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Borrowing in the Shadow of China[†]

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Abstract

Emerging economies' debt to China has considerably risen. We study the impact that such borrowing from China–which is almost completely official debt–has on the equilibrium quantities and prices for *marketable* sovereign debt. We do so by using a standard sovereign debt model with long-term debt augmented with subsidized Chinese loans that are subject to rollover risk. These model predictions are consistent with our panel-data evidence from emerging and low-income economies. Following a positive inflow from China, the borrower re-balances its debt portfolio away from market debt, thereby enjoying lower spreads. On the other hand, when facing a capital outflow vis-à-vis China, the economy taps international debt markets, levers up on defaultable market debt, and ends up paying higher and more volatile spreads. Finally, the model informs the welfare gains from having access to such rollover-prone Chinese loans and the consequences of counterfactual geopolitical shocks in the form of retrenchment risk vis-à-vis China.

KEYWORDS: Sovereign Debt, Defaults, Chinese Overseas Lending.
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1 Introduction

Recent contributions starting with the seminal work of Horn, Reinhart, and Trebesch (2021) document that Chinese lending to emerging economies is (i) large, (ii) non-marketable, and (iii) noisy (data is scarce and terms are opaque). Is having such debt vis-à-vis China a good or a bad signal for private international investors? Does it mean the country has a "trusted partner" on whom to rely during bad times? Or is it an additional vulnerability? We study the impact that borrowing from China (which is almost completely official debt) has on the equilibrium quantities and prices for *marketable* sovereign debt.

Using data on emerging and developing economies, we have suggestive evidence on a positive and significant "China debt restructuring event" premium: on average, after a Chinese-debt restructuring episode countries face larger spreads on their market debt (i.e. less favorable borrowing terms) in the order of 300 basis points. We also uncover a significant effect on market borrowing: a "China funding event" is associated with a market debt deleveraging, in the order of 10 to 20 percent. Moreover, after a Chinese funding event, a borrowing country's international sovereign bond yields fall by 80 to 100 basis points.

We use a standard sovereign debt model with long term debt to rationalize the above mentioned facts. We model Chinese official lending as non-defaultable subsidized debt subject to rollover risk as it can become due at a random date. We find that following a positive inflow from China our model economy chooses to re-balance its debt portfolio by deleveraging from market debt. In the process it pays lower spreads and faces less volatile consumption. On the other hand, when facing a capital outflow vis-à-vis China, the economy taps private markets more heavily, levers up on defaultable debt, and ends paying higher and more volatile spreads in equilibrium. Most of these model dynamics are in line with our empirical findings.

These findings suggest that the gains from having access to Chinese loans may be dampened by the interaction between endogenous market debt issuance, default risk, and rollover risk. In fact, we find that welfare gains from Chinese loans are highly state-dependent.

We then extend our model to allow for the possibility of an exogenous 'retrenchment shock,' by which we aim to capture the varying international developments that may lead China to stop lending to a certain country (or block of countries). We use this extended model to compute the ex-ante optimal level of exposure to Chinese funding and find that it is increasing in the market indebtedness of the country and decreasing in the probability of a Chinese retrenchment. In an increasingly uncertain geopolitical context, these results are a cautionary tale for emerging economies that have volatile political environments and which have so far relied heavily on Chinese official lending.

Related literature. The official reporting of China's international financing terms and arrangements can be characterized as being vague and sporadic. A wide array of literature shed light on particular features of China's massive lending. Horn et al. (2021) construct a comprehensive dataset to identify that China lent $1.5 trillion to more than 150 countries worldwide which previously were unidentified by official sources. To investigate the same issue, Morris, Parks, and Gardner (2020) investigate this issue on AidData to find that China's lending terms has some degree of concessionality. Bräutigam and Gallagher (2014) bring an evidence of resource-secured financing from China to African and Latin American countries. The research also focuses on the effects of the funding inflows on the economies of recipient countries. Onjala (2018) and Hurley, Morris, and Portelance (2019) identify that getting funding from China puts countries under the risk of debt distress. Bandiera and Tsiropoulos (2020) assess the debt sustainability of countries which have China's infrastructure financing projects and find that in the medium term at least 50% of these countries will face elevated debt vulnerability. Horn et al. (2021) highlight that there should have been an additional 15 to 20 "missing" defaults in the post 2010 period that pose a great risk to debt sustainability analysis in loan recipient countries and challenge the market pricing of the sovereign risk. Mkhitaryan (2021) develops and quantitatively evaluates a sovereign debt model to explore China's overseas lending arrangements and predict the periods of unidentified defaults. In contrast, Dreher, Fuchs, Parks, Strange, and Tierney (forthcoming) show that China's funds boost the country's economic growth in the short-term and with additional funding projects the economy grows even two years after the commitment.

Our paper builds on the quantitative literature on sovereign defaults, following Eaton and Gersovitz (1981), Aguiar and Gopinath (2006), and Arellano (2008). In particular, we extend the model in Hatchondo and Martinez (2009) to allow for official, non-defaultable flows with China. These flows are sometimes positive (in which case the model behaves as if the country experienced a transitory windfall) and sometimes negative (in which case the economy faces adverse terms similar to sudden stops). Since we model long-term debt, the issue of debt dilution (Hatchondo, Martinez, and Sosa-Padilla, 2016) is key to understanding the mechanisms at play. Other papers in the literature that relate to our work are Hur and Kondo (2016), Johri, Khan, and Sosa-Padilla (2022), Bianchi, Hatchondo, and Martinez (2018), among many others.

Layout. Section 2 documents the motivating facts. Section 3 introduces the model. Section 4 explains the parametrization of the model. Section 5 presents the quantitative results, discusses the properties of the optimal policies, and studies the welfare effects of being 'under the Shadow' of China. Section 6 studies the consequences of Chinese retrenchment risk. Section 7 concludes.

2 Motivating facts

2.1 Data description

We assemble a dataset to document new facts on the effects of Chinese lending on the amount and the pricing on marketable external public debt. As noted by Horn et al. (2021), Chinese lending terms and amounts are difficult to measure. We build on their extensive work and supplement it with data sources on the amount and the timing of Chinese lending relationships from Acker, Bräutigam, and Huang (2020), Kratz, Feng, and Wright (2019), and Hurley et al. (2019).

For each country, using these data sources, we add to the yearly debt stock positions with China, indicators of events related to the Chinese loans such as new funding round or restructuring events. Horn et al. (2021) document Chinese loans between 2000 and 2017 to more than 100 emerging and developing economies. We derive Chinese funding events as large surges in the flow of funding from China. Specifically, for a country-year pair (i, t), we set CHN $\text{funding}_{i,t} = 1$ if the change in reported debt vis-à-vis China in that year is above country i's median value of positive debt changes vis-à-vis China. The median number of Chinese funding years per country is 3.[1]

We then construct measures of new market debt issuance and yields on marketable international debt using bond-level data extracted from Bloomberg. We also collect annual measures of external public debt help by bondholders from the World Bank International Debt Statistics (IDS) along with standard macroeconomic variables such as output and foreign reserves. These variables constitute our annual country-level dataset on China funding events and Chinese debt stocks, sovereign bond prices and marketable debt dynamics.

[1]While the data on Chinese debt stocks covers many countries, most developing countries do not issue foreign-currency debt in international capital markets. Restructuring events with China are also less common than funding. In fact, the countries with restructuring events in our dataset and bonds data in Bloomberg are: Angola, Côte d'Ivoire, Congo (Rep.), Ghana, Sri Lanka, Mongolia, Mozambique, Nigeria, Ukraine, Venezuela, and Zambia.

2.2 Empirical strategy

Using the annual dataset on Chinese funding events, Chinese debt stocks, and external public debt held by bondholders, we estimate the effects of Chinese lending events. To do so we estimate the following regression on our annual data:

$$\log \text{external debt}_{i,t} = \alpha + \beta \text{ CHN funding}_{i,t} + \theta \ \log \text{external debt}_{i,t-1} + \gamma \ X_{i,t} + \varepsilon_{i,t} \quad (1)$$

where i represents a country, t denotes a year, $\log \text{external debt}_{i,t}$ is the log of the external debt held by bondholders, CHN $\text{funding}_{i,t}$ indicates whether a China funding has occurred for country i in period t, and $X_{i,t}$ are additional controls including GDP growth, foreign reserves, time fixed effects, and country fixed effects. We report our estimates under various specifications in Table 1.

Furthermore, we explore more directly the effect of Chinese funding on marketable debt dynamics using sovereign bond issuance in international capital markets. We construct an issuance indicator from the Bloomberg bond-level extract and estimate a linear probability:

$$\text{new bond issuance}_{i,t} = \alpha + \beta \text{ CHN funding}_{i,t} + \theta \text{ new bond issuance}_{i,t-1} + \gamma \ X_{i,t} + \varepsilon_{i,t} \quad (2)$$

where i represents a country, t denotes a year, new bond $\text{issuance}_{i,t}$ indicates whether a long-term international foreign currency bond was issued by country i in year t, CHN $\text{funding}_{i,t}$ indicates whether a China funding has occurred for country i in period t, and $X_{i,t}$ are additional controls including GDP growth, foreign reserves, time fixed effects, and country fixed effects. We report our estimates under various specifications in Table 2.

Finally, using the yields constructed from the bond-level data, we estimate the effect of Chinese funding events on bond prices in the following regressions:

$$\text{sovereign yields}_{i,t} = \alpha + \beta \text{ CHN funding}_{i,t} + \theta \text{ sovereign yields}_{i,t-1} + \gamma \ X_{i,t} + \varepsilon_{i,t} \quad (3)$$

and

$$\text{sovereign yields}_{i,t} = \alpha + \beta \text{ CHN restructuring}_{i,t} + \theta \text{ sovereign yields}_{i,t-1} + \gamma \ X_{i,t} + \varepsilon_{i,t} \quad (4)$$

where i represents a country, t denotes a year, sovereign $\text{yields}_{i,t}$ is the average yields (yield-to-maturity) on long-term international bonds of country i in year t, CHN $\text{funding}_{i,t}$ indicates whether a China funding has occurred in period t for country i, , CHN $\text{restructuring}_{i,t}$ indicates whether a China restructuring has occurred in period t for country i, and $X_{i,t}$ are additional controls including annual GDP growth, lagged external debt, time fixed effects, and country fixed effects. We report our estimates in Table 3 and in Table 4.

