

# International Reserve Management under Rollover Crises

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The views expressed herein are those of the authors and not necessarily those of the Federal Reserve Bank of Minneapolis or the Federal Reserve System.

## Motivation

To reduce the vulnerability to a debt crisis:

- Should the government reduce the debt or increase reserves?

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Answer unclear:

- Reserves provide liquidity, but reducing debt may be more effective

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- Tractable model of rollover crises with long-duration bonds and reserves
  - Sunspot shocks, deterministic income
- How should the government exit crisis zone?

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- If heavily indebted, optimal to initially reduce debt and keep zero reserves
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- Borrowing to accumulate reserves can reduce spreads

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- **Today**: reserve management under rollover crisis
  - **Borrowing to accumulate reserves helps exiting the crisis zone**
- **Hernandez (2019)**: numerical simulations w/ fundamental and sunspot shocks

Cole-Kehoe (2001); Corsetti-Dedola (2016); Aguiar-Amador (2020); Bianchi-Mondragon (2022); Bianchi and Sosa-Padilla (2023); Corsetti-Maeng (2023ab)

# Model

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## Environment

- Discrete time, infinite horizon. Constant endowment:  $y_t = y$
- Government trades two assets ...
  - short-term risk-free reserves,  $a$
  - long-term defaultable debt,  $b$ 
    - a bond issued in  $t$  promises to pay  $\left(\frac{\delta+r}{1+r}\right) [1, (1-\delta), (1-\delta)^2, \dots]$
- Risk-neutral deep pocket international investors:
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- Risk-neutral deep pocket international investors:
  - Discount future flows at  $1+r$ , assume  $\beta(1+r) = 1$
- Markov equilibrium w/ Cole-Kehoe (2000) timing:
  - Borrowing at the beginning of the period
  - Repay/default at the end

## Government

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t [u(c_t) - \phi d_t]$$

where  $d_t = 0$  (1) denotes repayment (default)

- If the government repays:

$$c_t = y + a_t - \frac{\delta + r}{1 + r} b_t + q_t(a_{t+1}, b_{t+1})[b_{t+1} - (1 - \delta)b_t] - \frac{a_{t+1}}{1 + r}$$

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- If the government defaults:

$$c_t = y + a_t - \frac{a_{t+1}}{1 + r}$$

and faces permanent exclusion and utility loss  $\phi$

## Recursive Government Problem

- State is  $s \equiv (a, b, \zeta)$   
 $\zeta$  denotes an iid sunspot that coordinates the lenders
- The government chooses to repay or default

$$V(a, b, \zeta) = \max \{ V_R(a, b, \zeta), V_D(a) \}$$

If indifferent, assume repay



## Value of Default

$$V_D(a) = \max_{a' \geq 0} \{u(c) - \phi + \beta V_D(a')\}$$

subject to

$$c \leq y + a - \frac{a'}{1+r}$$

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- Given  $\beta(1+r) = 1$ , this is

$$V_D(a) = \frac{u(y + (1 - \beta)a) - \phi}{1 - \beta}$$

## Value of Repayment

$$V_R(a, b, \zeta) = \max_{a' \geq 0, b'} \{u(c) + \beta \mathbb{E} V(a', b', \zeta')\}$$

subject to

$$c = y + a - \left(\frac{\delta + r}{1 + r}\right) b - \frac{a'}{1 + r} + q(a', b', s) [b' - (1 - \delta)b]$$

## Equilibrium Bond Price

$$q(a', b', s) = \begin{cases} \frac{1}{1+r} \mathbb{E} \left[ (1 - d(s')) \left( \frac{\delta+r}{1+r} + (1 - \delta)q(a'', b'', s') \right) \right] & \text{if } d(s) = 0 \\ 0 & \text{if } d(s) = 1 \end{cases}$$

where  $a''(s')$  and  $b''(s')$  are the future choice of reserves and debt

## Multiplicity of Equilibria

- Coordination failure may lead to self-fulfilling crises (Cole-Kehoe)
- If lenders expect...
  - ... repayment, they lend, and the government repays
  - ... default, they don't lend, and the government defaults

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Next: incentives to default depending on initial portfolio and whether investors are willing to roll over or not

## Repayment value when government can rollover

$$V_R^+(a, b) = \max_{a' \geq 0, b'} \{u(c) + \beta \mathbb{E} V(a', b', s')\}$$

subject to

$$c = y + a - \left(\frac{\delta + r}{1 + r}\right) b - \frac{a'}{1 + r} + \tilde{q}(a', b') (b' - (1 - \delta)b)$$

where  $\tilde{q}(a', b')$  denotes fundamental bond price

## Repayment Value in a Run

$$V_R^-(a, b) = \max_{a' \geq 0} \{u(c) + \beta \mathbb{E}V(a', (1 - \delta)b, s')\}$$

subject to

$$c = y + a - \frac{a'}{1 + r} - \left(\frac{\delta + r}{1 + r}\right) b$$

To pay debt, need to use reserves or cut consumption



# Characterization

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## Safe zone, crisis zone and default zone

- Immediate:  $V_R^+(a, b) \geq V_R^-(a, b)$
- When  $V_R^-(a, b) < V_D(a) \leq V_R^+(a, b)$ , multiple equilibria

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$$\mathbf{S} = \{(a, b) : V_D(a) \leq V_R^-(a, b)\},$$

$$\mathbf{D} = \{(a, b) : V_D(a) > V_R^+(a, b)\},$$

$$\mathbf{C} = \{(a, b) : V_R^-(a, b) < V_D(a) \leq V_R^+(a, b)\}.$$

## The Value in the Safe zone

- If  $(a, b) \in \mathbf{S}$ : we assume gov. stays in safe zone

$$V^S(a - b) = \frac{u(y + (1 - \beta)(a - b))}{1 - \beta}$$

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For a high enough  $\delta$ : can establish that gov. finds it optimal to stay in  $\mathbf{S}$

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- If  $(a, b) \in \mathbf{C}$ , govt. seeks to exit in finite time (may default along the way if bad sunspot hits)
  - Staying in the crisis zone implies eventually costly default
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Continuation value:

$$\mathbb{E}V(a', b', \zeta') = \begin{cases} V^S(a' - b') & \text{if } (a', b') \in \mathbf{S} \\ (1 - \lambda)V_R^+(a', b') + \lambda V_D(a') & \text{if } (a', b') \in \mathbf{C} \\ V_D(a') & \text{if } (a', b') \in \mathbf{D} \end{cases}$$

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How to exit: raise  $a$  or lower  $b$ ?

