05; ***p<0.01.

Figure A.1 shows kernel densities of the β^{ERV} obtained from the set of regressions based on simulated data.

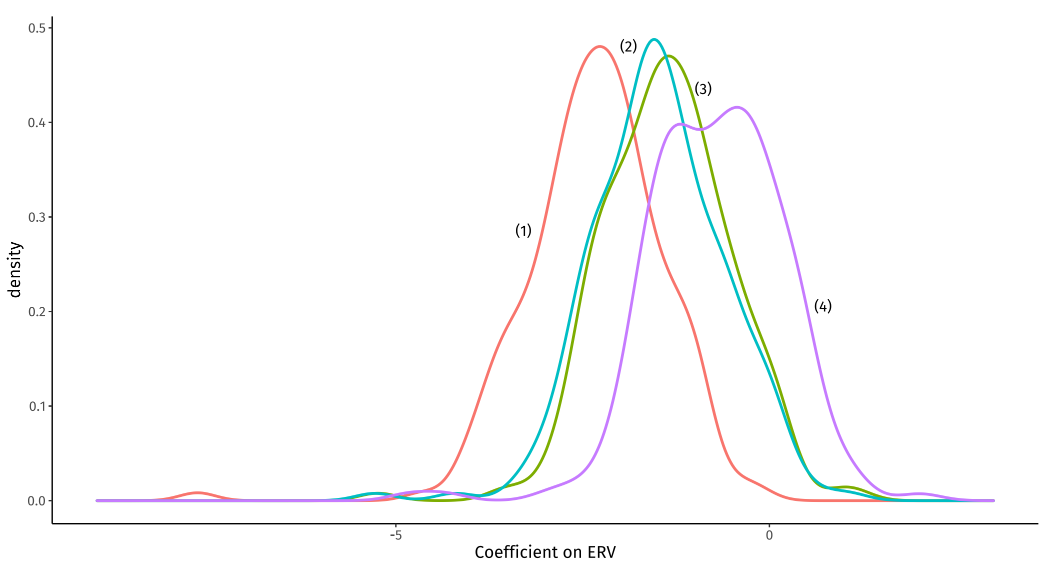


Figure A.1: Kernel densities for β^{ERV} on model simulated regressions.

B Appendix to Section 6

B.1 Commitment to Reserves

Consider the government problem at time t, starting from a given state (a, b, s), where the government can commit to a reserve accumulation policy next period, contingent on the shocks tomorrow. After the following period, we assume that the government continues to follow the Markov equilibrium strategies.

We denote by V_{t+1} the value function in the next period where the policy function for reserves was chosen in period t and by q_t^C the bond price the government faces at time t, which is now a function of the policy $\{a_{t+2}(s')\}$. We can then write the problem of the government as

$$\max_{a',b',c^T,e,h,\leq\mathcal{H}(c^T,e)\{a''(s')\}} \left\{U(c^T, F(h)) - \phi(e) + \beta\, \mathbb{E}_{s'|s}\left[V_{t+1}\left(a', b', s'; \{a''(s')\}\right)\right]\right\}, \tag{B.1}$$

subject to

$$c^T + q_a a' + \delta b \leq a + y^T + q_t^C\left(a', b', s; \{a''(s')\}\right)[b' - (1-\delta)b].$$

The first-order condition with respect to $a''(s')$ is given by

$$\beta f(s'|s) \frac{\partial\, V_{t+1}\left(a''(s'), b'', s'\right)}{\partial a''(s')} = u_T(c_t^T, c_t^N)\left(\frac{\partial q_t^C\left(a', b', s; \{a''(s')\}\right)}{\partial a''(s')}\right)[b' - (1-\delta)b] \tag{B.2}$$

for all s'. Here, we use f to denote the density function.

At time $t+1$, we have that a'' is now a state variable. The government problem is to choose $\{b'', c^T\}$ to solve

$$V_{t+1}^R\left(a', b', s'; a''\right) = \max_{b'',c^{T\prime},e',h'\leq\mathcal{H}(c^T,e)} \left\{U(c^{T\prime}, F(h')) - \Phi(e') + \beta\, \mathbb{E}_{s''|s'}\left[V\left(a'', b'', s''\right)\right]\right\}, \tag{B.3}$$

subject to

$$c^{T\prime} + q_a a'' + \delta b' \leq a' + y^{T\prime} + q\left(a'', b'', s'\right)[b'' - (1-\delta)b'].$$

Notice above that by the assumption of one-period commitment, V and q denote the continuation value and the price schedule in the Markov perfect equilibrium.