2.3 Stylized facts

We document our empirical results in Tables 1 and 2 for external debt dynamics, and in Tables 3 and 4 for international bond prices. We find four main stylized facts:

1. external public debt (held by bondholders) is lower following Chinese lending events
2. international sovereign bond issuance is reduced following Chinese lending events
3. sovereign bond yields decline following Chinese lending events
4. sovereign bond yields increase sharply after Chinese debt restructuring events

Table 1: External Debt Dynamics and China Debt Events

	(1)	(2)	(3)	(4)	(5)	(6)
	log External debt held by bondholders					
	level	level	change	change	change	level
CHN funding event	-0.183***	-0.129***	-0.177***	-0.136***		
	(0.0576)	(0.0400)	(0.0618)	(0.0404)		
CHN funding event (change)					-0.159***	
					(0.0409)	
CHN funding era						-0.124
						(0.0845)
adj. R^2	0.985	0.989	0.059	0.069	0.080	0.985
N	640	583	640	583	640	640

All regressions include country fixed effects and time fixed effects. Robust standard errors in parentheses. Regressions in levels include lagged debt values as controls. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

The external debt reduction effect of Chinese lending. Let us first discuss the evidence on the effect of Chinese lending on external public debt presented in Table 1. We estimate a lagged specification in log levels of the debt Equation 1 in columns (1), (2), and (6). We also estimate a specification in log changes in columns (4), (5), and (6).

In our baseline sample, we find in column (1) that China lending events are associated with approximately 18 percent reduction in total external debt. We find similar point estimates in specifications (3) and (5) that use log changes in debt as opposed to the baseline lagged specification in levels.

In specifications (2) and (4), given that most countries experience multiple China lending events, we exclude from our sample a country's first funding event with China. The estimates of the impact on China lending on external debt reduction falls slightly at around 13 percent.

Finally, we estimate the effect of the advent of Chinese loans in specification (6). We do so using a Chinese funding $\text{era}_{i,t}$ variable that equals 1 for country i in all the years t after a her first Chinese funding. Though not statistically significant, we find that a negative debt reduction effect estimate of approximately 12 percent. We conjecture that the lack of statistical significance is partly driven by the fact that most countries got their first Chinese loans at the beginning of the sample in the early 2000s.

Altogether, we conclude that China lending events are associated with an overall reduction in the external debt of the recipient country to other bondholders.

Table 2: Market Debt Issuance and China Debt Events

	(1)	(2)	(3)	(4)	(5)	(6)
	New Bond Issuance on International Markets					
	level	level	change	change	change	level
CHN funding event	-0.105**	-0.120*	-0.167**	-0.234***		
	(0.0510)	(0.0699)	(0.0668)	(0.0891)		
CHN funding event (change)					-0.123***	
					(0.0449)	
CHN funding event era						-0.143*
						(0.0745)
adj. R^2	0.282	0.286	0.002	0.008	0.004	0.281
N	698	652	698	652	698	698

All regressions include country fixed effects and time fixed effects. Robust standard errors in parentheses. Regressions in levels include lagged outcome values as controls. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

The international bond issuance effect of Chinese funding. We further assess the effects of Chinese lending on debt dynamics using sovereign bonds issuance indicator in Table 2. We estimate a lagged specification of the debt issuance linear probability Equation 2 in columns (1), (2), and (6). We also estimate a specification in log changes in columns (4), (5), and (6). These specifications mirror the ones in Table 1 with the notable difference that we add as control the log of (lagged) outstanding external public debt held by bondholders.

In our baseline sample, we find in column (1) that China lending events are associated with approximately 10 percent reduction in the (linear) probability of issuing new long term international bonds. We find slightly larger point estimates of approximately 17 percent and 12 percent in specifications (3) and (5) when we use changes as opposed to the baseline lagged specification in levels.

In specifications (2) and (4), given that most countries experience multiple China lending events, we drop a country's first China funding event. China lending events reduce the linear probability of debt issuance by 12 percent and 23 percent respectively.

Finally, we estimate the effect of the advent of Chinese loans in specification (6). We do so using a Chinese funding era$_{i,t}$ variable that equals 1 for country i in all the years t after a her first Chinese funding. Though only marginally statistically significant, we find that a negative debt issuance effect of approximately 14 percent.

Consistent with the evidence from World Bank IDS external debt, we conclude that China lending events are also associated with an overall reduction in the probability that the recipient country will issue debt in international capital markets.

Table 3: Yields and China Funding Event

	(1)	(2)	(3)	(4)	(5)	(6)
	Long Term (10+ years) Bond Yields					
	level	level	change	change	change	level
CHN funding event	-0.852***	-0.837**	-0.996***	-0.953**		
	(0.297)	(0.414)	(0.352)	(0.480)		
CHN funding event (change)					-0.391*	
					(0.225)	
CHN funding era						-0.497
						(0.427)
adj. R^2	0.842	0.846	0.243	0.248	0.224	0.838
N	299	275	299	275	299	299

All regressions include country fixed effects and time fixed effects. Robust standard errors in parentheses. Regressions in levels include lagged outcome values as controls. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

The China lending discount. We now turn to Table 3 to document how sovereign bond yields respond to new China funding events. While the framework is similar to the one used for external debt dynamics, the sample size is smaller. This is due to the fact (i) few countries issue bonds in international capital markets, and (ii) bond-level coverage in Bloomberg is not complete for emerging and frontier economies. We use the yield-to-maturity on long term bond with 10+ years of remaining maturity as yield measure in Equation 3.

In our baseline sample, we find in column (1) that China lending events are associated with a reduction of 85 basis points in the country's sovereign bond yields. We find a slightly larger point estimates of approximately 95 basis points in specification (3) when we use changes as opposed to the baseline lagged specification in levels. We obtain a more modest and only marginally significant reduction of around 40 basis points in specification (5) where we regress changes in the funding variable on changes in yields.

In specifications (2) and (4), given that most countries experience multiple China lending events, we drop a country's first China funding event. China lending events are associated with a reduction in sovereign yields by 83 percent and 95 percent respectively.

Finally, we estimate the effect of the advent of Chinese loans in specification (6). As in the previous tables, the standard errors are large here and find that a insignificant negative debt issuance estimate of approximately 50 basis points.

Overall, these various estimates suggest that China lending events are associated with an reduction in a sizeable sovereign bond yields debt for borrowing countries.

Table 4: Yields and China Debt Restructuring

	(1)	(2)	(3)	(4)	(5)	(6)
	Long Term (10+ years) Bond Yields					
	level	level	level	level	level	level
CHN restructuring event	3.220**	3.101**				
	(1.535)	(1.524)				
Non CHN restructuring event		2.780***	3.041***			
		(0.621)	(0.683)			
CHN restructuring event (lagged)				3.544***	3.661***	
				(1.203)	(1.209)	
Non CHN restructuring event (lagged)					3.572	3.502
					(2.396)	(2.428)
CHN funding rounds	-0.523***	-0.507***	-0.466**	-0.495***	-0.456***	-0.443***
	(0.183)	(0.182)	(0.181)	(0.184)	(0.171)	(0.170)
adj. R^2	0.850	0.852	0.845	0.845	0.854	0.849
N	298	298	298	298	298	298

All regressions include country fixed effects and time fixed effects. Robust standard errors in parentheses. Regressions controls include lagged yields . * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

The China debt restructuring premium. In Table 4, we document how China restructuring events affect sovereign bond yields. We use the yield-to-maturity on long term bond with 10+ years of remaining maturity as yield measure in Equation 4. Building on the China funding event discount, we control for past Chinese funding rounds also.

It is important to note that restructuring events are less common than funding events. In fact, the China debt restructuring events with available bond yields data in our data are: Angola (2015), Côte d'Ivoire (2011), Ghana (2007, 2015), Sri Lanka (2012), Nigeria (2003), Venezuela (2015, 2016, 2017), and Zambia (2017). To further corroborate and contrast the the effect of Chinese debt restructuring on marketable debt yields, we augment our dataset with Paris Club and commercial restructuring events from the updated data of Trebesch, Papaioannou, and Das (2012) and Asonuma and Trebesch (2016).

With these sample size considerations in mind, we find that Chinese debt restructuring events are associated with a large increase of nearly 320 basis points in the country's long term sovereign bond yields in our baseline specification in column (1). Even though the estimate is significant, the standard errors are large. This is not surprising given the small number of episodes restructuring events around which we have yields data. Nonetheless, the point estimate is similar in magnitude to the estimated bond premium of 300 basis points associated with a Paris Club or commercial restructuring event in specification (3). We find an increase of similar magnitude when we include both restructuring event variables in the regression.

Given the limited number of restructuring events with bond yields data, we also use the lagged value of the restructuring event in specification (4), (5), and (6) instead of the change specification used the China funding discount regressions. We find a positive, significant, and large increase in yields of around 350 basis points following Chinese debt restructuring events. We view these various results are strong evidence that China debt restructuring events are associated with an large increase sovereign bond yields debt for borrowing countries.[2]

3 Model

Our starting point is the canonical model of sovereign default with long-term debt (e.g., Hatchondo and Martinez, 2009, 2017). The main innovation is to introduce a process for capital flows vis-a-vis China: this process captures both new Chinese funding as well as restructuring events. These "Chinese flows" are taken as given by the borrowing economy, and they will be numerically disciplined using the empirical evidence we reviewed above.