## The Crisis Zone (ctd)

Consider portfolio  $(a, b) \in \mathbf{C}$ . If government exits in  $T(a, b)$  as long as  $\{\zeta_t\}_{t=0}^{T-1}$ :

$$q(a', b') = \frac{\delta + r}{1 + r} \sum_{t=1}^{T-1} \left( \frac{1 - \lambda}{1 + r} \right)^t (1 - \delta)^{t-1} + \left[ \frac{(1 - \lambda)(1 - \delta)}{1 + r} \right]^{T-1} \frac{1}{1 + r}$$

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### Proposition 1 (Monotonically increasing consumption path)

Consider an initial portfolio  $(a_0, b_0) \in \mathbf{C}$  such that the government exit time is  $T$ . Then, if  $\zeta_t = 0$  for all  $t \leq T - 1$ , we have  $c_{t+1} \geq c_t$  for all  $t \leq T$ .

## Debt Thresholds

$V^R(a, b)$  decreasing in  $b \Rightarrow$  for every  $a$ , there  $\exists$  unique thresholds  $b^-(a), b^+(a)$ :

$$V_R^-(a, b^-(a)) = V_D(a)$$

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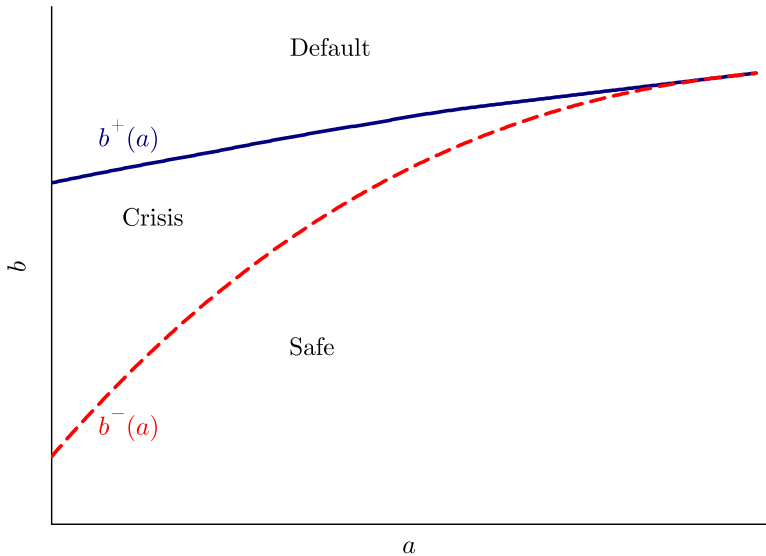
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Thresholds are such that:

1.  $(a, b) \in \mathbf{S}$  if and only if  $b \leq b^-(a)$
2.  $(a, b) \in \mathbf{C}$  if and only if  $b^-(a) < b \leq b^+(a)$
3.  $(a, b) \in \mathbf{D}$  if and only if  $b > b^+(a)$

## The Three Zones



## The slopes of the two boundaries

Recall:  $V_R^-(a, b^-(a)) = V_D(a)$  and  $V_R^+(a, b^+(a)) = V_D(a)$



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Differentiating with respect to  $a$

$$\frac{\partial b^-(a)}{\partial a} = \frac{\frac{\partial V_D(a)}{\partial a} - \frac{\partial V_R^-(a, b^-(a))}{\partial a}}{\frac{\partial V_R^-(a, b^-(a))}{\partial b}}$$

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**Proposition 2** establishes:  $\frac{\partial b^-(a)}{\partial a} \geq \frac{\partial b^+(a)}{\partial a} > 0$

## Lowest-NFA safe portfolio

$$(a^*, b^*) = \underset{a, b}{\operatorname{argmin}} a - b$$

s.t.  $(a, b) \in \mathbf{S}$

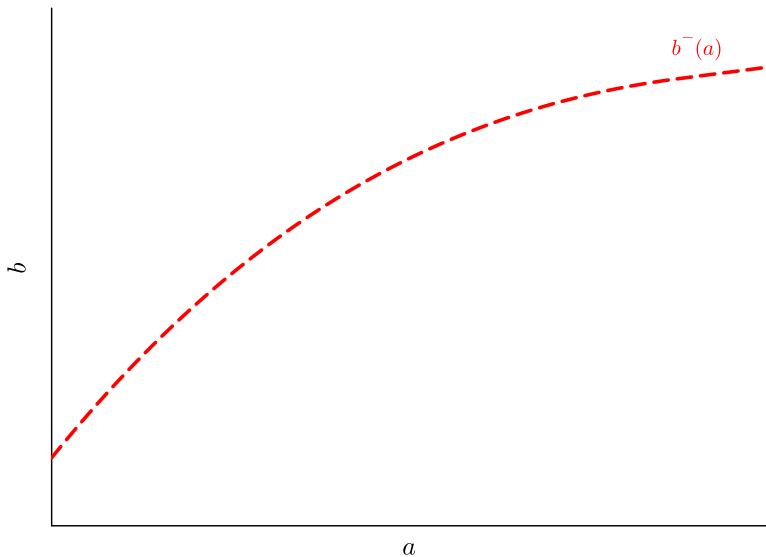
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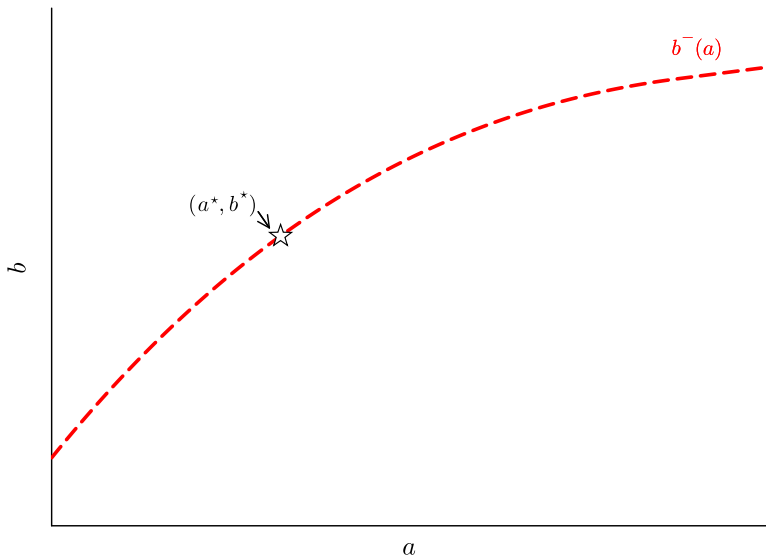
Using that  $(a, b) \in \mathbf{S}$  if  $b \leq b^-(a)$  and assuming a *strictly interior solution* for  $a^*$ , we obtain:

$$\frac{\partial b^-(a^*)}{\partial a} = 1$$

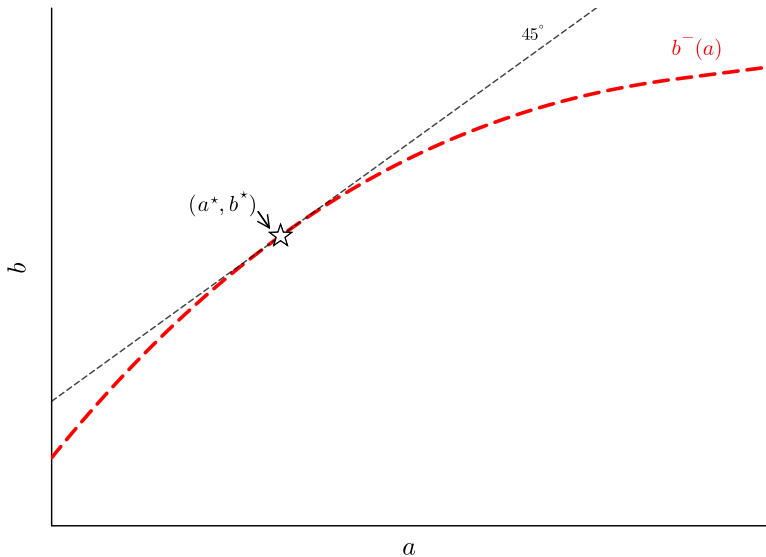
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### Proposition 3 (Positive reserves)

Suppose that the boundary of the crisis region at zero reserves  $b^-(0)$  satisfies

$$\beta(1 - \delta) \left[ u' \left( y - \left( \frac{\delta + r}{1 + r} \right) b^-(0) \right) - u' \left( y - (1 - \beta)(1 - \delta)b^-(0) \right) \right] > u'(y).$$

Then, the lowest-NFA safe portfolio has strictly positive reserves,  $a^* > 0$

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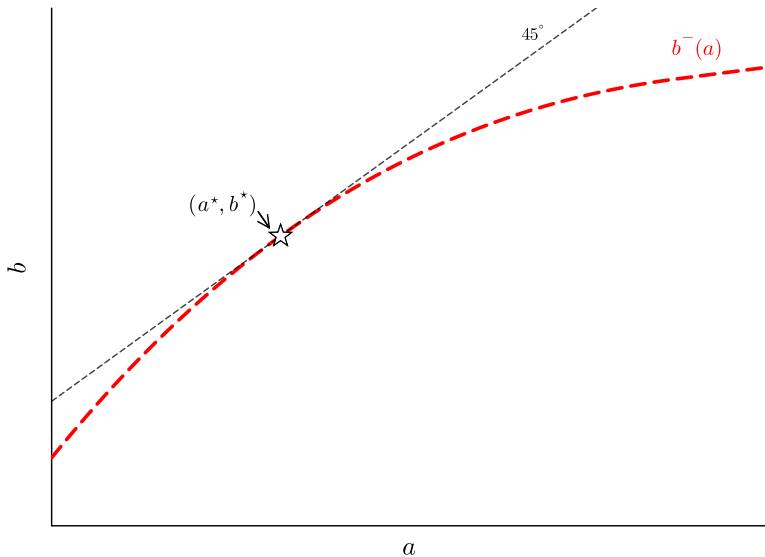
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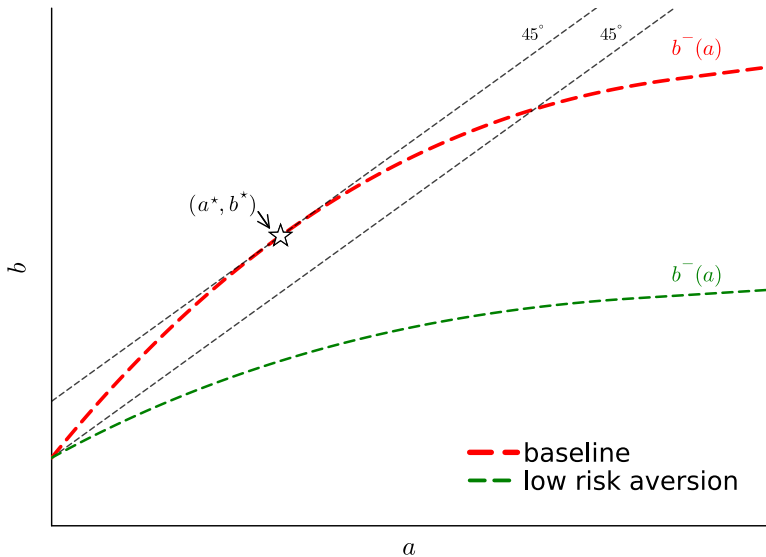
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- Proposition implies  $\left. \frac{\partial b^-(a)}{\partial a} \right|_{a=0} > 1$
- When does it fail? (i) low risk-aversion , (ii) one-period debt ( $\delta = 1$ ) [**Prop. 4**]