Assuming that the wage rigidity does not bind for simplicity. Under repayment, the envelope condition with respect to a'' is[62]

$$\frac{\partial V_{t+1}^R}{\partial a''} = u_T(c_{t+1}^T, c_{t+1}^N)\left(q_a - \frac{\partial q\left(a_{t+2}, b_{t+2}, s_{t+1}\right)}{\partial a''}[b_{t+2} - (1-\delta)b_{t+1}]\right) + \beta\mathbb{E}_{t+1}\frac{\partial V\left(a_{t+2}, b_{t+2}, s_{t+2}\right)}{\partial a''}. \tag{B.4}$$

[62] We use sequential notation for b, c^T, c^N to more easily compare the first-order conditions.

Replacing (B.4) in (B.2), we obtain

$$\beta f(s'|s)\left[u_T(c^T_{t+1},c^N_{t+1})\left(q_a-\frac{\partial q\left(a_{t+2},b_{t+2},s_{t+1}\right)}{\partial a''}(b_{t+2}-(1-\delta)b_{t+1})\right)+\beta\mathbb{E}_{t+1}\frac{\partial V\left(a_{t+2},b_{t+2},s_{t+2}\right)}{\partial a''}\right]=$$
$$u_T(c^T_t,c^N_t)\left(\frac{\partial q^C_t\left(a',b',s;a''(s')\right)}{\partial a''(s')}\right)(b_{t+1}-b_t(1-\delta)). \quad \text{(B.5)}$$

for all s_{t+1} in which the government repays.

Notice that in the Markov perfect equilibrium, the left-hand side of (B.5) is zero. However, when the government can commit, the sign depends on $\partial q^C_t\left(a',b',s;a''(s')\right)/\partial a''(s')$. To obtain an expression for this term, we can use that investors' optimality at time t

$$q^C_t\left(a',b',s;\{a''(s')\}\right)=\mathbb{E}_{s'|s}\left[m\left(s',s\right)\left(1-d_{t+1}\left(a',b',s';\{a''(s')\}\right)\right)\times\right.$$
$$\left.\left[\delta+(1-\delta)q\left(a''(s'),b''(a',b',s';\{a''(s')\}),s'\right)\right]\right], \quad \text{(B.6)}$$

where, with some abuse of notation, we use $b''(a',b',s';\{a''(s')\})$ to denote the value of borrowing that maximizes (B.3).

Equation (B.6) suggests that when the government promises a change in reserves in a future state s', the bond price at time t can change either because the default policy changes tomorrow or because the future price changes. Abstracting from changes in the default policy,[63] we then have that differentiating (B.6) with respect to $a''(s')$, we obtain

$$\frac{\partial q^C_t\left(a',b',s;a''(s')\right)}{\partial a''(s')}=f(s'|s)m(s',s)\left\{(1-\delta)\left[\frac{\partial q\left(a'',b'',s'\right)}{\partial a''(s')}+\frac{\partial q\left(a'',b'',s'\right)}{\partial b''}\frac{\partial b''}{\partial a''}\right]\right\} \quad \text{(B.7)}$$

for all s' where the government repays.

We can then replace (B.7) into (B.5) to obtain

$$\beta\left[u_T(c^T_{t+1},c^N_{t+1})\left(q_a-\frac{\partial q\left(a_{t+2},b_{t+2},s_{t+1}\right)}{\partial a}(b_{t+2}-(1-\delta)b_{t+1})\right)+\beta\mathbb{E}_{t+1}\frac{\partial V\left(a_{t+2},b_{t+2},s_{t+2}\right)}{\partial a}\right]=$$
$$m(s_{t+1},s_t)(1-\delta)u_T(c^T_t,c^N_t)\left(\frac{\partial q\left(a'',b'',s\right)}{\partial a''(s')}+\frac{\partial q\left(a'',b'',s\right)}{\partial b''}\frac{\partial b''}{\partial a''}\right)(b_{t+1}-b_t(1-\delta)). \quad \text{(B.8)}$$

Assuming that investors are risk neutral, $\beta=e^{-r}$, and using the definition of issuances $i_t=b_{t+1}-b_t(1-\delta)$, we obtain condition (23) in the text.