[2]It is important to note that the estimated China debt restructuring premium is similar in magnitude to the estimated effect of other restructuring events. Because our dataset is not fully balanced due to gaps in bond data coverage, our estimates are sensitive to the exclusion or inclusion of specific restructuring events. The results are particularly sensitive to the inclusion of Venezuela: it is the only country with multiple restructuring events in Venezuela and the near-complete bond data coverage. In a previous version of this paper, we found similar-but-not-so-large effects when we used monthly credit default swap data and monthly restructuring event dates.

3.1 Environment

Preferences and income process. The preferences are give by

$$\mathbb{E}_t \sum_{j=t}^{\infty} \beta^{j-t} u\left(c_j\right),$$

where $\mathbb{E}$ denotes the expectation operator, β stands for the subjective discount factor, and c_t represents domestic consumption. The utility function is strictly increasing and concave.

The economy's endowment of the single tradable good is denoted by $y \in Y \subset \mathbb{R}_{++}$. This endowment follows a Markov process.

Capital flows vis-à-vis China. We assume that the small open economy is a net borrower from China: its Chinese debt level can be either low ($b_c = L$) or high ($b_c = H$). Transitioning from low to high b_c implies a capital inflow (and transitioning from high to low implies a capital outflow). These transitions are governed by a random variable $a = \{0, 1\}$: most of the time $a = 0$ and the net flows vis-à-vis China are zero, when $a = 1$ the net flows are non-zero. These dynamics are independent of the country's default status.[3]

Market debt. The small open economy also borrows from a large pool of international investors by issuing long-duration bonds. As in Hatchondo and Martinez (2009), a bond issued in period t promises an infinite stream of coupons, which decreases at a constant rate δ. Hence, debt dynamics can be represented as follows:

$$b_{t+1} = (1-\delta)b_t + \ell_t,$$

where b_t is the initial debt level in period t, and ℓ_t is the number of long-term bonds issued in period t. The advantage of this payment structure is that it enables us to condense all future payment obligations derived from past debt issuances into a one-dimensional state variable: the payment obligations that mature in the current period, b_t. Bonds are priced in a competitive market inhabited by a large number of risk-neutral foreign investors that discount future payoffs at the risk-free rate, r.

[3] Note that we are assuming that debt obligations with China are non-defaultable. This is in line with the reported seniority of Chinese debt (Horn et al., 2021).

Defaults. The government cannot commit to future repayment (nor borrowing) decisions.[4] When the government defaults, it does so on all current and future debt obligations. This is consistent with the observed behavior of defaulting governments and it is a standard assumption in the literature.[5] A default event triggers exclusion from the debt market for a stochastic number of periods. Furthermore, income is given by $y - \phi(y)$ in every period in which the government is excluded from debt markets. Starting the first period after the default period, with a constant probability $\theta \in [0, 1]$, the government may regain access to debt markets. The government exits default without debt (a standard assumption in the literature).

Timing. The timing of events within each period is as follows. First, the government learns the economy's income and the realization of a (which controls the *net flows* vis-à-vis China). After that, the government chooses whether to default on its market debt. Before the period ends, the government may change its market debt positions, subject to the constraints imposed by its default decision.

3.2 Recursive formulation

There are four state variables: one endogenous and three exogenous. The endogenous state variable is the market debt level, b. The exogenous state variables are y (the income level), b_c (the Chinese debt level), and a (the random variable triggering non-zero Chinese net flow). Let us denote $s \equiv (a, y)$.

Let d denote the current-period default decision. We assume that d is equal to 1 if the government defaulted in the current period and is equal to 0 if it did not. Let V denote the government's value function at the beginning of a period, that is, before the default decision is made. Let V_0 denote the value function of a sovereign not in default. Let V_1 denote the value function of a sovereign in default. For any bond price function q, the function V satisfies the following functional equation:

$$V(b, b_c, s) = \max_{d \in \{0,1\}} \Big\{ dV_1(b_c, s) + (1-d)V_0(b, b_c, s) \Big\}, \tag{5}$$

[4]Thus, one may interpret this environment as a game in which the government making decisions in period t is a player who takes as given the (repayment and borrowing) strategies of other players (governments) who will decide after t.

[5]Sovereign debt contracts often contain an acceleration clause and a cross-default clause. The first clause allows creditors to call the debt they hold in case the government defaults on a payment. The cross-default clause states that a default in any government obligation constitutes a default in the contract containing that clause. These clauses imply that after a default event, future debt obligations become current.

where

$$V_0(b, b_c, s) = \max_{b'} \Big\{ u(c) + \beta \, \mathbb{E}_{s'|s} V(b', b'_c, s') \Big\}, \tag{6}$$

subject to

$$c + \kappa b = y + q(b', b'_c, s)(b' - (1 - \delta)b) + z(b_c, a) \tag{7}$$

with

$$b'_c = \begin{cases} H & \text{if } a = 1 \cap b_c = L \\ L & \text{if } a = 1 \cap b_c = H \\ b_c & \text{otherwise} \end{cases} \tag{8}$$

and

$$z(b_c, a) = b'_c - b_c = \begin{cases} H - L & \text{if } a = 1 \cap b_c = L \\ L - H & \text{if } a = 1 \cap b_c = H \\ 0 & \text{if } a = 0 \end{cases} \tag{9}$$

where κ represents size the market debt coupons and $z(b_c, a)$ in (9) represents the *net flows* vis-à-vis China. The value of default is:

$$V_1(b_c, s) = u\big(y - \phi(y) + z(b_c, a)\big) + \beta \, \mathbb{E}_{s'|s} \Big[\theta V(0, b'_c, s') + (1 - \theta) V_1(b'_c, s') \Big], \tag{10}$$

subject to (8) and (9).

The bond price satisfies the usual zero-profit condition and is given by the following functional equation:

$$q(b', b'_c, s) = e^{-r} \, \mathbb{E}_{s'|s} \left[1 - \hat{d}\left(b', b'_c, s'\right) \right] \left[\kappa + (1 - \delta) \, q \left(\hat{b}\left(b', b'_c, s'\right), b''_c, s' \right) \right], \tag{11}$$

where $\hat{d}$ and $\hat{b}$ denote the future default and borrowing rules that lenders expect the government to follow. The first term in the right-hand side of equation (11) equals the expected value of the next-period coupon payment promised in a bond. The second term equals the expected value of all other future coupon payments, which is summarized by the expected price at which the bond could be sold next period.

3.3 Equilibrium definition

A Markov Perfect Equilibrium is characterized by

1. a default rule $\hat{d}$ and a borrowing rule $\hat{b}$,

2. a bond price function q,

such that:

(a) given $\hat{d}$ and $\hat{b}$, the bond price function q is given by equation (11); and

(b) the default rule $\hat{d}$ and borrowing rule $\hat{b}$ solve the dynamic programming problem defined by equations (5)-(10), when the government can trade bonds at q.

4 Quantitative Analysis

Functional forms and stochastic processes. The utility function displays a constant coefficient of relative risk aversion, i.e.,

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma}, \text{ with } \gamma \neq 1.$$

The logarithm of the endowment follows an AR(1) process:

$$\log(y_t) = (1-\rho)\mu + \rho \log(y_{t-1}) + \varepsilon_t,$$

with $|\rho| < 1$, and $\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$. As in Chatterjee and Eyigungor (2012), we assume a quadratic loss function for income during a default episode:

$$\phi(y) = \max \left\{ y \left[\lambda_0 + \lambda_1 [y - \mathbb{E}(y)]\right], 0 \right\}.$$

We assume that $a \in \{0, 1\}$ follows an iid process such that $a = 1$ with probability π.

Calibration. Table 5 presents the parameter values in our benchmark model calibration. A period in the model refers to a year. The coefficient of relative risk aversion, the risk-free interest rate, and the discount factor β take standard values. We assume an average duration of sovereign default events of five years ($\theta = 0.20$), a number in the range of estimates reported in Gelos, Sahay, and Sandleris (2011) and Uribe and Schmitt-Grohé (2017).

The parameters governing the endowment process are estimated using real GDP data for the countries used in our empirical analysis in Section 2. We set the income process persistence and innovation volatility parameters to $\rho = 0.784$ and $\sigma_\varepsilon = 3.3\%$ using the average of cross-country estimates in our data. These values are in the range of typical

Table 5: Benchmark parameter values.

Parameter	Symbol	Value	Source
Risk aversion coefficient	γ	2.00	Standard
Risk-free rate	r	0.04	Standard
Discount factor	β	0.90	Standard
Market re-entry probability	θ	0.20	$\mathbb{E}$(exclusion) = 5 years
Debt duration	δ	0.168	Debt duration = 5 years
Bond coupon	κ	$(r+\delta)\,e^{-r}$	Risk-free bond price = e^{-r}
Income process - autocorrelation	ρ	0.784	GDP fluctuations (PWT)
Income innovations - standard deviation	σ_ϵ	0.033	GDP fluctuations (PWT)
Income process - average	μ	$-\frac{1}{2}\sigma_\epsilon^2$	$\mathbb{E}(y)=1$
Low China debt	L	0.00	Normalization
High China debt	H	0.05	Max. China debt flow = 5.0%
Rollover probability	π_a	0.10	China financing frequency
Default income cost - Intercept	λ_0	0.11	Avg. market debt = 30%
Default income cost - Slope	λ_1	0.945	Avg. spread = 3.55%

values used in studies focusing on emerging economies and low-income countries.[6]

We set $\delta = 0.168$. With this value, the risk-free duration of the debt is 5 years, which is close to the average duration found in previous literature.[7] The coupon is normalized to $\kappa = (r+\delta)e^{-r}$, which ensures that a default-free bond (with the same coupon structure as our sovereign bonds) trades at a price of e^{-r}.