## Lowest-NFA safe portfolio



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## **Simulations: Exiting the Crisis Zone**

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## Parametrization

$$u(c) = \frac{(c - \underline{c})^{1-\sigma}}{1-\sigma}$$

Parameter	Value	Description	Source
$y$	1	Endowment	Normalization
$\sigma$	2	Risk-aversion	Standard
$r$	3%	Risk-free rate	Standard
$1/\delta$	7	Maturity of debt	Italian Debt
$\underline{c}$	0.70	Consumption floor	Bocola-Dovis (2019)
$\beta$	$(1+r)^{-1}$	Discount factor	$\beta(1+r) = 1$
$\lambda$	0.5%	Sunspot probability	Baseline
$\phi$	0.34	Default Cost	Debt-to-income =90%

## Optimal Exit Strategy

**Q1:** How many periods until exiting?

- Inside the Crisis Zone we can define Iso-T regions

# Optimal Exit Strategy

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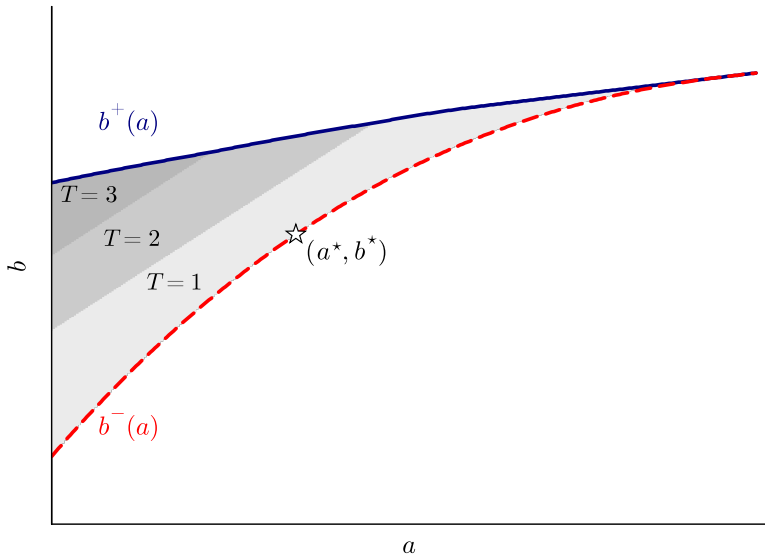
- Inside the Crisis Zone we can define Iso-T regions

**Q2:** What's the best strategy to exit the crisis zone?

- Should the government reduce its debt or increase reserves?
- If reserves are optimal, should gov. slowly build up its stock of reserves?

