The Perils of Bilateral Sovereign Debt*

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Abstract

Motivated by the emergence of new official creditors outside the Paris Club framework, we study the interaction between senior official lending and market debt. We develop a quantitative sovereign default model featuring a large senior lender with whom borrowing terms are negotiated. Obtaining more net financing from the market strengthens the government's bargaining position and improves bilateral terms. This endogenous cross-elasticity erodes the discipline of spreads and amplifies debt dilution, leading to welfare losses even as bilateral borrowing helps avert some defaults ex-post. With pre-specified rules, rewarding market issuance can be enough to generate such losses, and an optimal rule instead raises bilateral rates with net market financing. The direction of the cross-elasticity can thus guide in practice the assessment of new forms of bilateral sovereign debt.

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Introduction

A large fraction of sovereign borrowing in emerging-market economies takes the form of official debt, including loans from other governments, regional development banks, or multilateral institutions such as the IMF and the World Bank. The past few decades have witnessed the rise of new sovereign bilateral creditors operating outside of the Paris Club institutional framework (Horn et al., 2021b; Gelpern et al., 2021). The claims to de facto senior creditor status (relative to private creditors) by these new lenders have raised concerns about welfare implications for borrowing countries (Horn et al., 2021a, 2023).

This paper evaluates such concerns in the context of a quantitative sovereign debt and default model, augmented by the presence of a large lender who offers an alternative source of funds. Relative to private creditors in capital markets, this large lender has a superior enforcement technology. This assumption allows us to model the potential success of the new sovereign creditors in becoming senior to bondholders in international capital markets and trace out its implications. In addition, in our baseline setup, we assume that borrowing terms with the large lender are determined through bargaining, which gives a crucial role to the dynamics of the bilateral relationship.

We focus on the interaction between private markets and the large lender as two potential sources of financing.[1] While the availability of bilateral loans from the large lender affects the government's behavior in private debt markets, outcomes in these markets also influence threat points in the bilateral negotiation. The interest rate charged by the large lender is therefore constrained by implicit competition from private debt markets. But when default risk pushes up yields on marketable debt, the large lender is also able to charge a premium. Because there is no default risk on the bilateral loan, such a premium only reflects the borrower's outside options.

The baseline model yields two main results: the presence of the senior lender leads to a form of overborrowing in private debt markets and this creates welfare losses for the government relative to a benchmark in which only market funding is available. While bilateral loans can and are often used on the equilibrium path to avoid costly defaults, the borrowing country is worse off when the large lender is present. One reason for this is that the possibility of borrowing from the large lender while being excluded from private

[1] As most of the quantitative literature on sovereign debt, we focus on debt as a vehicle for consumption smoothing or frontloading and abstract from project financing or development lending.

markets raises the value of default, which, in turn, increases default incentives and lowers (marketable) debt prices. But there is another, more fundamental reason for the welfare losses. Even imposing the constraint that bilateral exposure cannot be increased during default, welfare for the government is still reduced by the presence of the large lender due to a "relational overborrowing" effect. Financing terms with the large lender are determined by both parties' threat points, and the government understands that reaching the bargaining stage with little cash on hand weakens its bargaining position. This effect is captured and summarized by an endogenous cross-elasticity of bilateral loan terms to market debt issuances. The cross-elasticity acts as a countervailing force for the market discipline of spreads and generates an extra incentive for the government to issue debt more aggressively and delever more slowly. The end result is an increase in the default frequency and therefore in spreads, which ultimately create welfare losses for the government.

The dynamics and welfare effects we describe result from three critical assumptions regarding the large creditor and the funding it offers. Compared to competitive markets, we assume that it is more difficult (or impossible) to default on the large creditor, that the terms of borrowing result from bilateral bargaining rather than competition and a zero-profit condition, and that these terms are renegotiated every period (in this case, because of short maturities). Central Bank swap lines constitute a prime real-world example, which Horn et al. (2021b) identify as a dominant channel through which official bilateral lending has reemerged since the 2000s.[2] These facilities are typically short-term and feature none of the cross-default clauses present in government bonds, facilitating seniority. They also involve no recorded defaults. Finally, while their terms are typically strictly confidential, anecdotal evidence suggests that a fair amount of bilateral negotiation is involved in determining amounts and interest rates, rather than market mechanisms or pre-specified rules.

Other forms of official debt share the seniority and/or duration features emphasized in our baseline model—including, for example, some types of IMF programs and repo or swap facilities offered by major central banks such as the Fed or the ECB. However, unlike the bilateral bargaining framework we study, these facilities typically feature borrowing

[2]While swap lines are important in our motivation to study the interaction of market debt and (non-defaultable) bilateral loans, our model is not geared to represent all aspects of swap lines. For example, we do not include a difference in the currency denomination of bilateral and market debt, conditionality (including requirements on the level of foreign reserves), or collateral requirements.

terms that are fixed in advance. For instance, IMF lending is subject to surcharges based on access and duration thresholds (IMF, 2024b), while the Fed's swap lines carry a standard 25 basis point spread over policy rates (Bahaj and Reis, 2023), publicly reported in real time on the FRBNY's website. To accommodate such institutional designs, we develop an extension in which bilateral loan pricing does not result from bargaining but rather follows fixed rules. This allows us to explore how rules-based lending affects debt dynamics and to compare welfare outcomes across alternative institutional arrangements.

The extension shows, in a much simpler setup, that when the interest rate on bilateral loans is relatively insensitive to the government's market debt—that is, when the interest rate is fixed or has a low elasticity to market debt—the relational overborrowing channel is muted or disappears altogether. In this case, the dominant effect of the large lender is to help the government avoid costly defaults on its marketable debt. As a result, the equilibrium features a lower default frequency and reduced sovereign spreads, improving the welfare of the borrowing country. We also find that an optimal design of bilateral lending facilities involves the *opposite* cross-elasticity: it makes the bilateral interest rate rise with the government's net market financing. This design, which is optimal within a parametric family of rules, reinforces the disciplining role of spreads and delivers higher welfare for the government relative to the equilibrium with bargaining and also relative to the equilibrium with only market funding.

The government's bargaining power relative to the large lender is a critical parameter. When the government can make take-it-or-leave-it offers (i.e., when it has full bargaining power), the large lender simply provides non-defaultable loans at the risk-free rate, recovering the model of Hatchondo, Martinez, and Önder (2017). In this case, the government's welfare is also higher relative to the equilibrium without the large lender. In our calibration, however, such gains quickly dissipate as the government's bargaining power declines.

In the baseline model, we assume that the large lender maximizes profits. In fact, it shares the objectives, risk attitudes, and intertemporal preferences of the competitive creditors who lend in debt markets. We make this assumption not to represent any specific bilateral lender accurately, but rather to isolate the pure effect of market structure. However, the feedback from debt issuance to cash on hand to the bilateral surplus, which gives rise to the cross-elasticity and the relational overborrowing effect, will be present unless the large lender has a strong specific motivation to avoid default on the government's marketable debt. As a consequence, we expect the core results to extend to alternative descriptions of the large bilateral creditor, including those with mixed policy, commercial, or financial stability objectives.

A key quantitative finding is that even relatively small bilateral loans can significantly affect sovereign debt outcomes when marketable debt has long maturity. The reason is that debt service on long-term debt is distributed over time, so the amount due in any given period is relatively small. By contrast, bilateral loans—modeled as short-term—must be repaid (or rolled over) in full every period. As a result, fluctuations in the bilateral loan interest rate have an impact on the current-period budget constraint that is comparable in magnitude to changes in the interest rate on a much larger stock of marketable debt.

The model delivers a simple heuristic for evaluating whether a particular form of bilateral lending is likely to be beneficial for the borrowing country. The key determinant is how the interest rate on bilateral loans responds to the government's choices in debt markets: if this cross-elasticity is such that bilateral loan terms improve after larger debt issuances, the relational overborrowing channel is likely to be active, as the large lender's willingness to lend cheaply amplifies the government's incentive to accumulate market debt. By contrast, if the terms of bilateral lending become more favorable when debt levels are low or falling, it is more likely to support debt sustainability and improve welfare. These effects operate alongside more familiar channels related to avoiding improvements in the value of default, as recognized in policies like the IMF's Financing Assurances and Sovereign Arrears framework (IMF, 2024a), which aim to limit (bilateral) financing at least until a "credible process for debt restructuring is underway."

Discussion of the Literature We contribute to a nascent literature on the interaction of different types of sovereign debt.[3] Hatchondo, Martinez, and Önder (2017) find that introducing a limited amount of non-defaultable debt improves the government's welfare but only for a short period of time, while Cordella and Powell (2021) show how the preferred creditor status of international financial institutions can arise endogenously. Dellas and Niepelt (2016) and Wicht (2025) focus on the coexistence of official and private creditors and show how an interior solution featuring both types of lending can arise as a choice of the borrowing government. Boz (2011) and Fink and Scholl (2016) study the interaction of

[3] Excellent surveys of the broader literature on sustainable public debt and sovereign default can be found in handbook chapters by Aguiar and Amador (2014), Aguiar, Chatterjee, Cole, and Stangebye (2016), D'Erasmo, Mendoza, and Zhang (2016), and Martinez, Roch, Roldán, and Zettelmeyer (2023).

market debt with a type of senior debt coming with conditionality. Kirsch and Rühmkorf (2017) and Roch and Uhlig (2018) investigate the role of official lending or bailout agencies in eliminating equilibrium multiplicity, as do Corsetti, Guimarães, and Roubini (2006). The model we consider does not feature this type of equilibrium multiplicity, which would open the door to welfare gains of bilateral loans if they could help rule out unfavorable equilibria. Importantly, we also abstract from conditionality and focus on the impact of market design itself. Also related are Kovrijnykh and Szentes (2007), Faria-e-Castro, Paul, and Sánchez (2024), or Bethune et al. (2024), who describe situations featuring strategic behavior of creditors in different contexts, like unsecured firm or household credit.

Models of bargaining have been used in the context of sovereign debt, but only to study the negotiations between debtor and creditors to resolve a default (Yue, 2010; Asonuma and Joo, 2020; Dvorkin et al., 2021; Fourakis, 2025; Almeida et al., 2025). The idea of issuing debt to affect the threat point in a subsequent negotiation is also present in classic contributions by Dasgupta and Sengupta (1993), Bronars and Deere (1991), or Perotti and Spier (1993), in the context of firms negotiating wage contracts. In these models, debt issuance is beneficial for the firm because it commits resources to debt repayment and thus shrinks the pie to be shared with workers, limiting wage increases and strengthening the firm's negotiating position. Similar to the dynamics we emphasize here, the firm then ends up with more debt than it would otherwise have chosen.

More recently, Arellano and Barreto (2025) combine market and official debt in a model of partial default (Arellano et al., 2023) and find that competitive pricing and longer maturities, like those associated with bilateral lending from the Paris Club, endogenously make official debts less risky despite more favorable restructuring terms. Liu, Liu, and Yue (2025) show how the best subgame-perfect equilibrium of a sovereign debt model with market, bilateral, and multilateral creditors decentralizes the constrained-efficient allocation with imperfect information and moral hazard. Similarly, Dovis and Kirpalani (2023) propose a design for a lender of last resort that improves the worst equilibrium. We view these papers as emphasizing how certain institutions could potentially improve welfare in a world with both market and official debts, while our results underscore the risks of such a market structure. Also on the perils side, Kondo, Sosa-Padilla, and Swaziek (2026) study a sovereign debt model augmented by exogenous capital flows from a large lender (in their case, China) and focus on the risk that it demands repayment unexpectedly.

Empirically, Perks et al. (2021), and Bahaj and Reis (2021, 2023) document the network of Central Bank swap lines. Cesa-Bianchi et al. (2022) study swap lines among advanced economies, where they can serve a different purpose.