The capital flows vis-à-vis China are characterized by three parameters: L, H, and π. Since we only have two levels of Chinese debt in our model, we normalize L to zero and set $H = .05$, the maximum annual change in Chinese debt stocks relative to GDP.[8] Looking at funding and restructuring events (vis-à-vis China) in our dataset we find a frequency of approximately 10% per year, and so we set $\pi = 0.10$.[9]

[6] We estimate the persistence and the volatility of the residuals from detrended annual real GDP series using the Hodrick-Prescott filter. We use real GDP data from the Penn World Table version 10.01 (Feenstra, Inklaar, and Timmer, 2015) for the countries borrowing both from China and in international capital markets.

[7] We use the Macaulay definition of duration that, with the coupon structure in this paper, is given by $D = (1+r)/(\delta+r)$, where r denotes the risk-free rate. Using a sample of 27 emerging economies, Cruces, Buscaglia, and Alonso (2002) find an average duration of 4.77 years, with a standard deviation of 1.52 years. Bai, Kim, and Mihalache (2017) report an average debt duration of 6.7 years in a panel of 11 emerging economies.

[8] $L = 0$ is a normalization in the sense that we can adjust the income process for the stream of payments associated with $L > 0$. We use two approaches to construct H: in the first approach, we compute the maximum of flows using changes in debt stocks reported in Horn et al. (2021). In the other approach, we use flows in Dreher et al. (forthcoming) constructed using micro-data on Chinese projects. These measures yield maximum inflow values of 3.8% and 5.8% of annual GDP. We set H to 5%, close to their midpoint.

[9] Given the initial rise in Chinese loans disbursements, we focus on the period after 2005 to measure the frequency of inflow and restructuring events. Inflow events occur more often than restructuring events. Depending on how we define "large" inflows, we find inflow probabilities of 5% to 15%, whereas restructuring probabilities range between 3% and 6.5%. We set the symmetric probability π to 10% and perform a robustness of our results using alternative values of π.

The parameters below the line are the ones that we set by simulating the model and matching certain moments in the data. The income cost of defaulting is governed by λ_0 and λ_1: these are calibrated to match the mean market debt (30%) and the mean spread (3.55%) in our data.

5 Results

We start by presenting the simulated moments produced by the benchmark calibration of our model and show that it is a reasonable approximation to data from low-income countries. We then study how the country's borrowing opportunities (as captured by the debt-spread menus) are affected by the presence of Chinese lending. Third, we show the typical dynamics around these funding events. Finally, we analyze the welfare implications of borrowing 'in the Shadow of China.'

5.1 Simulations

Table 6: Simulated moments.

Statistic	Baseline	Inflow	Outflow	$b'_c = H$	$b'_c = L$
Market Debt to GDP	30.06	31.14	29.04	29.25	30.9
Market Issuance to GDP	6.0	2.73	8.96	5.56	6.45
Consumption to GDP	98.95	101.04	96.55	99.27	98.62
Spread	3.55	3.06	4.0	3.34	3.76
S.D. Spread	2.22	1.8	2.47	2.05	2.36
Corr(Spread, GDP)	-0.64	-0.63	-0.67	-0.66	-0.64
P(Default t+1)	2.97	2.11	3.76	2.56	3.39
Default Frequency	2.52	2.81	2.2	2.27	2.78
S.D. Consumption/S.D. GDP	1.16	1.01	1.31	1.13	1.18

Note: Moments are computed for non-exclusion periods, except for the default frequency which uses all simulation periods. Units: percent.

The moments reported in 6 are chosen to illustrate the ability of the model to replicate distinctive business cycle properties of economies with sovereign risk as well as the dynamics emerging from the Chinese flows. The first column on this table (labeled 'Baseline') shows that our model approximates well the moments used as targets (average debt and spread levels) and is broadly consistent with non-targeted moments in the data. In both the model and data, consumption is more volatile than income and the sovereign spreads (computed as the difference in yields between the sovereign bonds and comparable default-free bonds) are volatile and countercyclical. Since we have long-term debt, it is not surprising that spreads are higher than the default frequency (which is approximately 2.5%).

The second and third columns in Table 6 report statistics conditioning on China 'funding events', i.e. conditioning on $a = 1$.[10] We can see that our model captures clear effects on both quantities and prices, reminiscent of the facts documented in section 2. First, the issuance of market debt shows that the country actively changes its portfolio when hit by an a shock. If experiencing a capital inflow, the economy chooses to rely less heavily on market debt, hence we see a decline in market issuances to roughly 3% (compared to an average of 6%). However, this decline in market debt is not one-to-one with the inflow from China: this implies that part of the Chinese inflow (coupled with the optimal issuance policy) is translating into higher consumption in those inflow periods (see also Figure 2). A similar behavior, with the opposite sign, is observed when the country undergoes a capital outflow vis-à-vis China: it taps private markets more heavily to offset the outflow but not fully – it also adjusts consumption down. Even though the average market debt issuance in these negative flow episodes is larger than the payments due to China, we see that consumption drops – this is due to the worse prices faced in these cases.[11]

With regards to sovereign spreads we see, as expected, that China outflow events are associated with higher and more volatile spreads than normal times. This premium paid during outflow events is consistent with the positive and significant effect of Chinese debt restructuring on CDS spreads (see section 2). In the case of China inflows, we see the opposite behavior: the level and the volatility of the spreads are 50 and 40 basis points lower than in normal times, respectively.

We use the one-period-ahead default probability to assess how the China funding events affect default incentives along the simulations. An outflow event is accompanied by an increase in the default probability of roughly 1%. After the typical outflow, the country ends the period with a debt level of around 38% of GDP, leaving it especially vulnerable to negative income realizations in the near future – hence, the spread and one-period-ahead default probability are close to each other. On the contrary, when the borrowing country receives an inflow from China, it borrows less from the market and its one-period-ahead default probability falls substantially (to roughly 2%). The spread, however, prices in all future default probabilities, and market participants, understanding that those funds the

[10] Recall, if $b_c = L \cap a = 1$ then the economy receives a capital inflow from China (second column of Table 6); if $b_c = H \cap a = 1$ then the economy experiences a capital outflow vis-à-vis China (third column of Table 6).

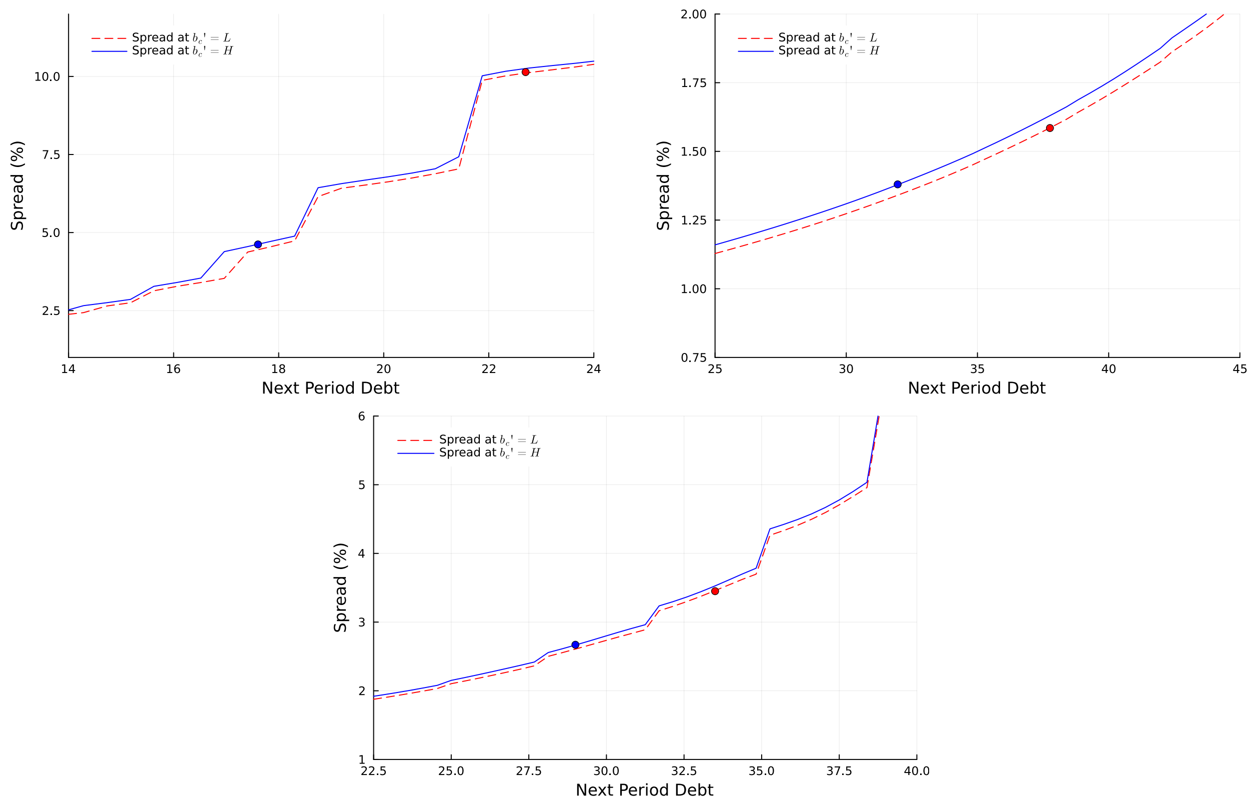
[11] Not only do consumption levels move with Chinese flows, but also the relative volatility changes: inflows make consumption less volatile while outflows make it more volatile than in normal times.

country obtained from China will need to be paid down later on, charge a spread that is 1% higher than the immediate default probability – this reflects the pricing of dilution risk (Hatchondo et al., 2016).[12]

5.2 Effects on borrowing opportunities and policy functions

Figure 1 shows the borrowing opportunities faced by the small open economy. It presents the spread–debt menus for different values of current income (below, at, and above the median) and for the two possible values of b_c', high or low.[13]

Figure 1: Spread–debt menus



Note: The top left panel shows the spread–debt menu available to the economy when current income is approximately 10 percent below mean income ($y \approx 0.9$), the top right panel is for when current income is approximately 10 percent above mean ($y \approx 1.1$), and the bottom panel is for mean income ($y = 1$). The blue solid line is for the case when end-of-period Chinese debt is high ($b_c' = H$), and the red dashed line is for when it is low ($b_c' = L$). The dots denote the optimal market debt chosen in each case.