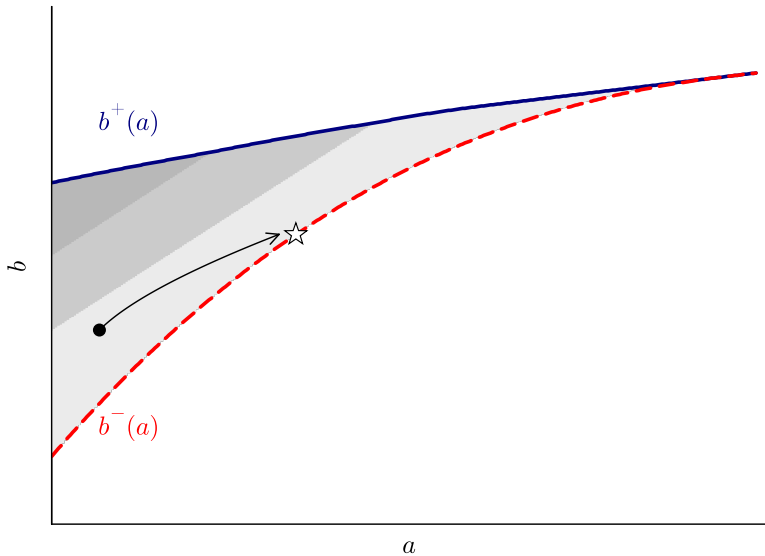
# How many periods until exit

Iso-T Regions



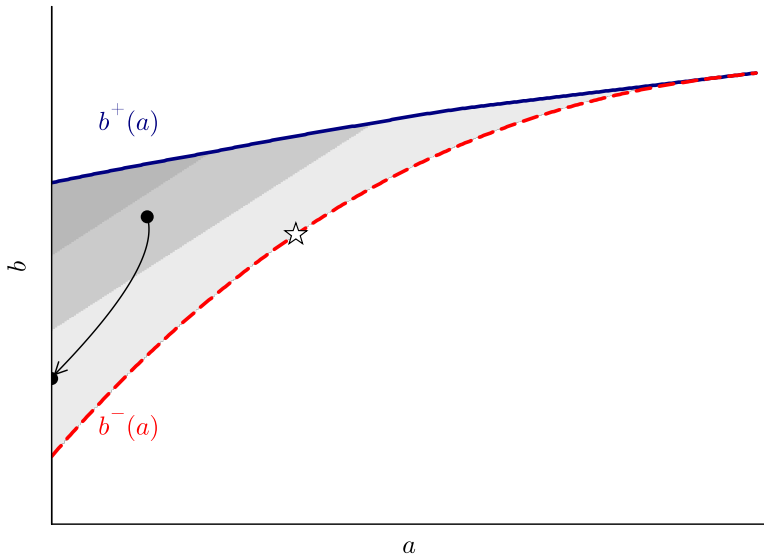
## Deleveraging Path

Safety in One Period  $\rightarrow (a^*, b^*)$



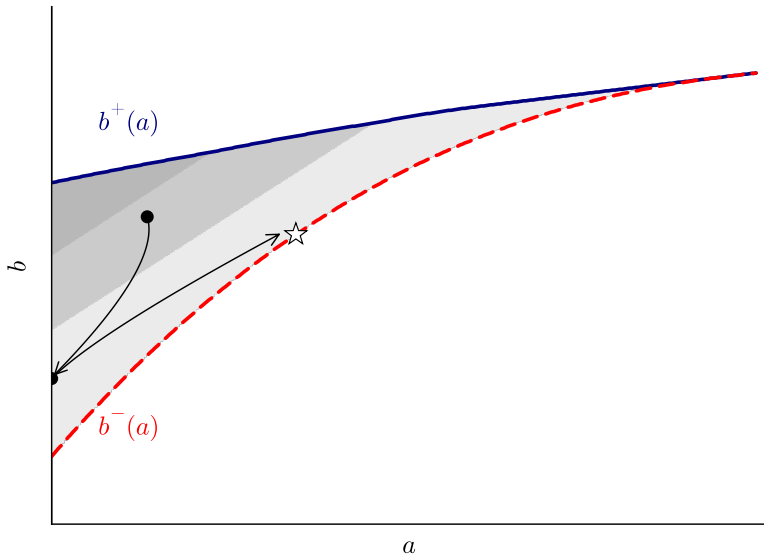
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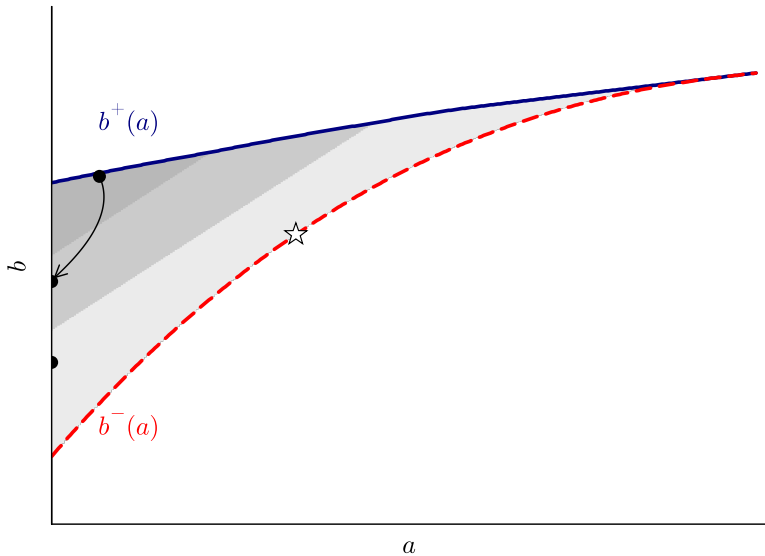
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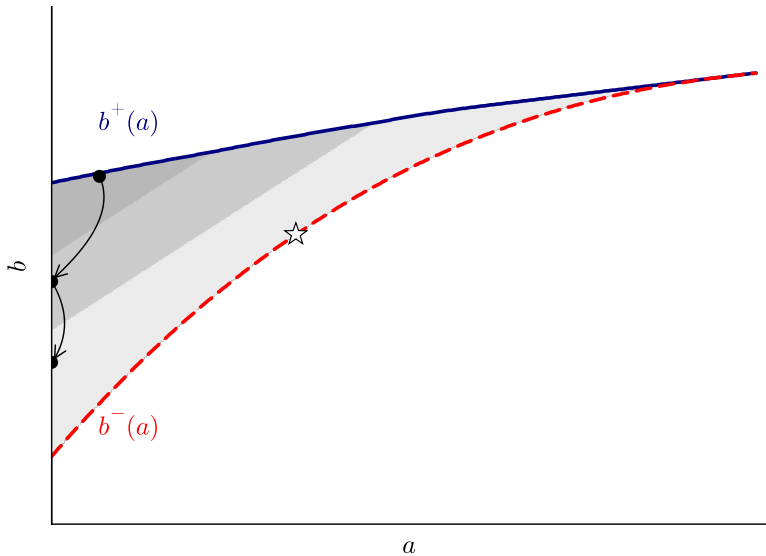
Safety in Three Periods  $\rightarrow (a^*, b^*)$