Layout The rest of the paper is structured as follows. Section 2 presents some motivating evidence on the increasing role of official external debt in emerging markets. Section 3 presents two minimal models which describe the emergence of the cross-elasticity and how it can increase incentives for debt dilution. Then, Section 4 describes the main model in which both types of debt coexist, Section 5 analyzes its equilibrium, and Section 6 describes the extension with fixed rules for the terms of bilateral loans. Finally, Section 7 concludes.

2. Motivating Evidence and Context

Figure 1 summarizes the evolution of official external debt across emerging-market and developing economies, using the World Bank's International Debt Statistics (IDS). The data include all public and publicly guaranteed (PPG) external sovereign debt, measured in constant 2023 USD and aggregated across debtor countries. We distinguish between three broad official creditor categories: multilateral lenders (including the IMF and World Bank), Paris Club bilateral lenders,[4] and other bilateral creditors—primarily non-Paris Club official lenders such as China.

The data show a clear shift in the composition of official external debt over the past two decades. While the IMF and World Bank, along with Paris Club members, accounted for the bulk of official lending in the early 2000s, other official creditors have grown rapidly in both absolute and relative terms. By 2023, the landscape is divided into three roughly equal parts: the IMF and the World Bank, other multilaterals, and bilateral creditors. Among bilaterals, China has emerged as the single largest lender.

While the rise of new bilateral creditors is evident from Figure 1, it only reflects liabilities included within the official perimeter of PPG debt and hence reported to the World Bank.[5] Arrangements with Central Banks (notably, swap lines) are often not included in

[4] As of 2026, the 22 permanent members of the Paris Club are Australia, Austria, Belgium, Brazil, Canada, Denmark, Finland, France, Germany, Ireland, Israel, Italy, Japan, Netherlands, Norway, Russia, South Korea, Spain, Sweden, Switzerland, the United Kingdom, and the United States.

[5] Even these PPG debt statistics suffer from systematic underreporting (Horn et al., 2026).

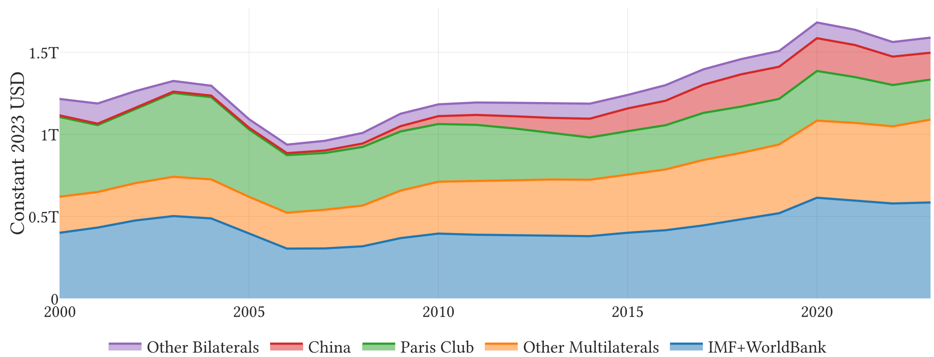

FIGURE 1: TOTAL OFFICIAL DEBT
Source: International Debt Statistics, The World Bank

such perimeters and therefore not captured by this or similar data sources.[6] This limitation in the data illustrates the opacity of some of these deals (other aspects, like interest rates and maturities, are also typically confidential), which underpins concerns about the welfare implications of these new lenders (Horn et al., 2021a, 2023; Gamboa, 2023). Moreover, the scarce data on certain lending channels underscores the need for analytical tools that go beyond reported debt stocks to capture strategic interactions in sovereign borrowing decisions. The rest of our paper addresses these concerns through a theoretical and quantitative evaluation of a model that incorporates a large bilateral lender operating alongside competitive market creditors.

3. Two Minimal Models

We begin our analysis by considering two stylized environments. These are designed to clarify the core mechanisms operating in the full model described in Section 4. The first stylized model describes how the cross-elasticity emerges from the government's attempts to improve its bargaining position. The second stylized model takes the cross-elasticity as given and shows how it magnifies the incentives for debt dilution, potentially leading to welfare losses.

[6] Bahaj and Reis (2021, 2023) compile data on the network of Central Bank swap lines and, in many cases, are only able to see the total committed size of the swap lines, not drawn or borrowed amounts.

3.1 How the Cross-Elasticity Emerges from Bargaining

A key feature of the quantitative model we introduce in Section 4 is a cross-elasticity of bilateral loan terms to market debt issuance. That full model features a within-period timing in which the government first chooses its market debt issuance and only afterward negotiates with the bilateral lender. As a result, the government can use market debt issuance to influence its cash on hand and, hence, its threat point at the bargaining stage.[7]

The simplified setup in this section abstracts from market debt itself and instead takes the government's cash on hand as a primitive. This allows us to isolate the bargaining mechanism and characterize how cash on hand (or, more generally, initial resources) affects the government's threat point and, through it, the amount and interest rate on the bilateral loan.

There are two dates, $t = 0, 1$. The government receives endowments $y_0 < y_1$ and seeks to maximize an expected-utility objective with a strictly concave per-period utility function u and discount factor $\beta \in (0, 1)$:

$$V = u(c_0) + \beta u(c_1)\,.$$

The large (and only) lender is risk-neutral and discounts time at $\beta_L \in (\beta, 1)$. All action occurs at the bargaining stage of period 0, when the government negotiates a transfer x in exchange for a repayment $m' = x(1+r)$ due in period 1. The government commits to this repayment. With a Nash bargaining weight of θ for the lender, the bargaining problem is

$$\max_{x,r} \mathscr{L}(x,r)^{\theta} \times \mathscr{B}(x,r;y_0,y_1)^{1-\theta} \tag{1}$$

where both parties' surplus functions are

$$\begin{aligned}\mathscr{L}(x,r) &= -x + \beta_L x(1+r) = x(\beta_L(1+r) - 1)\\ \mathscr{B}(x,r;y_0,y_1) &= \underbrace{u(y_0 + x) + \beta u(y_1 - x(1+r))}_{\text{agreement}} - \big(\underbrace{u(y_0) + \beta u(y_1)}_{\text{threat point}}\big)\,.\end{aligned}$$

The lender's surplus $\mathscr{L}$ reflects the provision of the transfer x in exchange for $x(1+r)$

[7] Section 5.3 discusses how this type of threat-point manipulation can be captured under alternative timing assumptions.

in the following period. Without loss of generality, we normalize the lender's outside option to 0. The government's surplus $\mathscr{B}$ is similar, except that if negotiations break down, the government can always consume the endowment.

The focus of this section is to study the comparative statics of borrowing quantities x and terms r to the government's cash on hand y_0. As mentioned above, in the quantitative model, the government is able to affect its cash on hand by choosing its issuance of market debt. How the outcome of bargaining changes with y_0 measures the strength of the incentive to engage in threat-point manipulation.

The solution to the bargaining problem (1) is characterized by two first-order conditions: an Euler equation,

$$u'(c_0) = \frac{\beta}{\beta_L} u'(c_1)\,,$$

and a surplus-sharing rule,

$$u'(c_0) = \frac{\theta}{1-\theta} \frac{\mathscr{B}(x, r; y_0, y_1)}{\mathscr{L}(x, r)}\,,$$

where $c_0 = y_0 + x$ and $c_1 = y_1 - x(1 + r)$ are the on-path consumption levels.

The Euler equation means that the size of the transfer x should satisfy the borrower frontloading motive when using the lender's discount rate β_L^{-1} as the interest rate. In other words, the transfer equates the government's marginal rate of substitution $\frac{u'(c_0)}{\beta u'(c_1)}$ to the marginal rate of transformation β_L^{-1} provided by the lender. In this sense, the intertemporal allocation of consumption is efficient. The surplus-sharing rule determines the terms at which the transfer is provided, setting the interest rate r to allocate surplus across lender and borrower.

Lemma 1. *Suppose the transfer size x is fixed at some $x^\star > 0$. Then, higher cash on hand y_0 leads to better terms when bargaining with the lender: $\frac{\partial r}{\partial y_0} < 0$.*

Proof of Lemma 1 In this case the first-order condition is

$$\theta \beta_L \mathscr{B}(x^\star, r; y_0, y_1) - (1-\theta)\beta \mathscr{L}(x^\star, r) u'(c_1) = 0$$

Omitting the arguments of $\mathscr{B}$ and $\mathscr{L}$ for readability, defining $F(r, y_0) = \theta\beta_L\mathscr{B} - (1 - \theta)\beta\mathscr{L}u'(c_1)$, and applying the implicit function theorem, we have

$$\frac{\partial r}{\partial y_0} = -\frac{\partial F/\partial y_0}{\partial F/\partial r} = -\frac{\theta\beta_L\frac{\partial\mathscr{B}}{\partial y_0}}{\theta\beta_L\frac{\partial\mathscr{B}}{\partial r} - (1-\theta)\beta\frac{\partial\mathscr{L}}{\partial r}u'(c_1) + (1-\theta)\beta\mathscr{L}u''(c_1)x}$$

For given (x, r), strict concavity of u means that higher cash on hand y_0 reduces the government's surplus, so we have $\frac{\partial\mathscr{B}}{\partial y_0} < 0$, and the numerator is negative. For the denominator, a higher interest rate r reduces the government's surplus, so $\frac{\partial\mathscr{B}}{\partial r} < 0$, and increases the lender's surplus, so $\frac{\partial\mathscr{L}}{\partial r} > 0$. Finally, the last term in the denominator is also negative, because of the concavity of u. In fact, the entire denominator being negative is the second-order condition for the maximization problem. We are then left with a negative numerator as well as a negative denominator, with a minus sign in front, which yields a negative sign for the derivative $\frac{\partial r}{\partial y_0}$. □

Lemma 2. *Suppose the interest rate r is fixed at some $r^\star > 0$. Then, higher cash on hand y_0 leads to a lower reliance on the lender: $\frac{\partial x}{\partial y_0} < 0$.*

Proof of Lemma 2 In this case the first-order condition is

$$\theta\mathscr{B}(x, r^\star; y_0, y_1) + (1-\theta)x\frac{\partial\mathscr{B}}{\partial x}(x, r^\star; y_0, y_1) = 0$$

Defining $F(x, y_0) = \theta\mathscr{B} + (1-\theta)x\frac{\partial\mathscr{B}}{\partial x}$, and applying the implicit function theorem once more, we have

$$\frac{\partial x}{\partial y_0} = -\frac{\partial F/\partial y_0}{\partial F/\partial x} = -\frac{\theta\frac{\partial\mathscr{B}}{\partial y_0} + (1-\theta)xu''(c_0)}{\frac{\partial\mathscr{B}}{\partial x} + (1-\theta)\left(u''(c_0) + \beta(1+r)^2u''(c_1)\right)}$$

Concavity of the utility function means that $(1-\theta)xu''(c_0) \leq 0$ and, as in the previous case, we have $\frac{\partial\mathscr{B}}{\partial y_0} < 0$. Again, the denominator being negative corresponds to the second-order condition for the maximization problem. Ultimately, the relevant expression is the ratio of two negative numbers, with a minus sign in front. □

When both x and r are free, this force can be split across the two margins. Higher cash on hand lowers the surplus generated by any given agreement, so the Nash solution restores surplus sharing through a lower transfer, a lower interest rate, or both.[8] In the quantitative model, the relevant equilibrium responses are to reduce both the transfer and the interest rate when cash on hand is higher, as we show numerically in Section 5.

Discussion In the quantitative model, this cash-on-hand comparative static maps into the government's market debt choice. The government first chooses market debt issuance and then enters bilateral negotiations. On the upward-sloping part of the debt Laffer curve, a higher choice of b' raises current resources and therefore strengthens the government's bargaining position vis-à-vis the large lender.[9] This is the origin of the cross-elasticity of bilateral loan terms to market debt issuance.