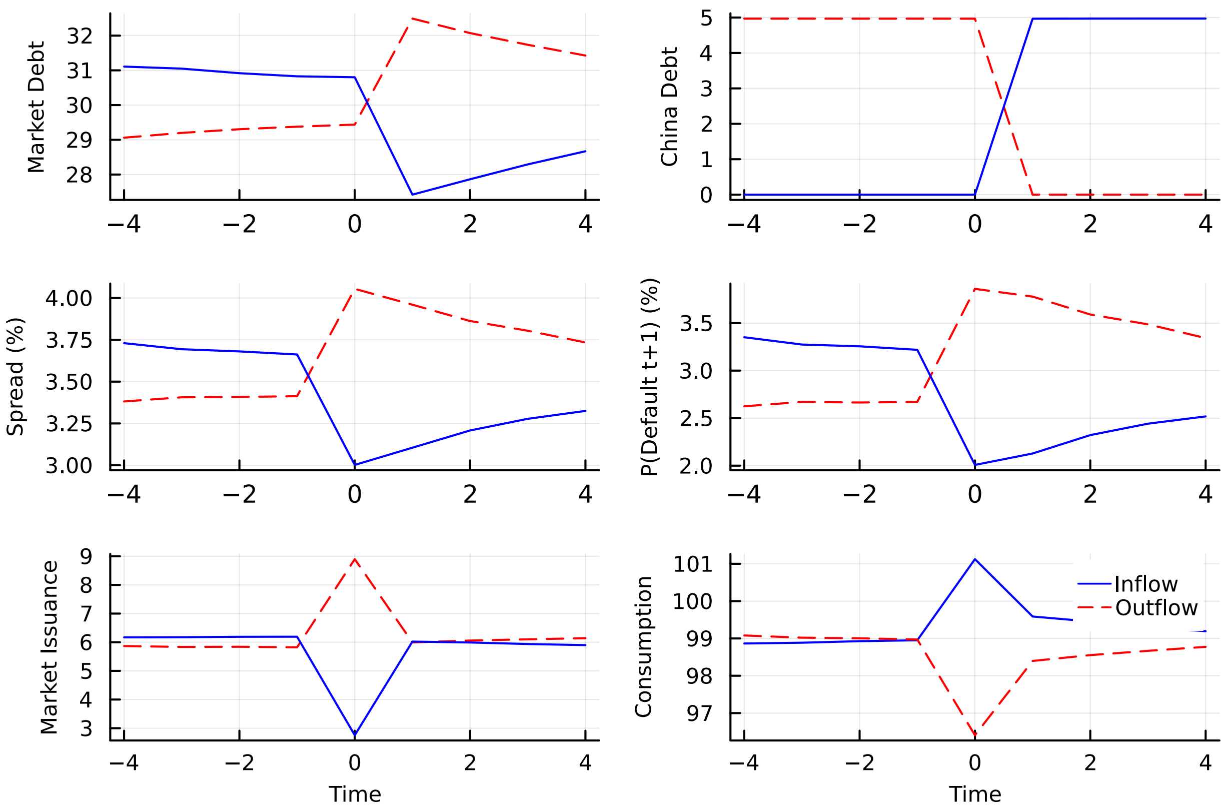
[12] The last two columns of Table 6 present the statistics of interest conditioning on the value of b_c', but without conditioning on the realization of a. We discuss them in the next subsection.

[13] Notice that what matters for the price schedules is not the current level of b_c nor period-t realization of a, but the end of period level of Chinese debt, b_c'.

The three panels of Figure 1 show that when $b'_c = H$ the country faces worse borrowing terms, i.e. for the same level of next-period market debt it needs to pay a higher spread. At the same time, the figure also shows that when the economy faces these worse price schedules, it endogenously chooses to rely less on market debt: the chosen portfolios (denoted with the dots in Figure 1) imply less market debt when $b'_c = H$. This 'deleveraging' is such that, in equilibrium, the economy pays lower spreads on market debt when the end-of-period Chinese debt is high. The last two columns of Table 6 show that this is indeed the case in the simulations. Overall, the substitution between market and Chinese debt that we see from the simulations and the spread-debt menus is in line with our findings in section 2.

5.3 Dynamics around Chinese funding events

Figure 2: Dynamics around Chinese funding events



Note: The plots show the average dynamics for 4 years before and after a Chinese funding event (conditioning on no exclusion). Blue solid lines are for inflow events ($L \to H$) and red dashed lines are for outflow events ($H \to L$).

Figure 2 shows the dynamics of our model around China funding events, both inflows (b_c moving from L to H) and outflows (b_c moving from H to L). The most clear message comes from the behavior of market debt: we see that the economy re-balances its portfolio right after these events. In the case of a capital inflow from China, the country lowers its market debt (from roughly 31% to less than 28% of mean GDP), and therefore the equilibrium spreads also decrease on impact (by roughly 75 bps). In the case of a capital outflow from China, the economy increases its market debt (roughly from 29% to slightly over 32% of mean GDP), and it ends up paying higher spreads (roughly 50 bps higher).

Another feature observed in these dynamics is a slight asymmetry between a positive and a negative China flow: after an outflow consumption decreases slightly more (in absolute terms) than what it increases after a positive flow. This implies that the economy finances more the positive shock than the negative shock: this is due to the pricing of default risk. As the economy increases debt to finance the Chinese outflow the market price for its bonds moves against it which limits the equilibrium borrowing and triggers a larger consumption adjustment. Naturally, this asymmetric response is captured in the equilibrium spreads.

After the aforementioned portfolio re-balancing the market debt issuances move back to their mean and the debt levels slowly revert back to the mean. Spreads remain elevated for longer after an outflow: this is because the economy is more vulnerable to a bad income realization in the early periods after having to borrow to repay Chinese loans (as reflected in the higher one-period-ahead default probability). After inflows, spreads increase back to mean as the debt increases and the lenders also price in the probability of an outflow coming.

5.4 Welfare

We next study the welfare implications of having access to Chinese funding. To do this we define a 'No-China' economy, which is identical to our baseline model except that it is not a possibility to receive (nor pay) funds from (to) China.

We measure welfare gains as the constant proportional change in consumption that would leave a consumer indifferent between continuing to live in the No-China economy and moving to the baseline economy (where China lending exists). The welfare gain of moving to the baseline economy (or the welfare gain of 'China financing') is given by[14]

$$\left[\frac{V^{\text{Bench}}}{V^{\text{No-China}}}\right]^{1/(1-\gamma)} - 1\,.$$

[14]We should note that given the taste shocks used to compute the equilibrium of the model, this formula is just an approximation. The use of taste shocks for computational convenience is explained in detail in Appendix A.

As Table 7 documents, these two economies feature very similar simulated moments. However, these averages mask interesting state-contingency present in the welfare effects.

Table 7: Simulated Moments: Baseline vs. No China

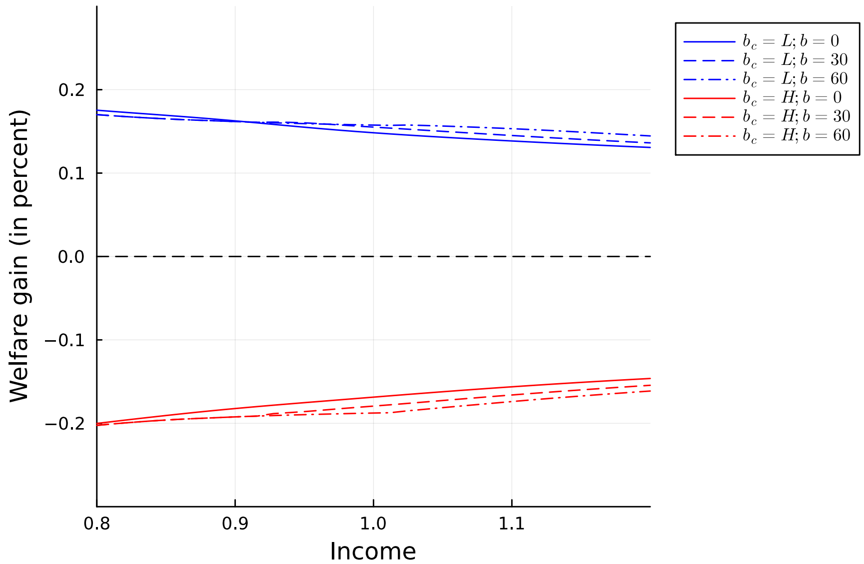
Statistic	Baseline	No China
Market Debt to GDP	30.06	30.01
Market Issuance to GDP	6.0	5.98
China Debt to GDP	2.52	0.0
Net Flow from China to GDP	-0.01	0.0
Spread	3.55	3.51
S.D. Spread	2.22	2.16
Corr(Spread, GDP)	-0.64	-0.66
P(Default t+1)	2.97	2.95
Default Frequency	2.52	2.52
S.D. Consumption/S.D. GDP	1.16	1.15

Note: Moments are computed for non-exclusion periods, except for the default frequency which uses all simulation periods.

Figure 3 plots the welfare gains as a function of initial income for different combinations of initial (b, b_c). We can see a clear pattern emerging: if the economy is starting from a position of low Chinese debt, then it is always desirable to be in the baseline economy and have this lending relationship with China. However, if the economy starts from the opposite position (with high Chinese debt), then the representative agent would prefer to live in the No-China economy (for the mean income and the mean market debt, the agents would be willing to pay 0.2 percent of permanent consumption to do that). As expected, these welfare gains and losses become smaller in absolute value as income increases.