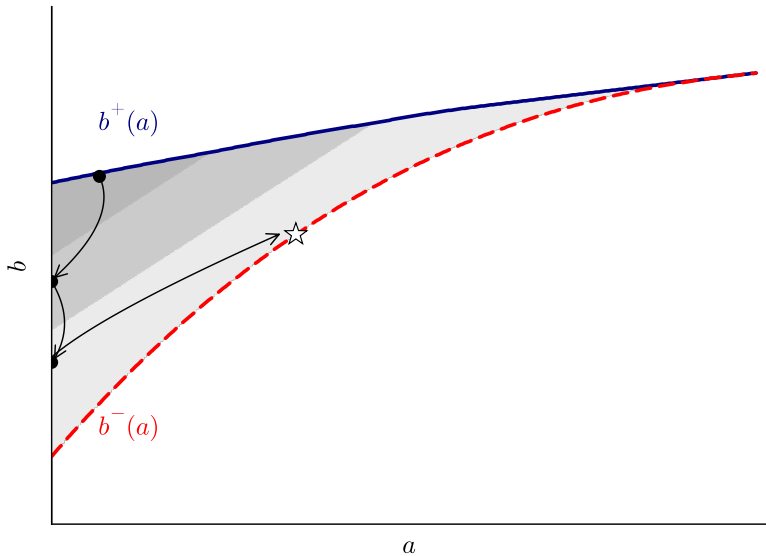
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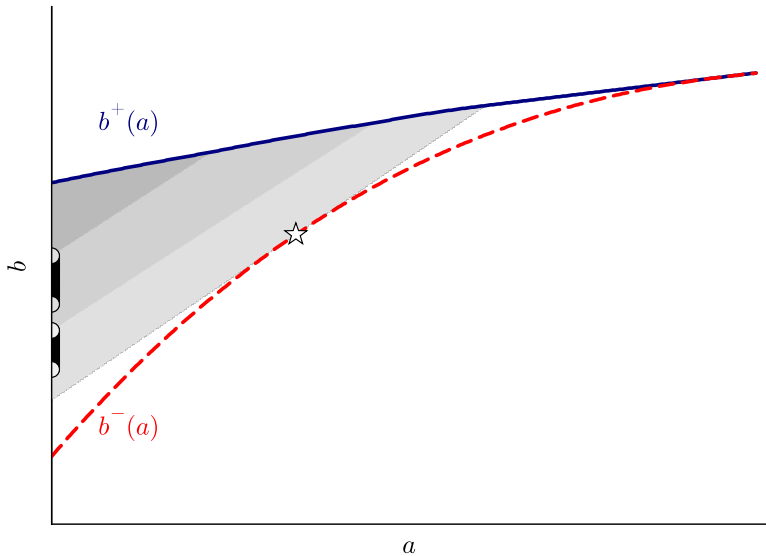
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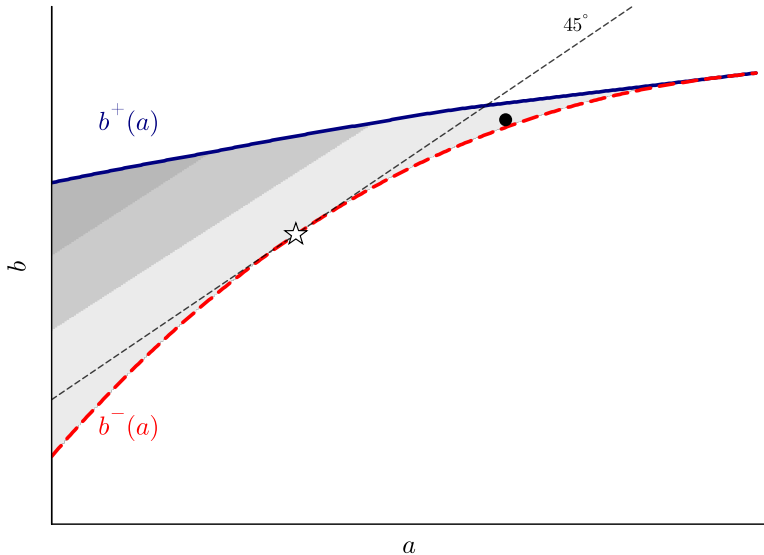
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Possible chosen portfolios for  $a - b < a^* - b^*$



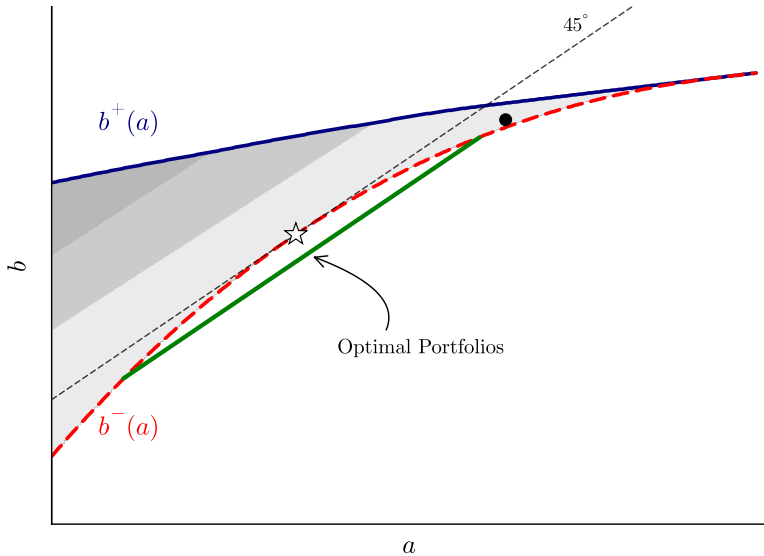
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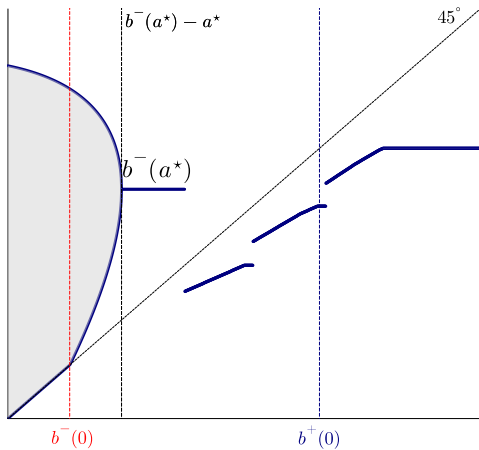
## Deleveraging Path