Equation (B.8) then says whether the government wants to commit to higher or lower reserves in state s' depends on the relative magnitude of two effects on the right-hand side. First, more

[63] If we assume that the government cannot commit to a reserve policy in states in which it chooses to default, this implies the default set cannot improve for a given (a',b') relative to the Markov perfect equilibrium. This is because ex post, the government maximizes the value from repayment. In this case, abstracting from changes in the default policy is natural.

reserves for given level of debt increase the bond price, an effect that calls for higher reserves. However, higher reserve accumulation in period $t+1$ imply that the government will also imply more debt to finance the accumulation of reserves. This second effect is illustrated in Figure B.2. The plot illustrates how the policy for borrowing in period $t+1$, b'', responds to higher reserve accumulation in period $t+1$ for a given value of the shocks.

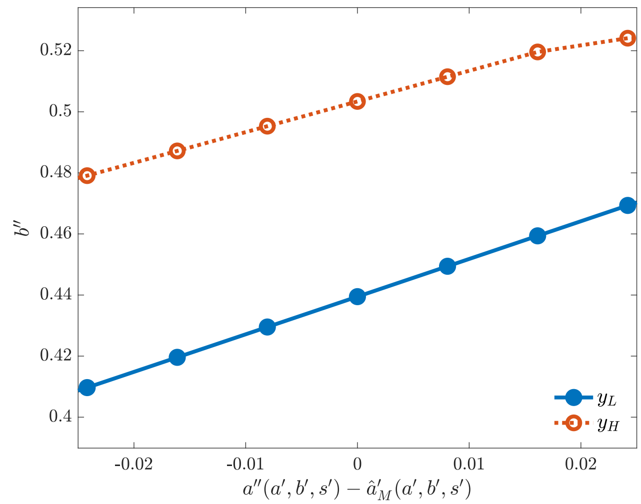


Figure B.2: Borrowing policies in $t+1$

Note: The figure shows the borrowing policies in $t+1$ for an initial portfolio equal to the mean in the simulations, assuming $\kappa = \kappa_L$ and two levels of income. Both axes are in percent of mean GDP.

Numerical implementation. Our numerical implementation of the optimal commitment assumes that the state contingency of the reserve accumulation is limited. In particular, (i) we assume that the governments sets $a''(a',b',s') = \hat{a}_M(a',b',s') + \epsilon(\kappa, y^L)$ for $y^{T'} = y^L$ and $a''(a',b',s') = \hat{a}'_M(a',b',s') + \epsilon(\kappa, y^H)$ for $y^{T'} = y^H$, where y^L (y^H) denotes income below (above) the mean; (ii) in states in which the government defaults, we set $\epsilon(s') = 0$; (iii) $\epsilon(\kappa^H, y^T) = 0$. The first condition establishes that the government can change by ϵ the amount of reserves relative to the Markov solution and that ϵ can take two values, depending on whether income is above or below the mean. The second condition precludes a commitment in the case in which the government defaults, and the third condition imposes that in a high risk premium shock, the government follows the Markov solution regardless of the income. As mentioned in the text, we find that for an initial portfolio equal to the mean in the simulations and an income level one-standard deviation below the mean, the government finds it optimal to commit to increasing the reserve accumulation by 0.8% of GDP in high income states tomorrow and to decreasing it by 1.6% of GDP in low income states tomorrow.

B.2 Government Problem with Reserve Accumulation Rule

In this section, we describe the problem of the government when it follows a reserve accumulation rule. We assume for simplicity a fixed exchange rate regime.

Every period the government has access to financial markets, the government solves

$$V_{rule}\left(b,a,s\right)=\max_{d\in\{0,1\}}\left\{(1-d)V^{R}_{rule}\left(b,a,s\right)+dV^{D}_{rule}\left(a,s\right)\right\},\tag{B.9}$$

where

$$\begin{aligned}V^{R}_{rule}\left(b,a,s\right)=&\max_{b',e,h\le\mathcal{H}(c^T,e),c^T}\left\{u(c^T,F(h))-\Phi(e)+\beta\mathbb{E}_{s'|s}\left[V_{rule}\left(b',a',s'\right)\right]\right\}\\&\text{subject to}\\c^T+q_a a'+\delta b=&\ a+y^T+q\left(b',a',\ y^T\right)\left(b'-(1-\delta)b\right),\\a'=&\ \beta^R_0+\beta^R_{debt}\,b+\beta^R_{spr}\,spread+\beta^R_{res}\,a+\beta^R_y\,y^T\,,\end{aligned}\tag{B.10}$$

for given coefficients β^R. In the rule for a', the spread is given by the difference between the constant yield to maturity implied in the bond price and the risk free rate.[64]

The value of default is given by

$$\begin{aligned}V^{D}_{rule}\left(a,s\right)=&\max_{c^T,e,h\le\mathcal{H}(c^T,e)}\left\{u\left(c^T,F(h)\right)-\psi_d\left(y^T\right)-\Phi(e)+\beta\mathbb{E}_{s'|s}\left[V_{rule}\left(0,a',s'\right)\right]\right\}\\&\text{subject to}\\c^T+q_a a'=&\ y^T+a,\\a'=&\ \beta^D_0+\beta^D_{res}\,a+\beta^D_y\,y^T\,,\end{aligned}\tag{B.11}$$

where β^D are the rule coefficients under default.