The intuition for these symmetric results comes from the nature of the Chinese flows and from discounting. If the country is initially facing low Chinese debt (which in our calibration is normalized to zero), then living in the baseline model implies that a Chinese flow is about to come first. For a sufficiently impatient agent, this front-loading of consumption increases trump the future austerity needed to repay China and the additional uncertainty induced by Chinese flows (which introduce additional rollover risk to the economy). On the other hand, an agent who is already indebted vis-a-vis China faces a repayment coming due first, naturally leading him/her to prefer the No-China model.

Figure 3: Welfare gains



Note: A positive number means that agents prefer the benchmark economy.

6 Extension: The Risk of Chinese Retrenchment

Having established how Chinese debt impacts the equilibrium price and quantity of market debt, we use the model to study the effects of alternative risk scenarios related to Chinese funding. In particular, it has been a recent concern that countries borrowing heavily from China may suffer a "Chinese retrenchment," and be faced with a sudden and permanent exclusion from Chinese funding. We study the implications of this form of geopolitical risk for market debt, prices, and welfare in this section.

6.1 Model with Retrenchment Risk

We augment the baseline model with the possibility that the country has to repay the debt owed to China (if any) and permanently lose access to Chinese flows. We denote by p_{SS} the probability of this sudden and permanent stop.

There are four state variables: one endogenous and three exogenous. The endogenous state variable is the market debt level, b. The exogenous debt variables are y (the income level), b_c (the Chinese debt level), and a (the realization of a non-zero Chinese net flow). Let us denote $s \equiv (a, y)$. The recursive representation of this problem follows:

Let d denote the current-period default decision prior to a Chinese retrenchment. We assume that d is equal to 1 if the government defaulted in the current period, and is equal to 0 if it did not. Below, we will denote with $\tilde{d}$ the default decision after the retrenchment has occurred.

$$V(b, b_c, s) = \max_{d \in \{0,1\}} \Big\{ dV_1(b_c, s) + (1-d)V_0(b, b_c, s) \Big\}, \tag{12}$$

where

$$V_0(b, b_c, s) = \max_{b'} \Big\{ u(c) + \beta(1-p_{SS})\, \mathbb{E}_{s'|s} V(b', b_c', s') + \beta p_{SS}\, \mathbb{E}_{y'|y} W(b', b_c', y') \Big\}, \tag{13}$$

subject to

$$c + \kappa b = y + q(b', b_c', s)(b' - (1-\delta)b) + z(b_c, a) \tag{14}$$

with

$$b_c' = \mathcal{B}_c'(b_c, a) = \begin{cases} H & \text{if } a = 1 \cap b_c = L \\ L & \text{if } a = 1 \cap b_c = H \\ b_c & \text{otherwise} \end{cases} \tag{15}$$

and

$$z(b_c, a) = \mathcal{B}_c'(b_c, a) - b_c = \begin{cases} H - L & \text{if } a = 1 \cap b_c = L \\ L - H & \text{if } a = 1 \cap b_c = H \\ 0 & \text{if } a = 0 \end{cases} \tag{16}$$

where κ represents the coupon, $z(b_c, a)$ in (16) represents the *net flows* vis-à-vis China, and $\mathcal{B}_c'(b_c, a)$ in (15) indicates how the debt level with China evolves. The value of default is:

$$\begin{aligned} V_1(b_c, s) = {} & u\big(y - \phi(y) + z(b_c, a)\big) \\ & + \beta(1-p_{SS})\mathbb{E}_{s'|s}\Big[\theta V(0, b_c', s') + (1-\theta)V_1(b_c', s')\Big] \\ & + \beta p_{SS}\mathbb{E}_{y'|y}\Big[\theta W(0, b_c', y') + (1-\theta)W_1(b_c', y')\Big], \end{aligned} \tag{17}$$

subject to (16) and (15).

The value functions W, W_0, W_1 satisfy:

$$W(b, b_c, y) = \max_{\tilde{d} \in \{0,1\}} \Big\{ \tilde{d} W_1(b_c, y) + (1-\tilde{d})W_0(b, b_c, y) \Big\}, \tag{18}$$

where

$$W_0(b, b_c, y) = \max_{b'} \Big\{ u(c) + \beta \, \mathbb{E}_{y'|y} W(b', 0, y') \Big\}, \tag{19}$$

subject to

$$c + \kappa b = y + \tilde{q}(b', y)(b' - (1-\delta)b) - b_c \tag{20}$$

The value of default is:

$$W_1(b_c, y) = u\big(y - \phi(y) - b_c\big) + \beta \mathbb{E}_{y'|y} \Big[\theta W(0, 0, y') + (1-\theta) W_1(0, s') \Big]. \tag{21}$$

The bond prices q and $\tilde{q}$ are jointly given by the following functional equations:

$$\begin{aligned} q(b', b_c', s) = {} & e^{-r}(1 - p_{SS}) \, \mathbb{E}_{s'|s} \left[1 - \hat{d}\,(b', b_c', s') \right] \left[\kappa + (1-\delta) \, q \left(\hat{b}\,(b', b_c', s') \, , b_c'', s' \right) \right] \\ + \quad & e^{-r} p_{SS} \, \mathbb{E}_{y'|y} \left[1 - \hat{\hat{d}}\,(b', b_c', s') \right] \left[\kappa + (1-\delta) \, \tilde{q} \left(\hat{\hat{b}}\,(b', b_c', y') \, , y' \right) \right], \end{aligned} \tag{22}$$

and

$$\tilde{q}(b', y) = e^{-r} \, \mathbb{E}_{y'|y} \left[1 - \hat{\hat{d}}\,(b', 0, y') \right] \left[\kappa + (1-\delta) \, \tilde{q} \left(\hat{\hat{b}}\,(b', 0, y') \, , y' \right) \right] , \tag{23}$$

where $\{\hat{d}, \hat{\hat{d}}\}$ and $\{\hat{b}, \hat{\hat{b}}\}$ denote the future default and borrowing rules that lenders expect the government to follow. The price in (23) is equivalent to the one observed in a standard sovereign model with long-term debt: this is because the 'retrenchment shock' induces a permanent loss of the Chinese funding.

6.2 Simulation Results under 'Retrenchment risk'

Table 8 shows the main moments of interest when the economy is living under the risk of a Chinese retrenchment. We find that for a 1% probability of the economy exogenously cutting ties with China, the moments before the retrenchment look similar to the baseline economy. The period of the retrenchment is a turbulent time: it is characterized by repayment of the Chinese debt, lower consumption, a higher issuance of market debt, and a higher level of spreads than in the median years before or after the retrenchment.

Table 8: Simulated Moments for the Chinese Retrenchment Risk Model: $p_{SS} = 0.01$

Statistic	Pre-Retrenchment			Retrench.	Post
	Average	Inflow	Outflow	Period	Retrench.
Market Debt to GDP	30.01	31.06	28.97	30.0	30.02
Market Issuance to GDP	5.98	2.71	8.95	7.26	5.97
China Debt to GDP	2.52	0.0	4.97	2.53	0.0
Net Flow from China to GDP	-0.01	4.96	-4.97	-2.53	0.0
Consumption to GDP	98.95	101.03	96.56	97.45	98.96
Spread	3.53	3.05	3.99	3.79	3.51
S.D. Spread	2.21	1.82	2.47	2.32	2.16
Corr(Spread, GDP)	-0.64	-0.63	-0.67	-0.66	-0.66
P(Default t+1)	2.94	2.1	3.77	3.44	2.95
Default Frequency	2.52	2.95	2.09	2.97	2.52
S.D. Consumption/S.D. GDP	1.16	1.01	1.31	1.25	1.15

Note: Moments are computed for non-exclusion periods, except for the default frequency which uses all simulation periods.

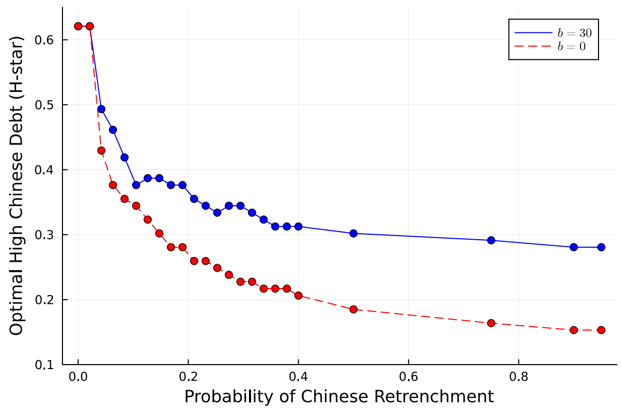
6.3 Ex-Ante Optimal Exposure to Chinese Debt

So far, we've treated the level of high Chinese debt as exogenous. We now ask, given this new risk of China going away, how much should a country 'expose' itself to China?

To answer this question, we define $H^*(b, p_{SS})$ as ex-ante optimal the level of high Chinese debt a country would choose, as a function of its market debt (b) and the probability of a Chinese retrenchment (p_{SS}). Figure 4 tells a clear story: (a) other things equal, if a country has a higher level of market debt, it prefers a higher level of Chinese loans, pointing to the importance of the substitution between these two funding sources; (b) regardless of the market debt level, the higher is p_{SS} the lower is the optimal exposure to China, pointing to the undesirability of this additional source of 'rollover' risk.

All in all, the findings in this section point to the possibility of boom-bust cycles fueled by China's evolving role in the international financial system. Changes in geopolitical alignments (captured by changes in p_{SS}) can trigger sharp changes in the optimal level of exposure a given country wants to take. It is entirely possible that a country like Argentina (for example), which went through a long period of close geopolitical proximity to China with the Peronist governments, increases its exposure and then a sudden (and unanticipated) domestic political shock (like the swing to right-wing policies under the Milei presidency) leaves it exposed to the material risk of having to pay down non-trivial Chinese loans. This case exemplifies the nuanced risks that countries face when they rely 'too much' on Chinese lending.

Figure 4: Optimal Chinese Debt by Market Debt Level



Note: This figure shows H^* given the sovereign has outstanding debt of 30% (blue) and 0% (red/dotted) of outstanding debt as a percent of mean income.