3.2 How the Cross-Elasticity Worsens Debt Dilution

We turn to analyzing the effect of the cross-elasticity on debt dilution. Debt dilution is a well-known feature of models with long-term debt (Hatchondo, Martinez, and Sosa-Padilla, 2016), in which new issuances affect the price of marginal and inframarginal debt issued currently, but also of outstanding debt issued in the past. As a consequence, with commitment the government would want to affect its future issuance strategy to boost current prices, leading to time inconsistency. In this section we show that the cross-elasticity magnifies the losses from this time-inconsistency problem: borrowing capacity with the large lender raises welfare for a government that can commit, but may reduce it for a government that decides sequentially.

There are three dates, $t = 0, 1, 2$. In the final period, the government receives a stochastic endowment $y(z)$, where $z \sim \mathcal{N}(0, \sigma_z^2)$, but wishes to consume in all periods to maximize an expected-utility objective

$$V = \mathbb{E}\left[\sum_{t=0}^{2} u(c_t)\right] \tag{2}$$

[8] In principle, the effect of reducing one of these variables could be so large as to require an increase in the other. The conditions required to rule out these pathological cases seem to be quite weak and are satisfied for most parametrizations of this problem.

[9] Initial market debt works differently: legacy obligations reduce current resources and generally weaken the government's bargaining position (some counter-productive dynamics can already be glimpsed here: what strengthens the government in the present also weakens it later). The cross-elasticity we emphasize here refers to the effect of new issuance on resources available before the bilateral negotiation.

with an increasing and concave utility function u, satisfying Inada conditions.

To finance consumption in all three periods, the government issues long-term bonds b at $t = 0$ as well as short-term bonds d and a bilateral loan m at $t = 1$. All three are expressed as promises to repay one unit of the good at $t = 2$. While the loan m is non-defaultable, the government faces a choice of whether to repay the bonds b and d, subject to some default costs $\xi(z)$. In other words, consumption in period 2 is given by

$$c_2 = \begin{cases} y(z) - m - (b + d) & \text{if the government repays} \\ y(z) - \xi(z) - m & \text{if the government defaults} \end{cases}$$

In this finite-horizon example, since the utility function is increasing, default occurs exactly in those states for which the costs of default fall short of the value of the debt, regardless of the bilateral loan which must be repaid either way. In other words,

$$c_2 = y(z) - m - \min\{b + d, \xi(z)\} \tag{3}$$

In the middle period, the government issues short-term debt d and loans m. Thanks to competition among risk-neutral investors, the price of debt reflects the probability of default, so $q(b, d) = \mathbb{P}\left(b + d < \xi(z)\right) = q(b + d)$, with a slight abuse of notation. Notice that the price of short-term debt also depends on the outstanding amounts of long-term debt (in a perfectly-substitutable way, as default affects both types of bonds equally).

In order to focus on how the cross-elasticity worsens the debt dilution problem, we assume an exogenous rule for the pricing of bilateral loans which takes the existence of the cross-elasticity as given: more legacy debt b may induce a lower price (higher interest rate) but more recently-issued debt d induces higher prices (the cross-elasticity), exactly as suggested by the bargaining model of Section 3.1. To keep things simple, we assume a linear pricing rule:

$$\phi(b, d, m) = \alpha_0 + \alpha_b b + \alpha_d d + \alpha_m m$$

where $\alpha_d > 0$ to represent in reduced form the negative response of the bilateral interest rate to cash on hand in Lemma 1. We also impose a limit on the bilateral loan, $\bar{m}$. Consumption in the middle period is

$$c_1 = dq(b + d) + m\phi(b, d, m) \tag{4}$$

Finally, in the initial period, the government issues long-term debt b at price q, but investors hold beliefs d^e about the issuance of short-term bonds d, which will happen later. Initial consumption is

$$c_0 = bq(b + d^e) \tag{5}$$

for the same price function q as above (notice that, since the bilateral loan m does not affect the default set, there is no need to anticipate m to decide q).

We consider different cases for the government's ability to influence beliefs d^e.

Commitment When the government can commit, it chooses $\{b, m, d^e, d\}$ to maximize (2) subject to (3), (4), (5), $m \leq \bar{m}$ and $d = d^e$.

The first-order conditions for m (using λ to denote the multiplier on the debt limit constraint) and b are

$$u'(c_1)\left(\phi + m\frac{\partial \phi}{\partial m}\right) = \mathbb{E}\left[u'(c_2)\right] + \lambda \tag{6}$$

$$u'(c_0)\Big(bq'(b+d) + q(b+d)\Big) + u'(c_1)\left(dq'(b+d) + m\frac{\partial \phi}{\partial b}\right) = q(b+d)\mathbb{E}\left[u'(c_2^R)\right]$$

and the first-order condition for d is

$$u'(c_0)bq'(b+d) + u'(c_1)\left(dq'(b+d) + q(b+d) + m\frac{\partial \phi}{\partial d}\right) = q(b+d)\mathbb{E}\left[u'(c_2^R)\right] \tag{7}$$

where c_2^R denotes period-2 consumption conditional on repayment. Because of equal seniority of short- and long-term debt, the contribution of period 2 to the first-order conditions for b and d is the same. The loan, by contrast, is repaid in all states: its marginal cost in (6) is the unconditional expectation $\mathbb{E}\left[u'(c_2)\right]$, while the bonds' marginal cost is only incurred in repayment states.

Discretion When deciding sequentially, the government takes as given a value d^e and chooses $\{b, m, d\}$ to maximize (2) subject to (3), (4), (5) and $m \leq \bar{m}$.[10] Equilibrium then

[10]Here, investors form beliefs about d before observing the choice of b. A more natural sequential-equilibrium formulation could be that lenders form expectations about default and future borrowing simultaneously and hence set a schedule $d^e(b)$. This schedule would imply extra terms appearing in the relationship between b and c_0, relating to the derivative of d^e. Since these terms have no equivalent in the commitment solution, the formulation here separates the two setups more cleanly. The full quantitative model of Section 4 features bond pricing which incorporates all relevant information.

requires investors to be correct in their assessment, so that the government does choose $d = d^e$. As a result, the first-order conditions for both b and m remain unchanged, but the first-order condition for d is now 'missing' the term reflecting the manipulation of d^e by the committed government:

$$u'(c_1)\left(dq'(b+d) + q(b+d) + m\frac{\partial\phi}{\partial d}\right) = q(b+d)\mathbb{E}\left[u'(c_2^R)\right] \tag{8}$$

How the cross-elasticity worsens debt dilution Debt dilution manifests itself in the different issuance choices under commitment and discretion. We therefore compare three solutions, which Figure 2 reports as functions of the maximum amount $\bar{m}$ that can be drawn from the bilateral loan: the commitment solution (b_C, d_C); the equilibrium with discretion (b_D, d_D); and an intermediate object (b_{DC}, d_{DC}), defined as the choices of a sequential government that reoptimizes while bond prices still reflect commitment beliefs, $d^e = d_C$. The intermediate solution isolates the temptation to deviate from the commitment plan; the comparison between d_{DC} and d_D then measures the amplification that comes from the equilibrium adjustment of prices. The result of this section is that the gap between d_C and d_D grows with $\bar{m}$: more bilateral borrowing capacity can worsen dilution, hurting welfare.

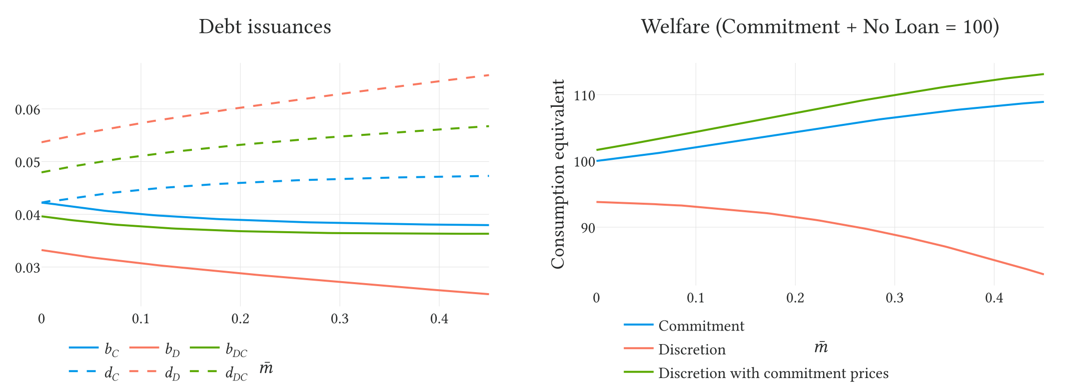

FIGURE 2: DEBT DILUTION AND THE CROSS-ELASTICITY

Note: The figure assumes $(\alpha_0, \alpha_b, \alpha_d, \alpha_m) = (-0.025, -1.5, 2.5, 0)$, a CRRA utility function with risk aversion $\gamma = 2$, income volatility $\sigma_z = 10\%$, and output costs of default $\xi(z) = 0.1 \cdot y(z)$

A useful benchmark is the commitment solution with $\bar{m} = 0$. With no discounting and the same default technology applying to both bonds, the committed government treats the two issuance periods symmetrically: the cross-price terms $u'(c_0)\,bq'$ and $u'(c_1)\,dq'$ appear in both (6) and (7), so subtracting one condition from the other leaves

$$u'\Big(\underbrace{q(b+d)\,b}_{=c_0}\Big)\,q(b+d) = u'\Big(\underbrace{q(b+d)\,d}_{=c_1}\Big)\,q(b+d) \qquad \Longrightarrow \qquad c_0 = c_1 \ \text{ and } \ b = d\,.$$

Against this benchmark, the committed and the sequential government differ in exactly one term of the first-order condition for d: comparing (7) with (8), the sequential government does not perceive the *dilution wedge*

$$\Delta \equiv -u'(c_0)\,b\,q'(b+d) > 0\,, \tag{9}$$

the marginal damage that anticipated period-1 issuance inflicts on the period-0 price of long-term debt. The committed government internalizes Δ and stops issuing d while the flow benefit of issuance still exceeds its period-2 cost by exactly Δ. The sequential government lacks the wedge Δ in its first-order condition and compensates by increasing d to restore equality between marginal benefits and costs. It therefore issues more, and this comes at the expense of b: the higher default probability worsens debt prices and induces the government to curb long-term issuance. At $\bar{m} = 0$, the gap $d_D - d_C$ measures the extent of debt dilution coming only from the presence of long-term debt.

The bilateral loan magnifies these effects. Suppose the borrowing limit binds, so that $m = \bar{m}$. More m raises period-1 resources directly through the proceeds $m \cdot \phi$ as well as indirectly because, with $\alpha_d > 0$, each unit of d now also improves the price ϕ and adds $m \cdot \alpha_d$. These extra resources at $t = 1$ lower the marginal utility $u'(c_1)$. The wedge Δ, in contrast, is predetermined and thus unaffected by m. Hence, the compensating effect of d on the first-order condition (8) is weaker, because the wedge is unchanged but the effect of d, which is scaled by $u'(c_1)$, is lower. As a consequence, the sequential government relies on quantities and chooses a larger d as m grows. The temptation to dilute, measured by $d_{DC} - d_C$ (ignoring for now the equilibrium response of expectations), is therefore increasing in m, as the left panel of Figure 2 confirms.