Given the optimal decision to default, we have a bond price schedule that must satisfy

$$q\left(b',a',s\right)=\mathbb{E}_{s'|s}\left\{m\left(s',s\right)\left[1-\hat{d}(b',a',s')\right]\left[\delta+(1-\delta)q\left(b'',a'',s'\right)\right]\right\},\tag{B.12}$$

and an analogous definition of Markov perfect equilibrium where the government optimization problem solves the problems above.

Table B.1 presents the results of the economy under the rule for reserve accumulation. Our optimization strategy proceeds by first using simulated data from the model to estimate a re-

[64] The constant yield to maturity implied in the bond price, i_b, is the one that satisfies:

$$q_t=\sum_{j=1}^{\infty}\delta(1-\delta)^{j-1}e^{-ji_b}.$$

Finally, we have that spread $=i_b-r$.

gression from

$$a_{t+1} = \beta_0 + \beta_{debt}\, b_t + \beta_{spr}\, spread_t + \beta_{res}\, a_t + \beta_y\, y_t^T$$

under both fixed and flexible exchange rates. Using this rule, we perform a grid search over the coefficients for the case of repayment, centered on the estimated values, to maximize welfare. This exercise results in the following coefficients: $\beta_0 = 0.336,\ \beta_{debt} = 2.535,\ \beta_{spread} = -1.69, \beta_{res} = 0.422,\ \beta_y = 0.418$.

Table B.1: Key statistics under reserve accumulation rules

	Benchmark	Rules	
		Best Rule	Greenspan-Guidotti
Targeted			
Mean debt (b/y)	44	42	19
Mean r_s	2.9	2.8	2.4
Δr_s w/ risk-prem. shock	2.0	1.9	1.7
Δ UR around crises	2.0	2.0	1.8
Mean y^T/y	41	41	40
Non-Targeted			
$\sigma(c)/\sigma(y)$	1.0	1.0	1.0
$\sigma(r_s)$ (in %)	3.1	3.0	2.7
$\rho(r_s, y)$	-0.7	-0.6	-0.7
$\rho(c, y)$	1.0	1.0	1.0
Mean Reserves (a/y)	16	15	6
Mean Reserves/Debt (a/b)	37	38	31
$\rho(a/y, r_s)$	-0.4	-0.7	0.5
Reserves/S.T. liabilities	112	139	100
Welfare gain (vs. No-Reserves)	0.18	0.07	-0.22

Note We assume the same parameters as in our baseline calibration. The Greenspan-Guidotti rule describes a rule where $a_t = \delta b_t$ for all t.

C Appendix to Section 7

C.1 Devaluations and Default

In this section, we present the details of the government problem faced by a government that has the option to abandon a fixed exchange rate, as described in Section 7.1.

We denote by $\bar{e}$ the fixed value for the exchange rate. Recall that the value of a government that has the option to abandon a currency peg is denoted by V^o and is given by (24). The value from choosing to stay in the currency peg is given by

$$V_{stay}(a,b,s) = \max_{d\in\{0,1\}} \left\{(1-d)\,V^R_{stay}(a,b,s) + d\,V^D_{stay}(a,s)\right\}\ , \tag{C.1}$$

where

$$\begin{aligned} V^R_{stay}(a,b,s) = \max_{a',b',h,c^T} &\left\{u(c^T,F(h)) + \beta\,\mathbb{E}_{s'|s}\left[V^o(a',b',s')\right]\right\} \\ &\text{subject to} \\ c^T + q_a a' + \delta b = a + y^T &+ q_{stay}\left(a',b',\ y^T\right)[b'-(1-\delta)b],\ \text{and} \\ h \leq \mathcal{H}&(c^T;\bar{e}); \end{aligned} \tag{C.2}$$

and

$$\begin{aligned} V^D_{stay}(a,s) = \max_{c^T,h,a'} &\left\{u\left(c^T,F(h)\right) - \psi_d\left(y^T\right) + \beta\,\mathbb{E}_{s'|s}\left[V^o(0,a',s')\right]\right\} \\ &\text{subject to} \\ c^T + q_a a' &= y^T + a,\ \text{and} \\ h &\leq \mathcal{H}(c^T,\bar{e}). \end{aligned} \tag{C.3}$$

Notice that the continuation values in (C.2) and (C.3) reflect that the government has the option to leave in the future.