7 Conclusion

We use a standard sovereign debt model with long-term debt to rationalize a set of facts about emerging economies borrowing from private markets and the impact that Chinese official lending has on it. We find that following a positive inflow from China our model economy chooses to re-balance its debt portfolio by deleveraging from market debt. In the process it pays lower spreads and faces less volatile consumption. On the other hand, when facing a capital flow vis-à-vis China, the economy taps private markets, levers up on defaultable debt, and ends paying higher and more volatile spreads in equilibrium. All these model dynamics are in line with panel-data evidence from emerging and low income economies. Furthermore, we use the model to study the welfare gains from having access to Chinese loans and find that they are modest and highly state-dependent.

We use our model to compute the ex-ante optimal level of exposure to Chinese funding and find that it is increasing in the market indebtedness of the country and decreasing in the probability of a Chinese retrenchment (i.e., decreasing in the probability that China severs ties with the borrowing country). In an increasingly uncertain geopolitical context, these results are a cautionary tale for emerging economies that have volatile political environments and which have so far relied heavily on Chinese official lending.

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A Computational Appendix

A.1 Baseline Model with Preference Shocks

We utilize i.i.d. Type 1 Extreme Value preference shocks to ensure convergence of the pricing schedule and value functions, similar to the method used by Dvorkin et al. (2021) and several other quantitative sovereign default papers. The basic computational strategy is described in Mihalache (2025).

Conditional on choosing to repay their outstanding debt, the sovereign receives an additive preference shock $\epsilon(b')$ for every possible value of debt b'.[15] Let $\mathbb{B} \equiv \{\underline{b}, b_1, ..., b_{N-1}, \bar{b}\}$ where $\underline{b} < b_1 < ... < b_{N-1} < \bar{b}$, the nodes are equally spaced, and positive values indicate borrowing. This grid represents all debt positions the sovereign could take on.

The cumulative density function of the vector of preference shocks, ϵ, is distributed according to the Type 1 Extreme Value distribution:

$$F(\epsilon) = \exp\left[-\sum_{b' \in \mathbb{B}} \exp\left(-\frac{\epsilon(b') - \mu_\eta}{\rho_\eta}\right)\right]$$

where μ_η is the location parameter, ρ_η is the scale parameter, and $\tilde{\gamma}$ is the Euler-Mascheroni constant. We choose ρ_η so that it is small enough to maintain the economic content of the sovereign's borrowing decision but large enough to ensure that computation of the value and price functions occur in a timely manner.[16] We fix $\rho_\eta = 0.0005$ and $\mu_\eta = -\tilde{\gamma}\rho_\eta$.[17]

The sovereign's problem in the baseline model is given by:

$$V(b, b_c, s) = \max_{d \in \{0,1\}} \Big\{ dV_1(b_c, s) + (1-d)EV_0(b, b_c, s) \Big\}, \tag{24}$$

where the ex-ante value of not defaulting is

$$EV_0(b, b_c, s) = \int V_0(b, b_c, s, \epsilon) dF(\epsilon) \tag{25}$$

with ϵ representing a vector of i.i.d. preference shocks; one for each value of $b' \in \mathbb{B}$. That is, $\epsilon \equiv (\epsilon(\underline{b}), \epsilon(b_1), ..., \epsilon(b_{N-1}), \epsilon(\bar{b}))$. The value function conditional on a particular realization of the preference shock vector ϵ is

$$V_0(b, b_c, s, \epsilon) = \max_{b' \in \mathbb{B}} v_0^{b'}(b, b_c, s) + \epsilon(b') \tag{26}$$

with

$$v_0^{b'}(b, b_c, s) = u(c^{b'}(b, b_c, s)) + \beta\, \mathbb{E}_{s'|s} V(b', b_c', s'), \tag{27}$$

[15] Note that the default/repayment choice is not subject to preference shocks. It would be straightforward to extend the model to include default choice shocks, but it is not necessary to achieve convergence.

[16] All else equal, increasing the scale parameter makes the pricing function more well-behaved, which aids in computation. But this comes at the cost of infusing more noise into the sovereign's borrowing decision.

[17] This implies that the expectation of an individual $\epsilon(b')$ is equal to 0.

where $c^{b'}(b, b_c, s)$ denotes current period consumption given the incoming state (b, b_c, s) and a particular value of b'. Thus $c^{b'}(b, b_c, s)$ equals

$$c^{b'}(b, b_c, s) = y + q(b', b'_c, s)(b' - (1 - \delta)b) - \kappa b + z(b_c, a) \tag{28}$$

with

$$b'_c = \begin{cases} H & \text{if } a = 1 \cap b_c = L \\ L & \text{if } a = 1 \cap b_c = H \\ b_c & \text{otherwise} \end{cases} \tag{29}$$

and

$$z(b_c, a) = b'_c - b_c = \begin{cases} H - L & \text{if } a = 1 \cap b_c = L \\ L - H & \text{if } a = 1 \cap b_c = H \\ 0 & \text{if } a = 0 \end{cases} \tag{30}$$

where κ represents the coupon, $z(b_c, a)$ in (30) represents the *net flows* vis-à-vis China, and b'_c in (29) indicates how the debt level with China evolves.

With $v_0^{b'}(b, b_c, s)$ in hand, we know the analytic solution for the ex-ante[18] choice probabilities and the ex-ante value of not defaulting. For states where $d(b, b_c, s) = 0$, the probability the sovereign chooses debt position $b' \in \mathbb{B}$ is:

$$\mathbb{P}^{b'}(b, b_c, s) = \frac{\exp\left(\frac{v_0^{b'}(b,b_c,s)}{\rho_\eta}\right)}{\sum_{b' \in \mathbb{B}} \exp\left(\frac{v_0^{b'}(b,b_c,s)}{\rho_\eta}\right)} \tag{31}$$

In other words, conditional on choosing not to default, $\mathbb{P}^{b'}(b, b_c, s)$ is the probability that the sovereign chooses b' given state (b, b_c, s). The lenders take these choice probabilities as given and use them to price the bonds.

We can calculate the ex-ante value of not defaulting as:

$$EV_0(b, b_c, s) = \mu_\eta + \tilde{\gamma}\rho_\eta + \rho_\eta \log\left(\sum_{b' \in \mathbb{B}} \exp\left(\frac{v_0^{b'}(b, b_c, s)}{\rho_\eta}\right)\right) \tag{32}$$

The Bellman equation representing the value of defaulting is unchanged relative to that shown in the main body of the paper:

$$V_1(b_c, s) = u\big(y - \phi(y) + z(b_c, a)\big) + \beta \mathbb{E}_{s'|s}\Big[\theta V(0, b'_c, s') + (1 - \theta)V_1(b'_c, s')\Big], \tag{33}$$

subject to (30) and (29).

Let $\hat{d}(b', b'_c, s')$ denote the period-ahead default policy and $\hat{b}(b', b'_c, s', \epsilon')$ the period-ahead borrowing policy. Note that while $\hat{b}(\cdot)$ depends on the realization of the preference shock vector ϵ', the period-ahead default policy $\hat{d}(\cdot)$ does not. This is because the preference

[18] Ex-ante as in prior to the realization of ϵ.

shocks are only realized if the sovereign chooses not to default; that is $\hat{d}(b', b'_c, s') = 0$. The equilibrium pricing function then follows:

$$q(b', b'_c, s) = e^{-r}\, \mathbb{E}_{s'|s} \left[1 - \hat{d}\,(b', b'_c, s')\right] \left[\kappa + (1-\delta) \int_{\epsilon'} q\left(\hat{b}\,(b', b'_c, s', \epsilon')\,, b''_c, s'\right) dF(\epsilon')\right]\,, \tag{34}$$

Let $\mathbb{P}^{b'}(b, b_c, s)$ denote the probability a borrower will choose b' given that it entered the period with states (b, b_c, s) *and* chose to repay its current period debt. We can replace the integral in the second bracketed term with a summation because of the type 1 extreme value shocks:

$$q(b', b'_c, s) = e^{-r}\, \mathbb{E}_{s'|s} \left[1 - \hat{d}\,(b', b'_c, s')\right] \left[\kappa + (1-\delta) \sum_{\hat{b}'' \in \mathbb{B}} q\left(\hat{b}'', b''_c, s'\right) \times \mathbb{P}^{\hat{b}''}(b', b'_c, s')\right]\,, \tag{35}$$

A.2 Chinese Retrenchment Risk Model with Preference Shocks

To incorporate preference shocks, the retrenchment risk model needs to be amended in a similar manner as the baseline model.

The government's value function at the beginning of the period is:

$$V\,(b, b_c, s) = \max_{d\,\in\,\{0,1\}} \Big\{ dV_1(b_c, s) + (1-d)EV_0(b, b_c, s) \Big\}, \tag{36}$$

The value of not defaulting, prior to the realization of ϵ, is:

$$EV_0(b, b_c, s) = \int V_0(b, b_c, s, \epsilon) dF(\epsilon) \tag{37}$$

After ϵ is realized, the ex-post no-default value function is:

$$V_0(b, b_c, s) = \max_{b' \in \mathbb{B}} v_0^{b'}(b, b_c, s) + \epsilon(b') \tag{38}$$

where

$$v_0^{b'}(b, b_c, s) = u(c^{b'}(b, b_c, s)) + (1 - p_{SS})\beta \mathbb{E}_{s'|s} V(b', b'_c, s') + p_{SS}\beta \mathbb{E}_{y'|y} W(b', b'_c, y'), \tag{39}$$

where $c^{b'}(b, b_c, s)$ equals

$$c^{b'}(b, b_c, s) = y + q(b', b'_c, s)(b' - (1-\delta)b) - \kappa b + z(b_c, a) \tag{40}$$

with $z(b_c, a)$ defined above. The choice probabilities are denoted by $\mathbb{P}_V^{b'}(b, b_c, s)$.