Equilibrium expectations amplify the deviation further. When investors anticipate the higher issuance, d^e rises and the period-0 price of long-term bonds falls; the government responds by reducing b, offsetting this effect only partially. Lower prices pull time-1 issuance in two directions: each bond raises less revenue, which discourages issuance through the substitution effect, but the implied fall in revenues raises the marginal utility of period-1 consumption, which encourages issuing more through the income effect. With CRRA utility and relative risk aversion above one, the income effect dominates and the equilibrium response adds to the deviation: $d_D > d_{DC}$, with this gap also widening in $\bar{m}$.[11]

The right panel of Figure 2 summarizes the welfare implications. Under commitment, the bilateral loan is unambiguously valuable as a direct consequence of free disposal. The value of reoptimizing while prices still reflect commitment beliefs is even higher, reflecting the time-inconsistent nature of the problem. Once investors anticipate this deviation, which is increasing in m as described above, equilibrium welfare under discretion falls as the debt-dilution problem worsens. The quantitative model of Section 4 embeds this mechanism in an infinite-horizon setting in which the cross-elasticity emerges endogenously from bargaining.

4. Model with bilateral loans and defaultable market debt

We now present the full version of the model, in which the borrowing government has access to the large lender as well as to a competitive fringe of private investors (or bondholders). Default on the debt b held by bondholders is possible, subject to standard income costs of default and temporary market exclusion. However, just as before, bilateral loans m cannot be defaulted on.

The timeline of events within a period is illustrated in Figure 3. At the start of t, the government owes b to the market, m to the large lender, and draws the exogenous state z, which evolves according to a Markov process with transition F. Let $v(b, m, z)$ and $h(b, m, z)$ represent the government's and the large lender's value functions at the beginning of a period, when the government has market access (we describe below how market access is lost and regained). The government accesses the market at the beginning (morning) of each period t and chooses whether to default or repay. If it chooses to repay, the government can issue new market debt. Later, in the afternoon of period t, the government contacts the large lender and bargains over the terms of a bilateral loan, as illustrated in Figure 3.

[11] The coefficient α_b matters for the gap and also for the commitment benchmark itself. In the quantitative model, legacy market debt weakens the government at the bargaining stage, which maps into $\alpha_b < 0$ here. Taking this into account, we have that a larger m tilts even the committed government away from long-term and toward short-term issuance, consistent with the behavior of b_C and d_C in Figure 2.

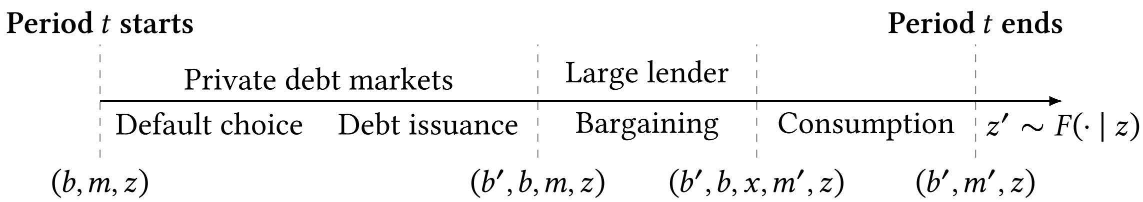

FIGURE 3: TIMELINE OF EVENTS WHILE NOT IN DEFAULT

Private markets In the morning of t, conditional on market access, the first decision the government makes is the default/repayment choice.

$$v(b, m, z) = \max \{v_R(b, m, z) + \epsilon_R, v_D(m, z) + \epsilon_D\}, \tag{10}$$

where the ϵ's follow a Type 1 Extreme Value distribution with scale parameter χ, yielding closed forms for $v(b, m, z)$ and the default probability $P(b, m, z)$

$$\begin{aligned} v(b, m, z) &= \chi \log \big(\exp(v_D(m, z)/\chi) + \exp(v_R(b, m, z)/\chi)\big) \\ P(b, m, z) &= \frac{\exp(v_D(m, z)/\chi)}{\exp(v_D(m, z)/\chi) + \exp(v_R(b, m, z)/\chi)} \end{aligned}$$

We assume that defaults on market debt are total (i.e., all outstanding bonds become worthless) and they trigger exclusion from international capital markets. Access to markets is then recovered stochastically with constant probability ψ. While the government remains in default, income is reduced by $\xi(z)$, where ξ is increasing and convex. In addition, the maximum amount that can be borrowed from the large lender m' while in default on market debt is constrained by $m' \leq \Gamma(m)$. We begin our analysis without restrictions by setting $\Gamma(m) = +\infty$, but describe below different possibilities for the value of $\Gamma(m)$.

If default is not chosen in the current period, the government issues new debt b' to the fringe of investors, understanding the value of entering negotiations with the senior lender after having issued this debt level b',

$$v_R(b, m, z) = \max_{b'} w_R(b', b, m, z), \tag{11}$$

where debt is a perpetuity with geometrically-decaying coupons (Leland, 1998; Hatchondo and Martinez, 2009; Arellano and Ramanarayanan, 2012). A unit of debt issued in period t promises to repay $\kappa(1 - \delta)^{s-1}$ units of the tradable good in period $t + s > t$, effectively making a unit issued at $t-1$ a perfect substitute for $(1-\delta)$ units issued at t. In this sense, δ controls the maturity of debt while κ controls the average size of the coupon payments. We refer to the resulting policy function for the market debt choice as $g_b(b, m, z)$.[12]

The price faced by the borrower government reflects the bondholders' expectations of repayment, discounted with a risk-neutral kernel,

$$q(b', b, m, z) = \frac{1}{1+r^*}\mathbb{E}\left[\left(1 - \mathbb{1}_D(b', m', z')\right)\left(\kappa + (1-\delta)q(b'', b', m', z')\right) \mid z\right], \tag{12}$$

where $r^* = \frac{1}{\beta_L} - 1$ is the international risk-free rate,[13] $\mathbb{1}_D(b, m, z)$ denotes the government's default policy as perceived by bondholders, $m' = m'_R(b', b, m, z)$ is the expected result of negotiations with the large lender, to happen later in the period, and $q(b'', b', m', z')$ is the expected future debt price, taking into account the government's issuance strategy g_b in the following period.

Notice that private investors must anticipate several future actions and outcomes when pricing bonds, in addition to the default probability itself: the government's future debt b'' (as is standard with long-term debt), but also the result of the bargaining stage m', which happens in the afternoon and affects both repayment and debt choices in the following period. Since legacy debt b is informative about these objects, it becomes relevant for pricing newly issued debt b'. This contrasts with most models of sovereign default, in which legacy debt is irrelevant for pricing.

Bilateral loan In the afternoon of t, the government meets with the large lender to negotiate the bilateral loan. As in Section 3.1, the large lender can provide funds up to some limit $\bar{m}$. The outcome of their negotiation is a transfer x and a new loan size m' which solve the following Nash bargaining problems, depending on default status vis-à-vis the

[12] As has become the norm in many studies of sovereign debt, we include preference shocks for the issuance decision, making $g_b(b, m, z)$ a random variable. However, we keep the preference shocks' variance small to avoid impacting the equilibrium, and omit them from the exposition of Bellman and pricing equations to ease notation.

[13] We set $\kappa = r^* + \delta$, so that the price of debt equals 1 when the government repays with certainty.

market

$$\max_{m',x} \mathscr{L}_R(b',x,m,m',z)^{\theta} \times \mathscr{B}_R(b',b,x,m,m',z)^{1-\theta}$$
$$\text{subject to} \quad x \geq -m; \quad 0 \leq m' \leq \bar{m}$$
$$\text{or} \tag{13}$$
$$\max_{m',x} \mathscr{L}_D(x,m,m',z)^{\theta} \times \mathscr{B}_D(x,m,m',z)^{1-\theta}$$
$$\text{subject to} \quad x \geq -m; \quad 0 \leq m' \leq \min\{\Gamma(m), \bar{m}\}$$

As in the stylized model of Section 3.1, the large lender's surplus reflects the provision of the transfer x in exchange for a new balance m'. Some new terms appear, reflecting (*i*) the dynamic nature of this infinite-horizon environment and (*ii*) interactions with private debt markets. These are captured by the surplus functions $\mathscr{L}_R$, $\mathscr{L}_D$, $\mathscr{B}_R$, and $\mathscr{B}_D$:

$$\begin{aligned} \mathscr{L}_R(b',x,m,m',z) &= -x + \beta_L \mathbb{E}\left[h(b',m',z') \mid z\right] - \left[m + \beta_L \mathbb{E}\left[h(b',0,z') \mid z\right]\right] \\ \mathscr{L}_D(x,m,m',z) &= -x + \beta_L \mathbb{E}\left[\psi h(0,m',z') + (1-\psi) h_D(m',z') \mid z\right] - \\ &\qquad \left[m + \beta_L \mathbb{E}\left[\psi h(0,0,z') + (1-\psi) h_D(0,z') \mid z\right]\right] \end{aligned}$$

and

$$\begin{aligned} \mathscr{B}_R(b',b,x,m,m',z) &= u(y(z) + B(b',b,m,z) + x) + \beta \mathbb{E}\left[v(b',m',z') \mid z\right] - \\ &\qquad \left[u(y(z) + B(b',b,m,z) - m) + \beta \mathbb{E}\left[v(b',0,z') \mid z\right]\right] \\ \mathscr{B}_D(x,m,m',z) &= u(y_D(z) + x) + \beta \mathbb{E}\left[\psi v(0,m',z') + (1-\psi) v_D(m',z') \mid z\right] - \\ &\qquad \left[u(y_D(z) - m) + \beta \mathbb{E}\left[\psi v(0,0,z') + (1-\psi) v_D(0,z') \mid z\right]\right] \end{aligned}$$

The lender's surplus depends on the initial balance m, which must be repaid in full if negotiations break down and the threat point is exercised (this is the sense in which bilateral loans are non-defaultable). The lender's continuation value ($h(b',m',z')$ in case of repayment or $h_D(m',z')$ in case of default) summarizes the large lender's outcomes at the future state (b',m',z').[14] When the threat point is exercised, we assume that, in addition to the lender receiving the full m, future rounds of bargaining can still happen, with a reset balance $m' = 0$. Finally, if the government is not in default, the bargaining stage occurs with knowledge of the debt issuance b', while if the government is in default, continuation values reflect that private market access is restored with probability ψ and

[14] In the stylized model of Section 3.1, with only two periods, $h(m,z) = m$.

zero market debt.

The government's surplus functions $\mathscr{B}_R$ and $\mathscr{B}_D$ and its state transitions also reflect these effects, where $y_D(z) = y(z) - \xi(z)$ is income in default and $B(b', b, m, z)$ summarizes net financing from the competitive investors received in the morning. Our assumptions on debt maturity yield $B(b', b, m, z) \equiv q(b', b, m, z)(b' - (1-\delta)b) - \kappa b$. In default, opportunities to reaccess markets arrive with probability ψ. The bargaining problems yield outcomes for the bilateral loan $\left\{x_R(b', b, m, z), m'_R(b', b, m, z)\right\}$ and $\left\{x_D(m, z), m'_D(m, z)\right\}$ in repayment and in default, respectively.

This bargaining problem is cast in terms of a transfer $x \geq -m$ in exchange for a new balance m'. The implicit interest rate r can be recovered via the identity

$$(x + m)(1 + r) = m', \tag{14}$$

which maps into the stylized two-period setup of Section 3.1.

The most important way in which the presence of debt markets affects the bargaining stage is through the net flows from debt repayment and new issuance, $B(b', b, m, z)$. These flows modulate the government's threat point: after a successful issuance which raises large revenues, the government is in a strong position to negotiate as repaying m is less costly.