As argued above, under a flexible exchange rate regime, the government could choose a depreciation large enough to ensure full employment, which would be the optimal policy. Therefore, in what follows, the value functions for the case of abandoning the peg (which implies flexible exchange rates going forward) will be written imposing that $h=\bar{h}$:

$$V_{flex}(a,b,s) = \max_{d\in\{0,1\}} \left\{(1-d)\,V^R_{flex}(a,b,s) + d\,V^D_{flex}(a,s)\right\}\ ; \tag{C.4}$$

$$V^{R}_{flex}(a,b,s)=\max_{a',b',c^T}\left\{u(c^T,F(\bar{h}))+\beta\,\mathbb{E}_{s'|s}\left[V_{flex}(a',b',s')\right]\right\} \tag{C.5}$$
$$\text{s.t.:}\quad c^T+q_a a'+\delta b=a+y^T+q_{flex}\left(a',b',\ y^T\right)\left[b'-(1-\delta)b\right].$$

$$V^{D}_{flex}(a,s)=\max_{c^T,a'}\left\{u\left(c^T,F(\bar{h})\right)-\psi_d\left(y^T\right)+\beta\,\mathbb{E}_{s'|s}\left[V_{flex}(0,a',s')\right]\right\} \tag{C.6}$$
$$\text{s.t.:}\quad c^T+q_a a'=y^T+a$$

The bond prices are given by

$$\begin{aligned} q_{stay}(a',b',s)=\mathbb{E}_{s'|s}\Big\{&m(s',s)\left[1-\mathcal{X}(b',a',s')\right]\left(1-d'_{stay}\right)\left[\delta+(1-\delta)q_{stay}\left(b''_{stay},a''_{stay},s'\right)\right]+\\ &m(s',s)\,\mathcal{X}(b',a',s')\left(1-d'_{flex}\right)\left[\delta+(1-\delta)q_{flex}\left(b''_{flex},a''_{flex},s'\right)\right]\Big\} \end{aligned} \tag{C.7}$$

and

$$q_{flex}(a',b',s)=\mathbb{E}_{s'|s}\Big\{m(s',s)\left(1-d'_{flex}\right)\left[\delta+(1-\delta)q_{exit}\left(b''_{flex},a''_{flex},s'\right)\right]\Big\}, \tag{C.8}$$

where $b''_i=\hat{b}_i(a',b',s')$, $a''_i=\hat{a}_i(a',b',s')$, and $d'_i=\hat{d}_i(a',b',s')$ are the borrowing, reserves and default decisions for $i\in\{stay,\ flex\}$, respectively.

The variable $\mathcal{X}$ represents the decision to exit a peg and is defined as

$$\mathcal{X}(a,b,s)=\begin{cases}1, & \text{if } V_{flex}(a,b,s)-\Lambda(a,b,s)>V_{stay}(a,b,s)\\ 0, & \text{otherwise .}\end{cases}$$

C.2 Alternative Nominal Rigidities

Rearranging the households' optimality for consumption (4), we arrive at the following expression for non-tradable consumption:

$$c^N_t=\left(\frac{1-\omega}{\omega}\frac{e_t}{P^N}\right)^{\frac{1}{1+\mu}}c^T_t. \tag{C.9}$$

Market clearing for non-tradables implies that $h^\alpha=c^N$. Then, we obtain that the amount of employment consisitent with household demand for consumption is given by

$$h_t=\left[\left(\frac{1-\omega}{\omega}\frac{e}{P^N}\right)^{\frac{1}{1+\mu}}c^T\right]^{\frac{1}{\alpha}} \tag{C.10}$$

We modify the nominal rigidities and now place them on the price of non-tradable goods. In particular, we assume that

$$\underline{P}^N \leq P^N \leq \overline{P}^N \tag{C.11}$$

Therefore, the problem the government solves under repayment is given by

$$V^R(a,b,s) = \max_{a',b',h,c^T,e} \left\{ u(c^T, F(h)) + \varphi \frac{\left(\bar{h}-h_t\right)^{1-1/\phi} - 1}{1-1/\phi} - \Phi(e) + \beta\, \mathbb{E}_{s'|s}\left[V\left(a',b',s'\right)\right] \right\} \tag{C.12}$$

subject to

$$c^T + \delta b + q_a a' = a + y^T + q(a',b',s)(b' - (1-\delta)b),$$

$$h \leq \left[\left(\frac{1-\omega}{\omega} \frac{e}{\underline{P}^N} \right)^{\frac{1}{1+\mu}} c^T \right]^{\frac{1}{\alpha}},$$

$$h \geq \left[\left(\frac{1-\omega}{\omega} \frac{e}{\overline{P}^N} \right)^{\frac{1}{1+\mu}} c^T \right]^{\frac{1}{\alpha}}.$$