The value of default is:

$$\begin{aligned} V_1(b_c, s) = {} & u\big(y - \phi(y) + z(b_c, a)\big) \\ & + \beta(1 - p_{SS})\mathbb{E}_{s'|s}\Big[\theta V(0, b_c', s') + (1-\theta)V_1(b_c', s')\Big] \\ & + \beta p_{SS}\mathbb{E}_{y'|y}\Big[\theta W(0, b_c', y') + (1-\theta)W_1(b_c', y')\Big], \end{aligned} \tag{41}$$

A new set of value functions and pricing schedules are required for the periods on and after retrenchment shock realization. The value functions W, EW_0, W_0, W_1 satisfy:

$$W\,(b, b_c, y) = \max_{\tilde{d}\in\{0,1\}} \Big\{\tilde{d}W_1(b_c, y) + (1-\tilde{d})EW_0(b, b_c, y)\Big\}, \tag{42}$$

where

$$EW_0(b, b_c, y) = \int W_0(b, b_c, y, \epsilon)dF(\epsilon) \tag{43}$$

with

$$W_0(b, b_c, y, \epsilon) = \max_{b'\in\mathbb{B}} w_0^{b'}(b, b_c, y) + \epsilon(b'), \tag{44}$$

where

$$w_0^{b'}(b, b_c, y) = u(c(b', b_c, y)) + \beta\,\mathbb{E}_{y'|y}W(b', 0, y') \tag{45}$$

subject to

$$c(b', b_c, y) = y + \tilde{q}(b', y)(b' - (1-\delta)b) - \kappa b - b_c \tag{46}$$

The b' choice probabilities are denoted by $\mathbb{P}_W^{b'}(b, b_c, y)$.
The value of default post-retrenchment is:

$$W_1(b_c, y) = u\big(y - \phi(y) - b_c\big) + \beta\mathbb{E}_{y'|y}\Big[\theta W(0, 0, y') + (1-\theta)W_1(0, y')\Big]. \tag{47}$$

The bond prices q and $\tilde{q}$ are jointly given by the following functional equations:

$$\begin{aligned} q(b', b_c', s) = {} & e^{-r}(1 - p_{SS})\,\mathbb{E}_{s'|s}\left[1 - \hat{d}\,(b', b_c', s')\right]\left[\kappa + (1-\delta)\sum_{\hat{b}\in\mathbb{B}} q\left(\hat{b}, b_c'', s'\right) \times \mathbb{P}_V^{\hat{b}}(b', b_c'', y')\right] \\ & + \quad e^{-r}p_{SS}\,\mathbb{E}_{y'|y}\left[1 - \hat{\tilde{d}}\,(b', b_c', s')\right]\left[\kappa + (1-\delta)\sum_{\hat{\tilde{b}}\in\mathbb{B}} \tilde{q}\left(\hat{\tilde{b}}, y'\right) \times \mathbb{P}_W^{\hat{\tilde{b}}}(b', b_c', y')\right], \end{aligned} \tag{48}$$

and

$$\tilde{q}(b', y) = e^{-r}\, \mathbb{E}_{y'|y}\left[1 - \hat{\hat{d}}\,(b', 0, y')\right]\left[\kappa + (1-\delta) \sum_{\hat{\hat{b}} \in \mathbb{B}} \tilde{q}\left(\hat{\hat{b}}, y'\right) \times \mathbb{P}_W^{\hat{\hat{b}}}(b', 0, y')\right], \tag{49}$$

where $b_c^{''} = \mathcal{B}_c^{'}(b_c^{'}, a')$ and where $\{\hat{d}, \hat{\hat{d}}\}$ denote the period-ahead default rules the lenders expect the government to follow.

A.3 Computational Strategy

We discretize both the bond grid and the income grid. The bond grid has 225 equally-spaced nodes between 0 and 1. We use the Rouwenhorst method to approximate the log income process, and the income grid has 75 nodes. To compute the Markov Perfect Equilibrium, we utilize a strategy that is similar to the method outlined in Mihalache (2025).

A.4 Computation of the Baseline Model

We will describe the solution method starting from arbitrary iteration $k \geq 1$. For $k = 0$ (i.e. at the beginning of the algorithm), we set the initial values of the pricing, value, and policy functions as follows:[19]

$$\begin{aligned}
V_1^0(b_c, s) &= \frac{u(y - \phi(y) + z(b_c, a))}{1-\beta} \\
\mathbb{P}^{0,b'=0}(b, b_c, s) &= 1 \\
q^0(b', b_c', s) &= 0 \\
EV_0^0(b, b_c, s) &= \frac{u(y - \kappa b - q^0(b', b_c', s)(b' - (1-\delta)b) + z(b_c, a))}{1-\beta} \\
V^0(b, b_c, s) &= \max\{V_1^0(y, b_c, a), EV_0^0(b, b_c, s)\} \\
d^0(b, b_c, s) &= \mathbf{1}_{\{V_1^0(\cdot) > EV_0^0(\cdot)\}}
\end{aligned}$$

At the start of an iteration, we enter with pricing schedule $q^k(\cdot)$, choice probabilities $\mathbb{P}^k(\cdot)$, default policy $d^k(\cdot)$, and value functions $V_1^k(\cdot)$, $EV_0^k(\cdot)$, $V^k(\cdot)$.

1. Use the choice probability $\mathbb{P}^k(\cdot)$ and default policy $d^k(\cdot)$ to calculate $q^{k+1}(\cdot)$ according to:

$$q^{k+1}(b', b_c', s) = e^{-r}\, \mathbb{E}_{s'|s}\left[1 - d^k\,(b', b_c', s')\right]\left[\kappa + (1-\delta) \sum_{\hat{b}'' \in \mathbb{B}} q^k\left(\hat{b}'', b_c'', s'\right) \times \mathbb{P}^{k,\hat{b}''}(b', b_c', s')\right] \tag{50}$$

[19]Models of long-term debt are known to suffer from multiple equilibria. Different initial guesses may lead to different equilibrium objects.

2. Use $q^{k+1}(\cdot)$ and the iteration k value functions to compute $EV_0^{k+1}(\cdot)$, $\mathbb{P}^{k+1,b'}(\cdot)$, and $V_1^{k+1}(\cdot)$.

$$
\begin{aligned}
v_0^{b',k+1}(b,b_c,s) &= u(c^{b',k+1}(b,b_c,s)) + \beta\,\mathbb{E}_{s'|s}V^k(b',b_c',s') \\
\text{s.t } c^{b',k+1}(b,b_c,s) &= y + q^{k+1}(b',b_c',s)(b'-(1-\delta)b) - \kappa b + z(b_c,a)
\end{aligned}
$$

$$
EV_0^{k+1}(b,b_c,s) = \rho_\eta \log\left(\sum_{b'\in\mathbb{B}} \exp\left(\frac{v_0^{b',k+1}(b,b_c,s)}{\rho_\eta}\right)\right)
$$

$$
\mathbb{P}^{k+1,b'}(b,b_c,s) = \frac{\exp\left(\frac{v_0^{b',k+1}(b,b_c,s)}{\rho_\eta}\right)}{\sum_{b'\in\mathbb{B}}\exp\left(\frac{v_0^{b',k+1}(b,b_c,s)}{\rho_\eta}\right)}
$$

$$
V_1^{k+1}(b_c,s) = u\big(y-\phi(y)+z(b_c,a)\big) + \beta\mathbb{E}_{s'|s}\Big[\theta V^k(0,b_c',s') + (1-\theta)V_1^k(b_c',s')\Big]
$$

3. Use $EV_0^{k+1}(\cdot)$ and $V_1^{k+1}(\cdot)$ to calculate $V^{k+1}(\cdot)$ and $d^{k+1}(\cdot)$.

$$
V^{k+1}(b,b_c,s) = \max\{V_1^{k+1}(b_c,s), EV_0^{k+1}(b,b_c,s)\}
$$

$$
d^{k+1}(b,b_c,s) = \mathbf{1}_{\{V_1^{k+1}(b_c,s) > EV_0^{k+1}(b,b_c,s)\}}
$$

4. Check for convergence: if $\max|q^{k+1}(\cdot)-q^k(\cdot)| < 10^{-9}$ and $\max|V^{k+1}(\cdot)-V^k(\cdot)| < 10^{-9}$, convergence is achieved. If not, return to step 1 with the updated functions.

A.5 Computation of the Retrenchment Risk Model

In the version of the model with retrenchment risk, the sovereign can be in one of three stages: (i) prior to the retrenchment shock, (ii) the period of the retrenchment shock, (iii) all periods after the retrenchment shock (post-retrenchment). To compute the Markov Perfect Equilibrium, we start with the last stage. We first compute the post-retrenchment equilibrium objects (stage (iii)). We then use these post-retrenchment functions to compute the retrenchment shock period objects (stage (ii)). Finally, we use the compute functions from stages (ii) and (iii) to compute the equilibrium objects prior to the shock.

We compute the retrenchment risk version of the Markov Perfect Equilibrium as follows:

1. Follow the steps described in section A.4 to compute the post-retrenchment (i.e. $b_c = 0$ for the rest of time) equilibrium objects: $W_1(0,y)$, $EW_0(b,0,y)$ $W(b,0,s)$, $\tilde{d}(b,0,y)$, $\mathbb{P}_W^{b'}(b,0,y)$, and $\tilde{q}(b',y)$.

2. Use the post-exclusion equilibrium objects from step (1) to compute these same functions but for $b_c > 0$. Use equations 36 through 41 to calculate these objects.

3. Given the already computed $W_1(b_c, y)$, $EW_0(b, b_c, y)$ $W(b, b_c, s)$, $\tilde{d}(b, b_c, y)$, $\mathbb{P}_W^{b'}(b, b_c, y)$,$\tilde{q}(b', y)$, use the strategy described in section A.4 to compute the pre-retrenchment equilibrium objects $V(\cdot)$, $EV_0(\cdot)$, $V_1(\cdot)$, $d(\cdot)$, and $\mathbb{P}_V^{b'}(\cdot)$.