Consumption After the negotiation is done and transfers settled, consumption takes place.

$$\begin{aligned} c_R(b', b, m, z) &= y(z) + B(b', b, m, z) + x_R(b', b, m, z) \\ c_D(m, z) &= y(z) - \xi(z) + x_D(m, z) \end{aligned} \tag{15}$$

Given transfers $x_R(b', b, m, z)$ and $x_D(m, z)$, we can now return to the beginning of the period and determine the values of repaying and defaulting

$$\begin{aligned} w_R(b', b, m, z) &= u(c_R(b', b, m, z)) + \beta \mathbb{E}\left[v\left(b', m'_R(b', b, m, z), z'\right) \mid z\right] \quad \text{and} \\ v_D(m, z) &= u(c_D(m, z)) + \beta \mathbb{E}\left[\psi v\left(0, m'_D(m, z), z'\right) + (1 - \psi) v_D\left(m'_D(m, z), z'\right) \mid z\right], \end{aligned} \tag{16}$$

while for the large lender we have

$$
\begin{aligned}
h(b,m,z) &= \mathbb{E}\left[\mathbb{1}_D(b,m,z)h_D(m,z) + (1-\mathbb{1}_D(b,m,z))h_R(g_b(b,m,z),b,m,z)\right] , \\
h_R(b',b,m,z) &= -x_R(b',b,m,z) + \beta_L \mathbb{E}\left[h(b',m'_R(b',b,m,z),z') \mid z\right] \text{ , and} \\
h_D(m,z) &= -x_D(m,z) + \beta_L \mathbb{E}\left[\psi h(0,m'_D(m,z),z') + (1-\psi)h_D(m'_D(m,z),z') \mid z\right] .
\end{aligned}
\tag{17}
$$

Equilibrium We focus on *recursive equilibria*, which consist of government value functions $\{v, v_R, w_R, v_D\}$, large-lender value functions $\{h, h_R, h_D\}$, a default probability P, a bond price q, and policy functions for market issuance g_b, bilateral terms $\{x_R, m'_R, x_D, m'_D\}$ and consumption $\{c_R, c_D\}$ such that:

1. Given the bond price q and the value functions $\{h, h_D, v, v_D\}$, the bilateral loan terms $\{x_R, m'_R\}$ and $\{x_D, m'_D\}$ solve the Nash-bargaining problems (13) in repayment and in default, respectively;

2. Given the bargained outcomes, the value v_R satisfies (11) with the maximum attained by the issuance g_b, the repay/default choice delivers v and P through (10), and w_R, v_D satisfy (16);

3. The large lender's value functions h, h_R, h_D satisfy (17); and

4. The bond price q satisfies the competitive lenders' zero-profit condition (12).

5. Quantitative results

We parametrize our model at a quarterly frequency following standard strategies in the sovereign default literature. Most parameters are set externally, such as those governing the stochastic process for income, the risk-free rate, risk aversion, the length of exclusion after default, and the duration of market debt (Chatterjee and Eyigungor, 2012), as well as the scale parameter for the default-choice preference shocks.[15]

The critical parameter θ, which governs the bilateral lender's bargaining power, is set to 0.5 in our baseline, but we study comparative statics around it later on. In all cases

[15]As has become standard for numerical performance, we also include preference shocks for the debt issuance choice. We set both variances to small values to avoid impacting the equilibrium.

we fix the bilateral loan capacity $\bar{m}$ to be 20% of the quarterly income. The remaining parameters (the sovereign's discount rate and those governing the cost of default[16]) are calibrated internally to match the average debt-to-output ratio as well as the mean and standard deviation of spreads.[17] Our calibration is based on comparing data for Argentina between 1993q1 and 2001q4 to simulated data from the model. For the simulations, we use 35-period samples that end just before a default event, and condition on no default occurring within those 35 quarters or two years prior, as is standard in this literature when assuming no recovery after default (Arellano, 2008; Chatterjee and Eyigungor, 2012). Tables 1 and 2 summarize our parametrization and calibration.

TABLE 1: BASELINE PARAMETER VALUES

	Parameter	Value
Sovereign's discount factor	β	0.9607
Sovereign's risk aversion	γ	2
Preference shock scale: default	χ	0.025
Preference shock scale: issuance	χ_b	0.005
Bilateral loan cap	$\bar{m}$	0.2
Lender's bargaining power	θ	0.5
Risk-free interest rate	r^*	0.01
Duration of debt	ρ	0.05
Income autocorrelation coefficient	ρ_z	0.9484
Standard deviation of y_t	σ_z	0.02
Reentry probability	ψ	0.0385
Default cost: linear	d_0	-0.2514
Default cost: quadratic	d_1	0.292

Note: The government's utility function is assumed to be CRRA with parameter γ, the income process is an AR(1) in logs so that $y(z) = \exp(z)$ and $z' = \rho_z z + \sigma_z \epsilon'$ where $\epsilon' \overset{iid}{\sim} \mathcal{N}(0,1)$, the cost of default is $y_D(z) = y(z) - \xi(z) = y(z) - y(z)\max\{0, d_0 + d_1 y(z)\}$.

Table 3 presents some statistics from simulating the model with and without bilateral loans, for different values of the lender's bargaining weight θ. Statistics correspond to the ergodic distribution of each model, and the term 'loans' refers to bilateral loans m. Welfare gains are expressed as permanent-consumption equivalents of transitioning from the equilibrium with only market debt to the one with both funding sources, at a time with

[16] We assume a quadratic function for the cost of default $\xi(z) = y(z)\max\{0, d_0 + d_1 y(z)\}$.

[17] As most studies in this literature, we take the debt figures computed and reported by Chatterjee and Eyigungor (2012). These refer to external public debt (on average 100% of quarterly GDP), corrected for a recovery rate of 30% in case of default. We express these as a share of annual GDP, yielding 17.4%.

Table 2: Targeted moments

	Data	Model
Avg spread (bps)	815	740
Std spread (bps)	443	485
Debt to GDP (%)	17.4	18.1

Note: Simulation moments are based on 35-quarter samples before a default on market debt, conditional on no default occurring within those 35 quarters or two years prior, in a version of the model without bilateral loans. Data moments are computed for Argentina between 1993q1 and 2001q4.

market access.[18] The default frequency is calculated as the annualized ratio of the number of defaults to the number of periods of market access. The leftmost column corresponds to the equilibrium with only market debt upon which our calibration is based.[19]

The remaining columns show that even with a relatively large bargaining weight for the government ($\theta = 0.25$), the availability of bilateral loans substantially increases the frequency of default. As a consequence, the government pays higher and more volatile spreads on its market debt. In both cases, the government is able to place more market debt. This, combined with the financing from the large lender itself, allows more front-loading of consumption, which the relatively impatient government values. When the large lender's bargaining power is low ($\theta = 0.25$), the increase in amounts borrowed is enough to overcome the worsening of the equilibrium in terms of default frequency and spreads (i.e., the welfare gains are positive).

Since the large lender keeps a share of the surplus generated by the loan, the borrower government is somewhat reluctant to use it. In a typical simulation path, conditional on no default, the amount borrowed bilaterally is 2.4% of annual income with a standard deviation of 1.5%. Figure 4 shows that this changes significantly around default events: the

[18]We compute welfare gains by finding λ such that

$$\int c(v(b,z))(1+\lambda)\, d\mu(b,z) = \int c(v^\star(b, m=0, z))\, d\mu(b,z)\,,$$

where $c(x)$ is the level of a consumption stream which yields value x, v is the value function under the equilibrium with only market debt, $v^\star$ is the value function under the alternative equilibrium ($m = 0$ in the second argument emphasizes that we evaluate at a point without any balance on the bilateral loan), and μ is the ergodic distribution of the equilibrium with market debt only, conditional on repayment.

[19]Statistics in Table 3 are slightly different than those in Table 2, which is based on the pre-default samples used for calibration.

Table 3: Business cycle statistics with and without bilateral loans

	Only market	Market + Bilateral Loans	
		$\theta = 0.25$	$\theta = 0.5$
Avg spread (bps)	630	1,005	1,398
Std spread (bps)	474	620	894
$\sigma(c)/\sigma(y)$ (%)	104	99.5	99
Debt to GDP (%)	16.4	17.7	17.6
Loan to GDP (%)	0	2.79	2.44
Loan spread (bps)	–	22.3	-298
Corr. loan & spreads (%)	–	64.2	68.9
Default frequency (%)	5.03	7.67	9.55
Welfare gains (rep)	–	0.17%	-0.095%

Note: Statistics are based on simulations of the ergodic distribution of each model, conditional on repayment. 'Only market' refers to the model without bilateral loans, while the columns marked 'Market + Bilateral Loans' refer to versions in which bilateral loans can be used even during default without any restrictions (i.e., $\Gamma(m) = +\infty$). Welfare gains are given as equivalent increases in consumption from the model with only market debt to the full model at $m = 0$, calculated by averaging over the ergodic distribution of the model with only market debt, conditional on repayment.

bilateral loan m shoots up before the moment of default. This allows the government to, at least, postpone defaulting (Figure 4 does not show the defaults that were avoided as a consequence of the large lender's presence). During the default spell itself, the government quickly reaches the (unconditional) limit $\bar{m}$.

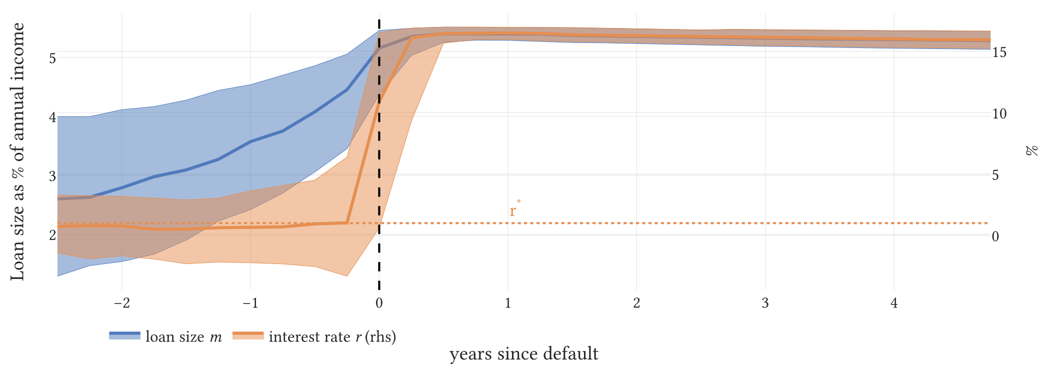

Figure 4: Loans around default events

Note: Event studies around default of market debt, with Unrestricted ($\Gamma(m) = +\infty$) loans and $\theta = 0.5$.

The large lender subsidizes the accumulation of bilateral debt before default, to then raise rates during the default spell. This behavior includes some risk-taking by the large lender: when the economy recovers market access, it immediately pays off the bilateral loan (see Figure 12 in the Appendix). If reentry to markets happens quickly, the large lender makes losses. In other words, the large lender is gambling on the exclusion period being long.[20]

Bilateral loans affect the economy in two ways: on the one hand, they provide extra financing when default risk makes borrowing in private markets costly. But they also provide funds in default, which raises the value of being excluded from private markets and consequently spreads. To disentangle these effects, we consider variants of the model in which the large creditor is less willing to lend when the economy is not in good standing with private markets. We capture this possibility via the constraint for the bargaining problem, which requires that $m' \leq \Gamma(m)$ while the government is in default (in addition to the overall limit $\bar{m}$). Notice that the Unrestricted version discussed so far corresponds to the case of $\Gamma(m) = \infty$. We consider two variants. One in which bilateral loans are Unavailable during default, such that $\Gamma(m) = 0$. In this version, the government must repay any existing balances in its bilateral loan upon defaulting, which effectively constitutes an extra cost of default.[21] Second, a Limited version in which $\Gamma(m) = m$, so that existing bilateral loans can be rolled over during default, but the government cannot obtain additional net financing. We view this Limited variant as the one that generates the least artificial changes in the effective cost of default and hence focus our discussion on this case. However, in what follows, we report results for all three variants.