The problem under default is analogous:

$$V^D(a,b,s) = \max_{a'h,c^T,e} \left\{ u(c^T, F(h)) + \varphi \frac{\left(\bar{h}-h_t\right)^{1-1/\phi} - 1}{1-1/\phi} - \Phi(e) + \beta\, \mathbb{E}_{s'|s}\left[V\left(a',0,s'\right)\right] \right\} \tag{C.13}$$

subject to

$$c^T + q_a a' = a + y^T,$$

$$h \leq \left[\left(\frac{1-\omega}{\omega} \frac{e}{\underline{P}^N} \right)^{\frac{1}{1+\mu}} c^T \right]^{\frac{1}{\alpha}},$$

$$h \geq \left[\left(\frac{1-\omega}{\omega} \frac{e}{\overline{P}^N} \right)^{\frac{1}{1+\mu}} c^T \right]^{\frac{1}{\alpha}}.$$

Table C.1 presents the results. As explained in the body of the paper, we consider the fixed exchange rate economy. We renormalize $\bar{h}$ to 3 and set φ so that the equilibrium labor in the flexible exchange economy is equal to 1, so that households work one-thrid of the time. This amounts to setting:

$$\varphi = (1-\omega)\,\alpha\,(\bar{h}-1)^{-1/\phi}.$$

In this way, the flexible exchange rate case has the same allocations as in the benchmark model. Given the above preferences, the Frisch labor supply elasticity is given by $\left(\frac{\phi(\bar{h}-h)}{h}\right)$. We set $\phi = 0.5$ so that, focusing on the efficient level of labor, the Frisch elasticity is 1, a value commonly used in the literature. All other parameters are kept at their benchmark values, except for the bounds on P^N and $\overline{W}$ (which we set equal to zero) . Following the description in Section 7.2, the calibration strategy delivers $\underline{P}^N = 1.112$ and $\overline{P}^N = 2.396$. In the last column, the upward and downward price rigidities bind 10% of the time.

Table C.1: Wage vs. price rigidities

	Wage Rigidity (Benchmark)	Price rigidity $\underline{P}^N \leq P^N$	$\underline{P}^N \leq P^N \leq \overline{P}^N$
Mean debt (b/y)	44	49	50
Mean r_s	2.9	2.7	2.7
Δr_s w/ risk-prem. shock	2.0	2.1	2.2
Mean y^T/y	41	42	43
Δ UR around crises	2.0	-.-	-.-
Δ Employment around crises	-.-	-2.0	-2.6
Mean Reserves (a/y)	16	15	13

Note: All parameters are kept at their benchmark values (except for $\overline{W}$ which we set to zero). In the first column for price rigidity, we set $\underline{P}^N$ such that the decrease in the employment rate around default events is 2%. In the last column, we keep $\underline{P}^N$ unchanged and set $\overline{P}^N$ such that both price rigidties bind 10% of the time.

C.3 Inflation Targeting

Table C.2: Model comparisons with inflation targeting

	Data	Model: Fixed Exchange Rate	Model: Inflation Targeting
Targeted			
Mean debt (b/y)	44	44	51
Mean r_s	2.9	2.9	2.8
Δr_s w/ risk-prem. shock	2.0	2.0	2.1
Δ UR around crises	2.0	2.0	0.5
Mean y^T/y	41	41	42
Non-Targeted			
$\sigma(c)/\sigma(y)$	1.1	1.0	1.1
$\sigma(r_s)$ (in %)	1.7	3.1	3.0
$\rho(r_s, y)$	-0.3	-0.7	-0.7
$\rho(c, y)$	0.8	1.0	1.0
Mean Reserves (a/y)	16	16	12
Mean Reserves/Debt (a/b)	35	37	23
$\rho(a/y, r_s)$	-0.2	-0.4	-0.3

C.4 Currency Mix

In this section, we study a version of the model in which debt is denominated in a bundle of domestic and foreign currencies. This feature is, of course, only relevant for the flexible exchange rate economy. We assume the government faces the devaluation cost Φ. Assume the government issues in period $t-1$ this bond, and then in period t, it issues again, only in foreign currency.