Safety in One Period  $\rightarrow (a - b > a^* - b^*)$

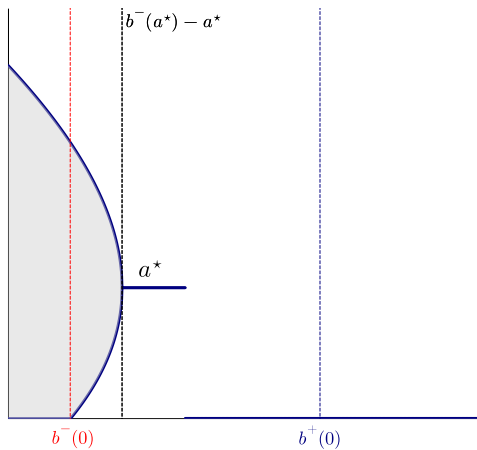


# Policies

Debt,  $b'$

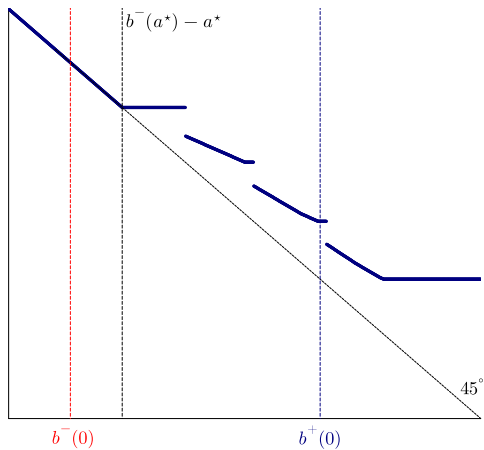


Reserves,  $a'$

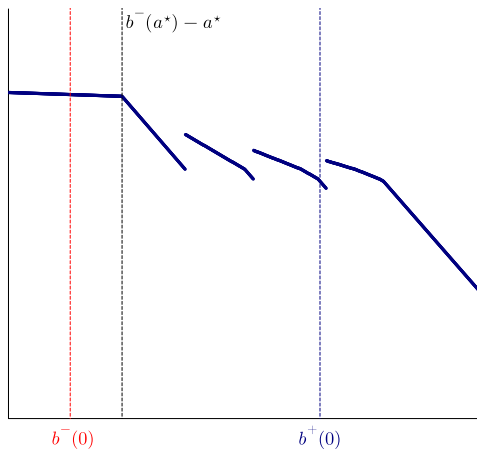


# Policies

Net Foreign Assets,  $a' - b'$

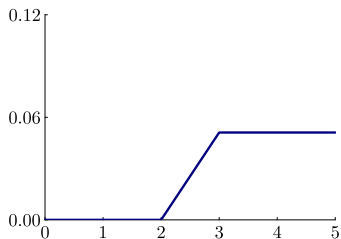


Consumption

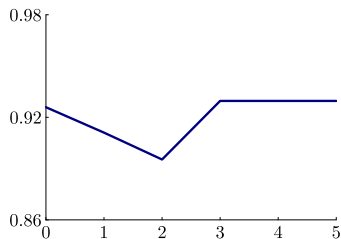


# Deleveraging Dynamics

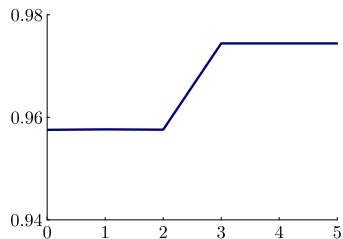
Reserves,  $a$



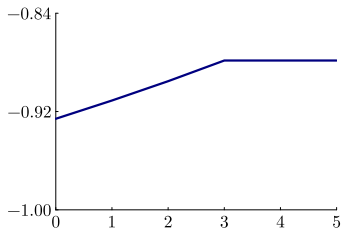
Debt,  $b$



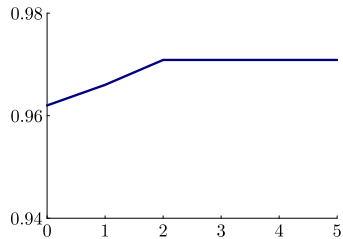
Consumption



Net Foreign Assets



Debt Price,  $q(a', b', s)$





## Taking Stock

To exit crisis zone, first deleverage, then raise debt and reserves

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- If initial portfolio  $(a, b)$  is such that  $(a', b') \in \mathbf{C}$ . Then, the optimal solution features  $a' = 0$ .

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Remark on maturity:

- **With one-period debt,  $\delta = 1$ :**  $V_R^-$  and  $V_R^+$  are unaffected by equal increases in debt and reserves  $\Rightarrow$  issuing debt to accumulate reserves increases spreads
  - Zero reserves are optimal

## Experiment – How reserves help exit crisis zone

- Assume gov. starts w/ portfolio  $(a, b)$ , **but** from  $t+1$  onward,  $a' = 0$

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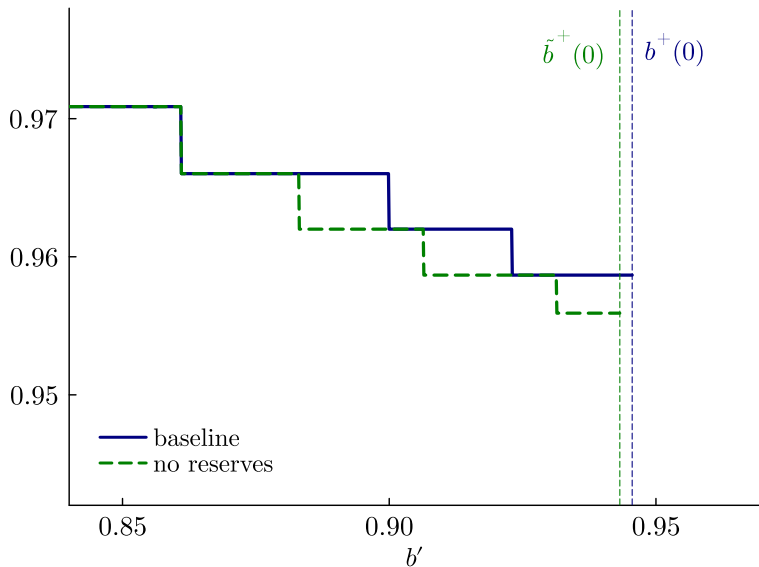
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Without reserves:  $\downarrow b^+$ . More costly to deleverage  $\Rightarrow$  lower debt-carrying capacity