Table 4 compares the three bilateral-lending specifications with one another and with the equilibrium without bilateral loans, using statistics computed under their respective ergodic distributions. The version in which bilateral loans are Limited during default removes some of the more mechanical incentives to default, as there is no possibility to obtain financing from the large lender to smooth or compensate the income costs of default. However, the decrease in default frequencies that obtains is rather small, with spreads still significantly higher than in the case with market financing only. When bilateral loans are

[20]This behavior also explains the negative average spread on the bilateral loan reported in Table 3. That statistic is a time average. The expected profits of the large lender correspond to an average of spreads weighted by borrowed amounts, and these two are positively correlated.

[21]In this variant, when repaying the debt becomes difficult, the government may want even to borrow from the large lender only to increase the cost of default, thereby ‘signaling’ to the market its willingness to pay.

fully Unavailable during default, they act as an extra cost of default, as any balance must be repaid in full immediately upon defaulting. In this version, it can also be seen that the government becomes much more reluctant to borrow from the large lender. In this case, the default frequency is still slightly higher than in the case without bilateral loans, but the government is able to place a bit more debt (both market and bilateral), which pushes welfare up.

Table 4: Business cycle statistics with bilateral loans

	Only market	**Unrestricted,** $\Gamma(m) = +\infty$	**Limited,** $\Gamma(m) = m$	**Unavailable,** $\Gamma(m) = 0$
Avg spread (bps)	630	1,398	1,209	650
Std spread (bps)	474	894	860	381
$\sigma(c)/\sigma(y)$ (%)	104	99	100	104
Debt to GDP (%)	16.4	17.6	17.6	18
Loan to GDP (%)	0	2.44	2.22	0.903
Loan spread (bps)	–	-298	-126	523
Corr. loan & spreads (%)	–	68.9	67.7	63.8
Default frequency (%)	5.03	9.55	9.04	5.28
Welfare gains (rep)	–	-0.095%	-0.038%	0.12%

Note: Statistics are based on simulations of the ergodic distribution of each model, conditional on repayment. 'Only market' refers to the model without bilateral loans; all others are computed with $\theta = 0.5$. Default frequencies are computed on the ergodic distribution, as are welfare gains, which are given as equivalent increases in consumption calculated over the ergodic distribution of the model without bilateral loans, conditional on repayment.

5.1 *Default probabilities and debt prices*

Figure 5 shows debt prices q as a function of the debt choice b' when the government owes nothing to either type of lender (i.e., $b = m = 0$). Solid lines represent the case without bilateral loans, while dotted and dashed lines correspond to the versions with Unrestricted, Limited, and Unavailable loans. Figure 5 highlights the negative impact of bilateral loans and the main consequence of the increase in the ergodic frequency of default observed when the large lender is present, reported in Table 4. In the Unrestricted and Limited versions, market debt prices are depressed relative to the case without bilateral loans, especially when debt is low (in the Limited case, there is an intermediate debt level which prices are improved). Finally, the case when loans are Unavailable in default is interesting, as spreads are *lower* at all debt levels, while the average spread level (and default frequency) reported in Table 4 was higher.

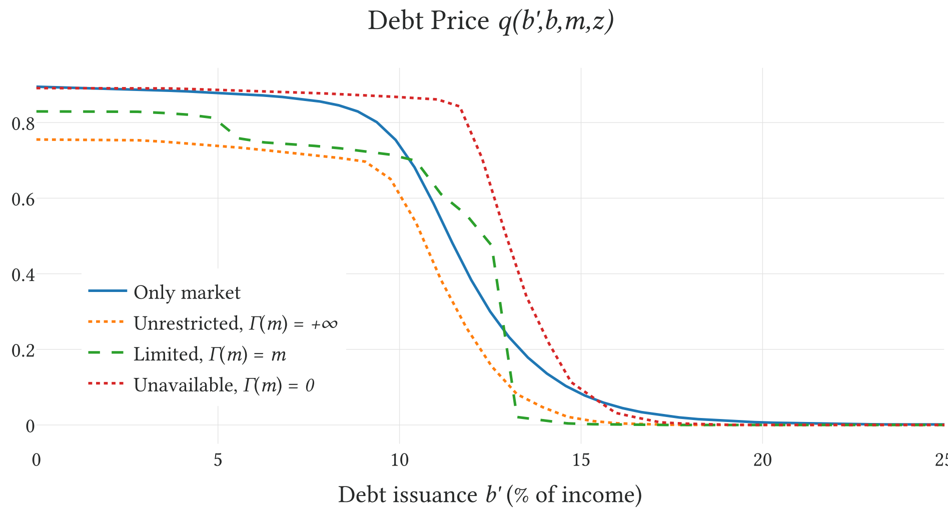

Figure 5: Debt prices

Note: Debt prices computed at $b = m = 0$, all models with bilateral loans parametrized with $\theta = 0.5$.

Figure 6 shows ex-post default regions for private debt when the bilateral loan $m = 0$. By and large, the presence of bilateral loans does not change debt capacity, with only marginal changes in the default sets. When loans are Unrestricted, the default region expands marginally, but not to an extent that would justify the increase in default frequency reported in Table 4. When loans are Limited, debt capacity actually increases marginally, while the default frequency is almost as large as in the Unrestricted case. Finally, when loans are Unavailable during default, the default region is virtually unchanged relative to the case without loans (the effect on the default barrier actually looks more like a rotation than a uniform inward or outward movement).

Combining the observations from Figures 5 and 6 yields some insight into the underlying forces and dynamics. For given debt levels, the (one-period-ahead) default probability does not increase substantially when the large lender is present. In other words debt-carrying capacity does not seem to be directly affected by the presence of the large lender. However, debt prices do fall noticeably, even for a given level of issuance. Because debt prices reflect a weighted average of current and future default probabilities, the combination of lower debt prices and similar default probabilities points to changes in expected future debt-issuance choices that generate more risk.

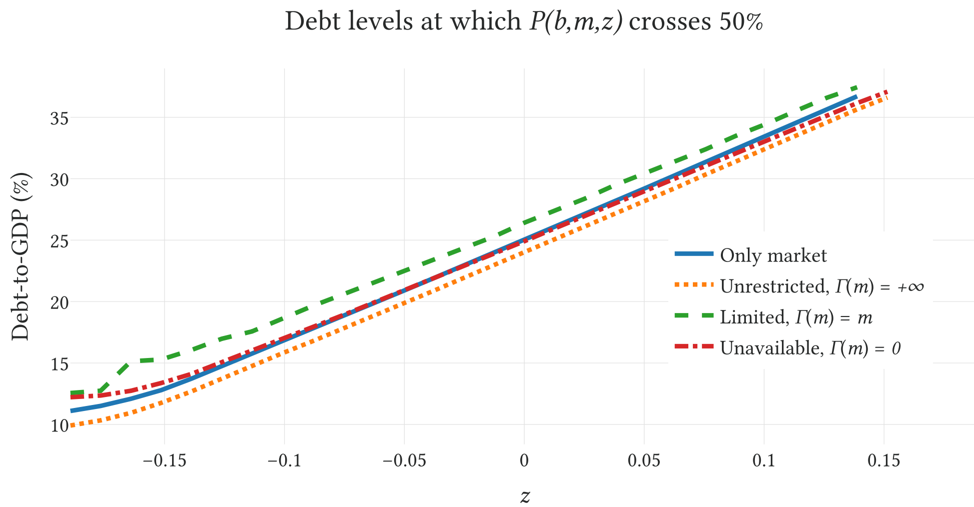

FIGURE 6: DEFAULT REGIONS
Note: Default regions computed at $m = 0$, all models with bilateral loans parametrized with $\theta = 0.5$.

5.2 *Dynamics with bilateral loans*

In the model with bilateral loans, the government issues debt in a riskier manner. Figure 7 presents the ergodic distribution of the debt-to-GDP ratio in simulations of the four models, conditional on repayment. When the large lender is present, in all variants, the economy spends more time in the region of the state space where debt is high and hence default risk is elevated.

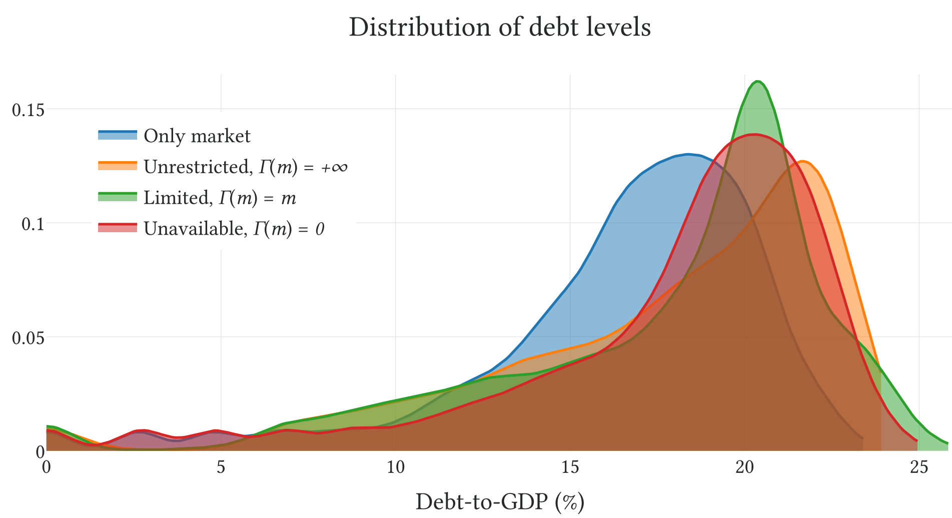

FIGURE 7: DISTRIBUTION OF DEBT LEVELS
Note: All models with bilateral loans parametrized with $\theta = 0.5$.

Figure 7 contains the key to the increase in default frequencies and spreads when the large lender is present. While debt capacity is not affected by the availability of bilateral loans, the government does spend much longer time in the region of the state space where default risk is heightened. This also explains why debt prices are generally depressed even though the default regions do not expand dramatically upon the introduction of bilateral loans (Figure 6). We now investigate why the equilibrium with the large lender is characterized by a more aggressive borrowing behavior from the government.

5.3 *Relational overborrowing*

The key to the government's market borrowing decisions in the morning lies in the afternoon. At the bargaining stage, the choice of market debt b' has already been made by the government. As a result, if bilateral negotiations break down and the threat point needs to be exercised, the borrower is left with a consumption level of $y(z) + q(b', b, m, z)(b' - (1 - \delta)b) - \kappa b - m$.

The term $B(b', b, m, z) = q(b', b, m, z)(b' - (1-\delta)b) - \kappa b$ captures net financing received from the market in the morning. It critically affects the government's threat point at the bargaining stage. In periods when the government is trying to reduce market debt b', net financing B is very low (or negative), and the government enters negotiations with the large lender in a 'weak' position as consumption at the threat point is low. This generates large gains (for the borrower) from any transfer x provided by the large lender. Thus, in order to split the surplus such transfers must come at a high interest rate. Conversely, when the government has received a large amount of net financing B, it expects a high level of consumption even if negotiations break down, which results in a strong position and good borrowing terms with the large lender. In the language of the stylized model of Section 3.1, net financing B increases y_0 and decreases y_1, strengthening the government's bargaining position and resulting in better terms from the large lender. The result is a negative cross-elasticity of the bilateral loan's interest rate to net financing B.

Figure 8 shows the interest rate that results from bilateral negotiations as a function of the choice of b',[22] when b is at an intermediate level (marked by the vertical dashed line) and $m = 0$. It confirms the discussion of the cross-elasticity above: bilateral borrowing terms quickly worsen when the government delevers in debt markets.

[22] At state (b, m, z), the choice of b' fully determines debt prices $q(b', b, m, z)$ and net financing $B(b', b, m, z)$.