Assume that a fraction ν is in domestic currency. In period t, the budget constraint is

$$c_t^T = y^T - \delta\left(\frac{\nu b_t}{e_t} + (1-\nu)b_t\right) + q(a_{t+1}, b_{t+1}, s)\left(b_{t+1} - b_t(1-\delta)\right). \tag{C.14}$$

The government's problem is now given by

$$V^R(a,b,s) = \max_{a',b',h,c^T,e} \left\{u(c_t^T, c_t^N)) - \Phi(e_t) + \beta\,\mathbb{E}_{s'|s}\left[V\left(a',b',s'\right)\right]\right\} \tag{C.15}$$

$$\text{subject to}$$

$$\begin{aligned} c^T + q_a a' + \left[\frac{\nu}{e} + (1-\nu)\right]\delta b =& a + y^T + q\left(a', b',\ y^T\right)[b' - (1-\delta)b], \\ h \le& \mathcal{H}(c^T, e). \end{aligned}$$

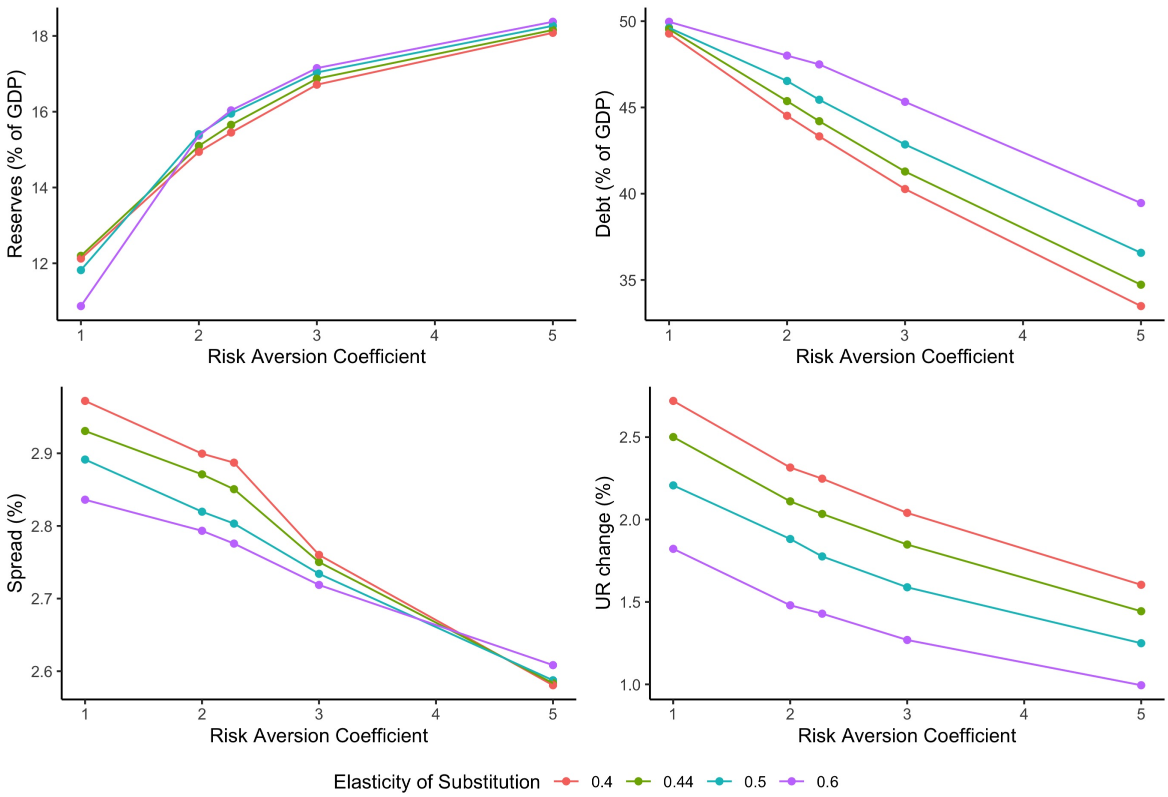
From the investors' side, optimality implies

$$q_t = \mathbb{E}_t\left\{m_{t,t+1}(1-\hat{d}_{t+1})\left[\frac{\nu}{e_{t+1}} + (1-\nu) + (1-\delta)q_{t+1}\right]\right\}, \tag{C.16}$$

and so now investors need to anticipate the optimal exchange rate policy of the government in future periods.