▶ Default zone expands

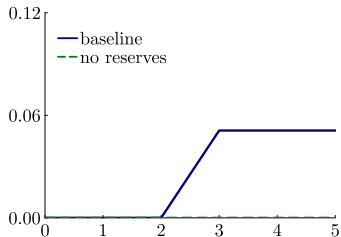
## Price Schedule, $q(0, b')$



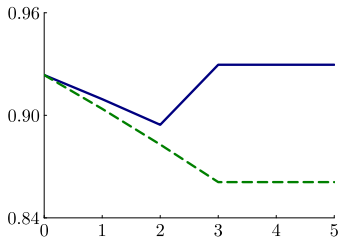


## Lower consumption without reserves

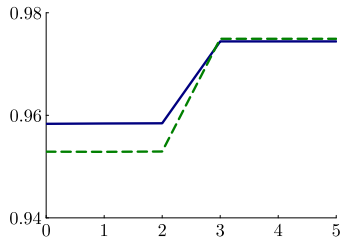
Reserves,  $a$



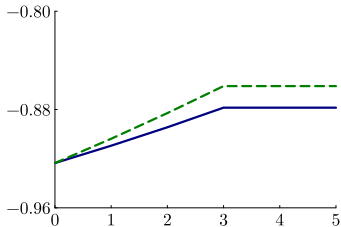
Debt,  $b$



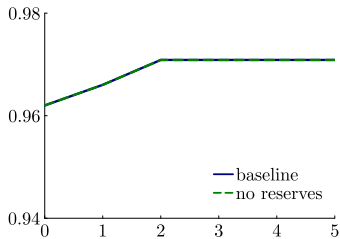
Consumption



Net Foreign Assets

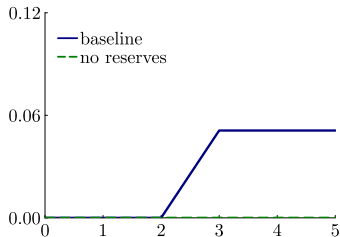


Debt Price,  $q(a', b', s)$

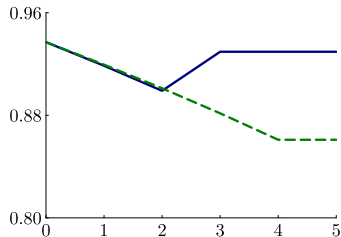


## Longer to exit without reserves

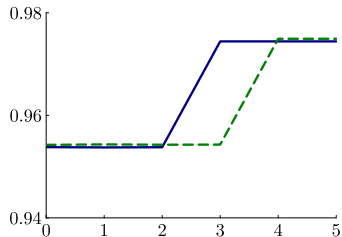
Reserves,  $a$



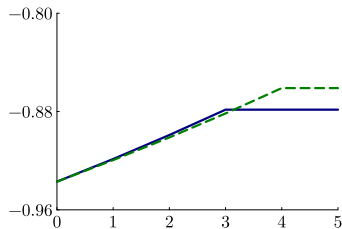
Debt,  $b$



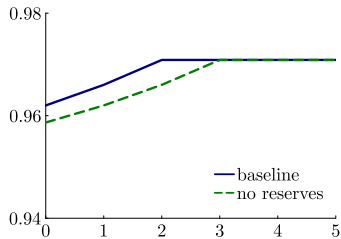
Consumption



Net Foreign Assets



Debt Price,  $q(a', b', s)$



## Conclusions

- Simple theory of optimal foreign reserve management under rollover risk
- Optimal to accumulate reserves to reduce vulnerability
  - However, only after debt has been reduced towards safe zone
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- Simple theory of optimal foreign reserve management under rollover risk
- Optimal to accumulate reserves to reduce vulnerability
  - However, only after debt has been reduced towards safe zone
- Issuing debt to accumulate reserves can reduce spreads
- Findings speak to policy discussions on appropriate level of FX reserves (e.g. IMF)
  - Following a debt crisis, IMF often prescribes increasing reserves
  - However, we find holding reserves not optimal at beginning of deleveraging process



Scan to find the paper!

**THANKS!**

If government not vulnerable tomorrow after repaying in a run:

$$\max_{a'} u \left( y - \frac{\delta + r}{1 + r} b + a - \frac{a'}{1 + r} \right) + \beta V^S(a' - (1 - \delta)b)$$

- **Solution:**  $a'(a, b) = \max[0, a - \delta b]$ .
  - With low initial reserves, government constrained  $\Rightarrow a' = 0$

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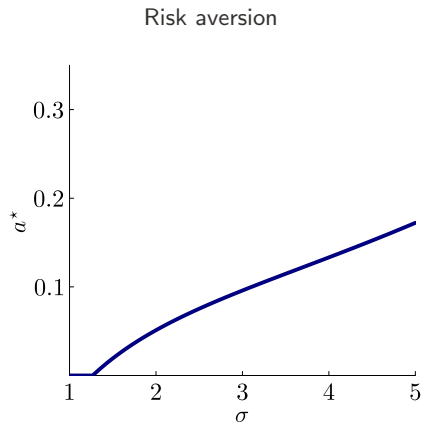
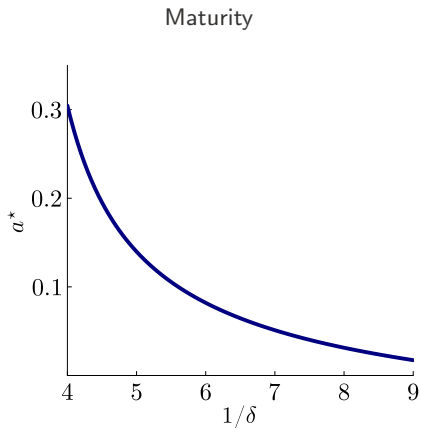
$$\max_{a'} u \left( y - \frac{\delta + r}{1 + r} b + a - \frac{a'}{1 + r} \right) + \beta V^S(a' - (1 - \delta)b)$$

- Solution:  $a'(a, b) = \max[0, a - \delta b]$ .
  - With low initial reserves, government constrained  $\Rightarrow a' = 0$
- If  $a \geq \delta b$  and  $(a - \delta b, (1 - \delta)b) \in \mathcal{S}$ , then  $V_R^-(a, b) = V_R^+(a, b)$ .
  - If high reserves, govt. can achieve unconstrained consumption even in a run
  - Note reserves enough to pay all coupons not needed!
    - Just enough to repay the fraction of the debt that would allow the keep the same NFA,



## Sensitivity: effect of maturity and risk-aversion on $a^*$

▶ back



Panels show the level of  $a^*$  for different values for  $\delta$  and  $\sigma$ . The value of  $\phi$  is recalibrated to match the same debt level  $b^-(0)$  as in baseline.

# Default zone expands