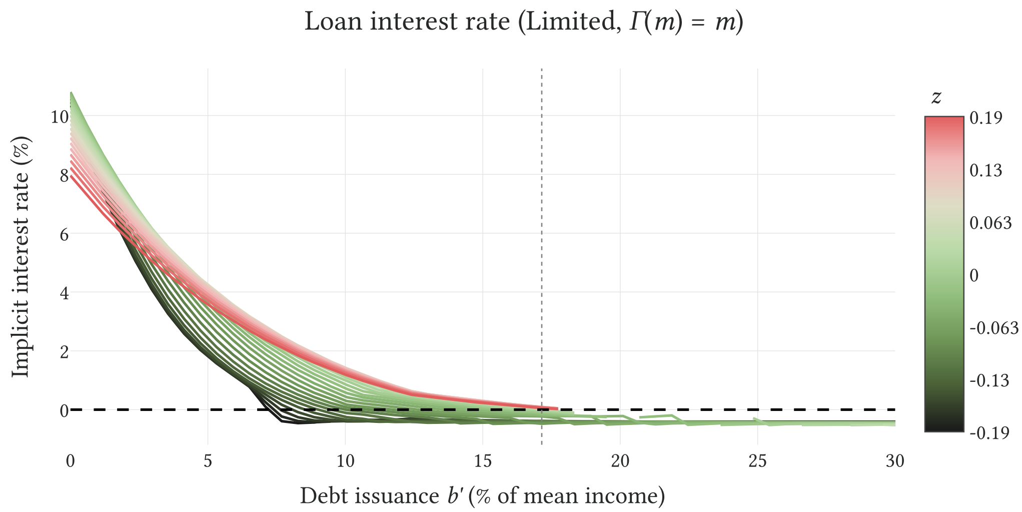

Figure 8: Interest rate on the bilateral loan

Note: Interest rate on the bilateral loan computed at $m = 0$, b at 17% of mean income, for Limited bilateral loans ($\Gamma(m) = m$) and $\theta = 0.5$.

As a result of this elasticity of loan terms to debt, new terms appear in the government's Euler equation for market debt b':

$$u'(c)\left(q + \frac{\partial q}{\partial b'}i + \frac{\partial x}{\partial b'}\right) = \beta\mathbb{E}\left[u'(c')(1 - \mathbb{1}_D)\left(\kappa + (1-\delta)q' - i'\left(\frac{\partial q'}{\partial b} + \frac{\partial q'}{\partial m}\frac{\partial m'}{\partial b'}\right) - \left(\frac{\partial x'}{\partial b} + \frac{\partial x'}{\partial m}\frac{\partial m'}{\partial b'}\right)\right)\right]$$
$$+ \beta\mathbb{E}\left[\mathbb{1}_D\frac{\partial v_D}{\partial m}\frac{\partial m'}{\partial b'}\right] \tag{18}$$

where $i = b' - (1-\delta)b$ represents new market debt issuances, $\frac{\partial q'}{\partial b}$ refers to the derivative of the following period's debt price with respect to the beginning-of-period debt level at that point (in other words, b' from the perspective of the current period). Similarly, the term $\frac{\partial q'}{\partial m}\frac{\partial m'}{\partial b'}$ captures the derivative of the following period's debt price q' to the amount owed to the large lender at that moment (next period's m, which is today's m'), times the derivative of the current choice of m' to the current choice of b', and similarly for $\frac{\partial x'}{\partial b}$ and $\frac{\partial x'}{\partial m}$. Relative to a world with only market debt, there are three new effects arising from the presence of the large lender.

First, issuing debt affects transfers x received from the large lender in the afternoon of the current period. Second, issuing debt also affects the desired amount of loans m' taken from the large creditor. This flows into debt prices q' and transfers x' in the following period through the chain-rule terms (and a different level of m next period). Finally, because debt prices and borrowing terms also depend on the initial level of indebtedness b, issuing new debt affects both the price at which market debt will be placed q' as well as transfers x' in the following period.

The left-hand side of (18) describes a countervailing force to the market discipline of spreads. In a model with only market debt, the benefit of issuing an extra unit of debt is $u'(c)\left(q+\frac{\partial q}{\partial b'}i\right)$: the government understands that issuing more debt generates revenues q but may increase spreads (decrease the price q) and this mitigates issuances. With the large lender, the benefit of issuing an extra bond includes additionally the term $\frac{\partial x}{\partial b'}$ as the extra issuance also affects the threat point at the bargaining stage. Notice that, by writing the transfer x as a price $\frac{1}{1+r}$ times the new balance m', net of the initial balance m, as in (14), we can re-express the left-hand side of the Euler equation as

$$u'(c)\left(q\underbrace{\frac{\partial i}{\partial b'}}_{=1}+\frac{\partial q}{\partial b'}i+\frac{1}{1+r}\frac{\partial m'}{\partial b'}+\frac{\partial\frac{1}{1+r}}{\partial b'}m'\right).$$

Importantly, the extra terms include the cross-elasticity $\frac{\partial\frac{1}{1+r}}{\partial b'}$, which mitigates the disciplining effect of spreads by making the bilateral loan cheaper as market debt increases. This mitigation of the total costs of debt issuance creates an extra marginal gain of issuing debt, which leads the government to accumulate debt more aggressively and to delever more slowly. As in the stylized model of Section 3.2, a relational overborrowing effect arises: with discretion, the government ends up diluting legacy bondholders more than it would choose to do with commitment.

Discussion Our timing assumptions allow the government to use the issuance of market debt to influence its threat point at the bargaining stage.[23] This assumption highlights a peril of bargained bilateral loans. We believe that this timing captures the relevant interactions and that our results would also emerge under different assumptions. However, with the opposite timing of bilateral-then-market, the government would only be able to issue market debt after bargaining with the large lender; therefore, the threat point could include a different issuance strategy than the agreement point, reducing the scope for the cross-elasticity we emphasize. In that case, the forces leading to the cross-elasticity would manifest as off-path threats. That said, a timing with market-then-bilateral-then-market would recover the on-path threat-point manipulation (issuing debt before meeting

[23] In the absence of a savings technology, all revenues must be consumed within the period, making any timing somewhat artificial.

the large lender to improve the bargaining position) which lies at the heart of our main results.

More generally, a model in which cash on hand could be carried across periods (for example, including reserves) would weaken the link between debt issuance and consumption, rendering the within-period timing irrelevant. In that more general model, reserves would correspond directly to our current notion of cash on hand and would be what strengthens the government's bargaining position. The government would then be able to use debt-financed reserves to manipulate its threat point,[24] plausibly leading to higher debt (as well as higher reserves) when the large lender is present, just like in the current model. This means that the cross-elasticity and the relational overborrowing effect would still be present, but both debt and reserves would be affected. The overall effect of the large lender's presence would then depend on the net effects of larger gross positions.

5.4 *Welfare effects of bilateral loans*

The welfare effects of bilateral loans result from the combination of the forces discussed above. Figure 9 shows the government's value function as a function of debt b when it owes $m = 0$ to the large lender. The government prefers bilateral loans to be Unavailable during default, except of course when a default in the current period is very likely.

The presence of bilateral loans reduces the government's value at virtually all debt levels (except in the 'Unavailable' version which hardwires an extra penalty for default). A natural question is why the government might enter in such an arrangement. Figure 10 shows the welfare gains of entering an agreement with the large lender by comparing the government's value at the equilibrium with only market financing to the value at the equilibrium with bilateral loans when $m = 0$, when loans are Limited during default, conditional on repayment (the default area is shaded). Small 'islands' of positive welfare gains appear when income is intermediate and the government is close to the default boundary. At these states, the gains from being able to postpone default by obtaining financing from the large lender overpower the losses from higher spreads in the future equilibrium.

We conclude our analysis by studying how welfare gains change with the bargaining

[24] Bianchi and Sosa-Padilla (2024) describe how the net effect of debt and reserves goes beyond the net foreign asset position.

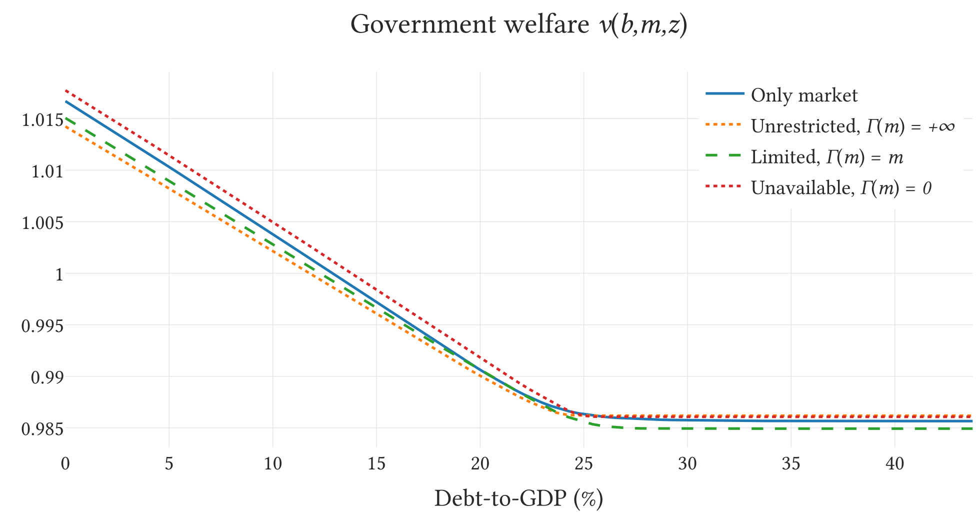

Figure 9: Value functions

Note: Value functions measured in consumption-equivalent terms and computed at $m = 0$ and average income, all models with bilateral loans parametrized with $\theta = 0.5$.

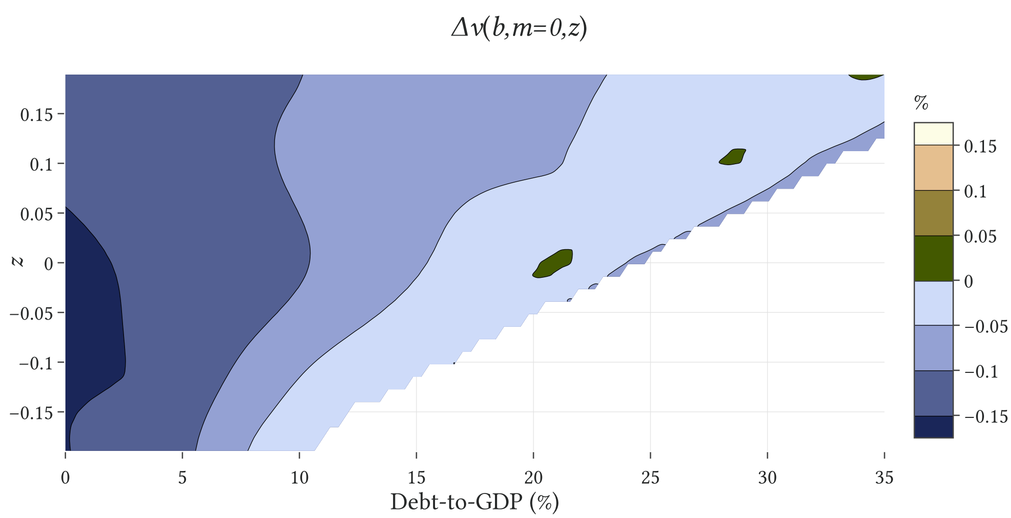

Figure 10: Gains from allowing bilateral loans (Limited)

Note: Welfare gains computed as the difference between the equilibrium with Limited bilateral loans ($\Gamma(m) = m$) and $\theta = 0.5$, at $m = 0$, and without bilateral loans, measured as consumption-equivalent increases.

weight θ, shown in Figure 11. In all cases, a government very skilled at bargaining can secure advantageous terms from the large lender, enough to extract welfare gains despite the cross-elasticity. As θ increases, such gains dissipate. The ordering of the different variants is mostly preserved relative to the $\theta = 0.5$ benchmark: Unavailable $\succcurlyeq$ Limited $\succcurlyeq$ Unrestricted.