C.5 Other Extensions/ Sensitivity

Figure C.1: Sensitivity to changes in the elasticities.



Note: The figure presents a sensitivity exercise with respect to risk aversion and the elasticity of substitution across goods. The figure shows reserves/GDP ratio, in addition to three moments we target in the calibration: the debt/GDP ratio, the spread, and the change in the unemployment rate around default events.

D Additional Figures

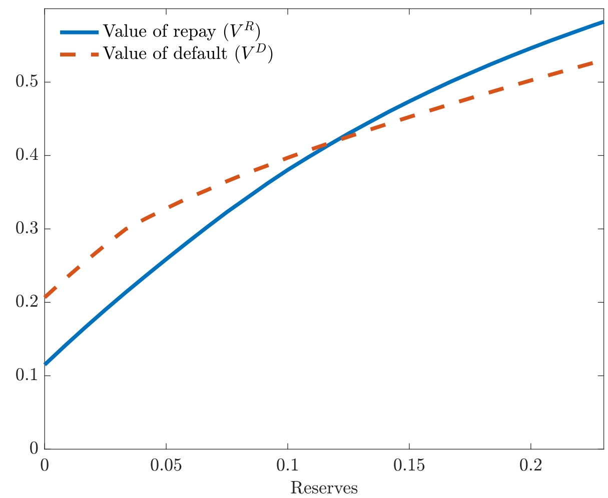


Figure D.1: Value functions for different levels of reserves.

Note: The initial states are such that debt equals 20% of GDP, tradable income is two standard deviations below its mean, and $\kappa = 0$ (no risk premium shock).

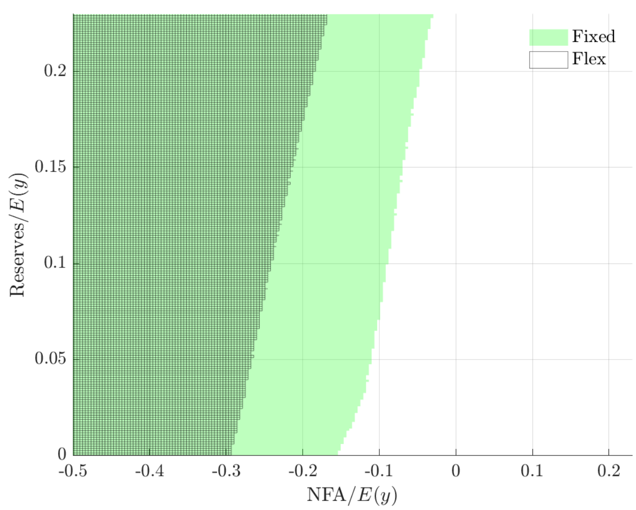


Figure D.2: Default regions for given reserves and NFA positions.

Note: The area with a black mesh is the default set for the economy under a flexible exchange rate regime, and the area with a green shading is the default set for the economy under a fixed exchange rate regime.

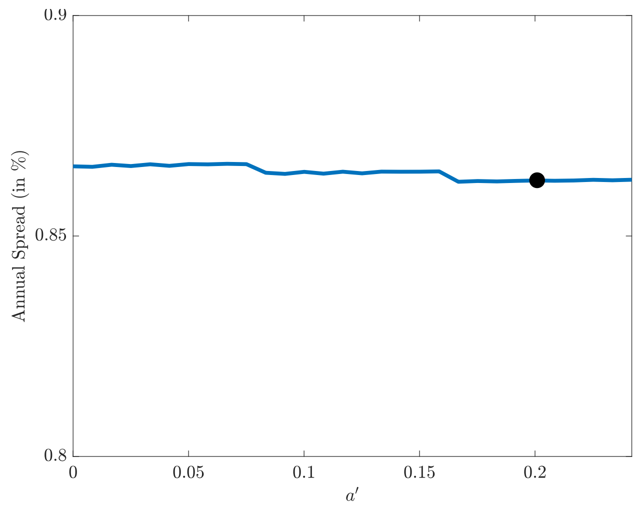


Figure D.3: A case in which spreads do not increase in gross positions under a fixed exchange rate

Note: The figure shows the spreads for different candidate levels of debt-financed reserves. The black dot denotes the optimal level of a'. The figure is computed ensuring that candidate portfolios keep c^T fixed at 80% of mean tradable income, and it assumes initial reserves equal 13% of GDP, initial debt is 49% of GDP, tradable income is one standard deviation above its mean and there is no risk premium shock.