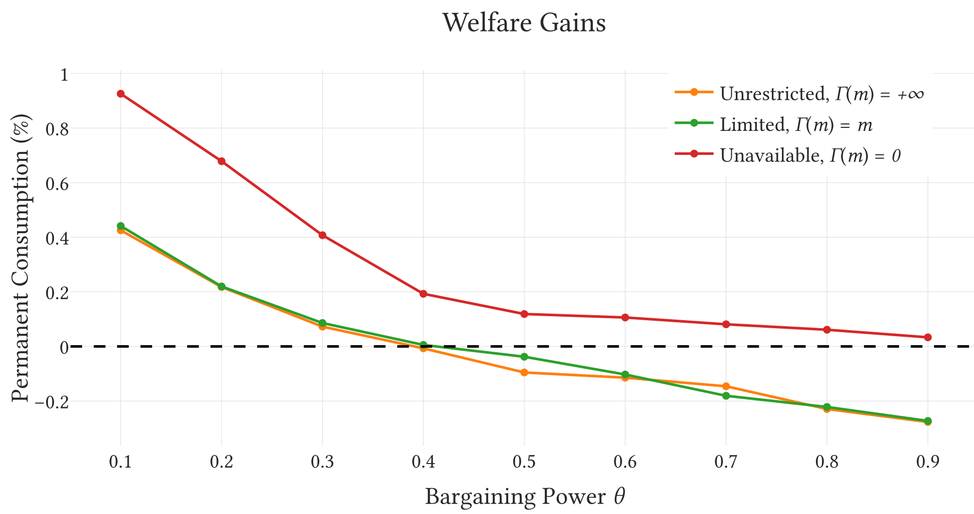

FIGURE 11: COMPARATIVE STATICS ON θ

Note: Welfare gains computed as consumption-equivalent increases averaging over the ergodic distribution of the model without bilateral loans, conditional on repayment.

6. PROGRAMMING THE LARGE LENDER

We have shown how the combination of market debt and bargained bilateral loans generates a cross-elasticity of bilateral terms to market debt issuances, how this cross-elasticity incentivizes risk-taking and leads to a relational overborrowing effect, and how this effect heightens the frequency of default, raises spreads, and can create welfare losses for the government.

In this section we replace the bargaining protocol with fixed rules for the terms of bilateral loans, with two objectives. First, to better understand the dynamics of the relational overborrowing effect in a simpler case. Second, to explore design questions surrounding bilateral loans. Can rules that curb spreads on market debt by containing sovereign default risk be found? What designs create welfare gains for the government? Are there possible Pareto improvements, under which the government attains a higher value than without bilateral loans and also the large lender obtains profits?

To investigate these questions, we consider rules for the interest rate charged by the large lender of the following form:

$$r(b', m', b) = \max\left\{r^*,\ \alpha_0 + \alpha_b(b' - (1-\delta)b) + \alpha_m m'\right\}$$

To keep the analysis focused, we conduct these exercises in the main version of the model with $\Gamma(m) = m$, corresponding to the Limited case in which bilateral loans can be refinanced during default but new drawings are forbidden. With different specifications of the Γ function, similar patterns would emerge.

With these rules, the government now faces the exogenous interest rate $r(b', m', b)$ on the bilateral loan when it issues $b' - (1-\delta)b$ in markets and obtains m' from the bilateral loan. Notice that because the bilateral loan always costs weakly more than the risk-free rate r^*, the large lender makes non-negative profits by construction. With this guarantee, we concentrate on outcomes for the government in what follows.

We consider three possible rules. First, a 'risk-inducing' rule with $\alpha_b < 0$, which features the cross-elasticity found in the equilibrium with bargaining. Second, a 'size-dependent' rule with $\alpha_b = 0$ but $\alpha_m > 0$, inspired by the IMF's surcharges policy,[25] which critically does not load on market debt and therefore does not create the cross-elasticity. Third, an optimally designed rule that optimizes the government's welfare over the parameters $(\alpha_0, \alpha_b, \alpha_m)$. Table 5 summarizes our findings.[26]

The risk-inducing rule creates similar dynamics to the equilibrium with bargaining and, thus, welfare losses for the government. However, the size-dependent rule exemplifies the possibility of rules which induce lower default and spreads, improving the government's welfare, while also generating profits for the large lender.

The rule that optimizes the government's welfare features $\alpha_b > 0$ and hence embodies the opposite cross-elasticity as the risk-inducing rule and the equilibrium with bargaining. The optimal rule curbs the government's incentives to accumulate debt through the same

[25]The cost of IMF programs increases with the size of the program via surcharges which apply after the amounts borrowed exceed certain thresholds. Surcharges also apply as a function of time, which we do not capture.

[26]The size-dependent rule is parametrized with $(\alpha_0, \alpha_b, \alpha_m) = (0.01, 0, 0.5)$, while the risk-inducing rule uses $(\alpha_0, \alpha_b, \alpha_m) = (0.085, -2.875, 0.05)$, and the optimal rule uses $(\alpha_0, \alpha_b, \alpha_m) = (0, 0.25, 0.125)$.

Table 5: Outcomes with exogenous rules for bilateral loans

	Only market	**Size dependent** r	**Risk inducing** r	**Optimal** r	**Limited,** $\theta = 0.5$
Avg spread (bps)	630	413	831	227	1,209
Std spread (bps)	474	240	447	130	860
$\sigma(c)/\sigma(y)$ (%)	104	105	104	106	100
Debt to GDP (%)	16.4	17.3	17.1	17.7	17.6
Loan to GDP (%)	0	0.727	0.486	2.56	2.22
Loan spread (bps)	–	662	296	516	-126
Corr. loan & spreads (%)	–	49.4	-29.2	38.9	67.7
Default frequency (%)	5.03	3.2	6.19	1.92	9.04
Welfare gains (rep)	–	0.15%	-0.097%	0.69%	-0.038%

Note: Statistics are based on simulations of the ergodic distribution of each model, conditional on repayment. 'Only market' refers to the model without bilateral loans, while 'Limited' refers to the version in which bilateral loan balances cannot be increased during default ($\Gamma(m) = m$) with $\theta = 0.5$. The middle columns denote the versions with exogenous terms for the bilateral loan: in 'Size dependent r', the bilateral interest rate increases with the bilateral loan m, while in 'Risk inducing r', it decreases with the market debt b. Welfare gains are given as equivalent increases in consumption calculated over the ergodic distribution of the model without bilateral loans, conditional on repayment.

logic that prevails in the equilibrium with bargaining, only reversed. By increasing the cost of entering or remaining in the risky region of the state space, the optimal rule helps the government avoid default but also helps it mitigate the debt dilution problem, raising prices for market debt. This benefits the government even when debt is low and default risk is contained.

Quantitatively, the optimal rule generates larger gains for the government than those obtained at very low values of θ in the equilibrium (when the government holds almost all of the bargaining power), while still generating profits for the large lender. Figure 11 shows that Limited bilateral loans generate welfare gains of about 0.45% of permanent consumption for the government at $\theta = 0.1$, meaning that the optimal rule's gains are 50% larger. Optimizing over a richer class of rules would boost these even more. These results underscore the amount of surplus that the equilibrium with bargaining leaves on the table. Sequential bargaining constantly creates incentives for the government to attempt to strengthen its negotiation position, in a way that is individually rational but ends up wasting resources or even hurting overall welfare.

7. Concluding remarks

Inspired by the rise of new bilateral lenders outside the Paris Club and with claims to de facto seniority relative to private creditors, we investigate the interaction between marketable debt and bilateral debt. We develop a model of sovereign debt in which the government can borrow from a large senior lender who bargains over terms as well as from a competitive fringe of private investors. We find that both sources of funds are linked by strong interactions, even when quantitatively the amounts borrowed from the senior lender remain an order of magnitude smaller than market debts.

The model features a 'relational overborrowing' effect, which arises in the presence of the senior lender. This effect is due to an endogenous cross-elasticity of bilateral borrowing terms to outcomes in debt markets, which erodes the disciplining role of spreads. Our model of bargaining generates this cross-elasticity with three key ingredients: loans from the senior lender are more difficult (or impossible) to default than market debt, their terms result from sequential bargaining, and they are of shorter maturity. These ingredients capture the defining features of Central Bank swap lines, which Horn et al. (2021b) identify as a key instrument of senior bilateral sovereign lending. The extension of the model with exogenous bilateral terms shows that the cross-elasticity is in itself sufficient to incentivize the government to overborrow, face more sovereign risk, and ultimately attain lower levels of welfare.

The model with exogenous bilateral terms illustrates a broader point. Policy discussions surrounding bilateral debts often focus on the price of the loans and on seniority itself. Our results emphasize that welfare losses are likely to come from a different source. In particular, we show how welfare losses can arise even when the government is free not to borrow from the bilateral lender if the terms offered are not advantageous enough relative to the market. In other words, our results hold in an environment in which seniority can actually help the government by completing markets. We also provide examples of pre-set pricing schedules for the senior debt, like size-dependent rules, which improve welfare for the government without losses for the senior lender. The optimal rule, in fact, creates large welfare gains for the government by raising bilateral rates in response to large market issuances (the opposite cross-elasticity as the one emerging from the equilibrium with bargaining). Our main result is that the perverse incentives created by the cross-elasticity, which have been more overlooked, are crucial to generating the welfare losses we find. Our main model then describes a set of plausible assumptions, particularly sequential bargaining over bilateral borrowing terms, under which the cross-elasticity would be expected to arise, making bilateral sovereign debt dangerous.

Our results suggest that increasing the number and type of financing sources may not be necessarily beneficial for the borrowing government. While bilateral loans can help a government fend off default, they can also make it more likely: either through reducing the effective costs of default or through the relational overborrowing effect.

The welfare impact of the senior lender's presence raises important policy questions and challenges. Limiting the use of bilateral debt during defaults is a clear welfare-enhancing policy in this model. Because the relational overborrowing effect operates through increased default risk, the gains of fiscal rules that constrain market borrowing should be larger for countries with access to the type of bilateral debts we describe.

The model implies a practical diagnostic to gauge the likely effects of a new bilateral creditor or instrument in practice. Bilateral loans whose interest rate is expected to be strongly decreasing in the amount issued (or spreads) of marketable debt will induce relational overborrowing and are thus likely to hurt welfare. More generally, it highlights the benefits of transparent and pre-specified rules for the terms of bilateral debts.

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A. More Results

Dynamics of loans around defaults Figure 12 shows that, conditioning on an exclusion period of 2 years, the economy pays off the bilateral loan as soon as it recovers market access.

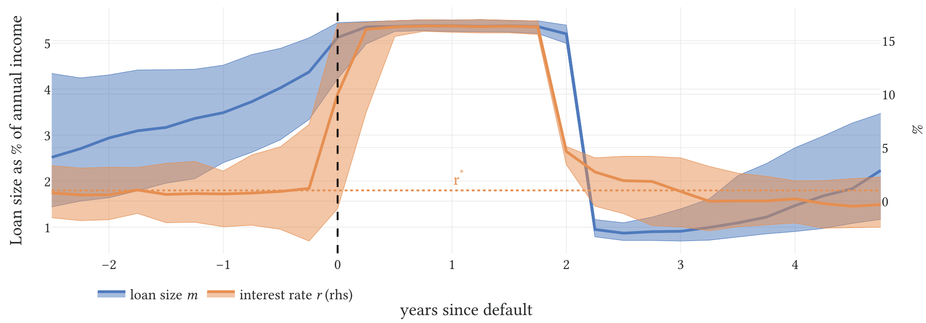

Figure 12: Loans around default events

Note: Event studies around default of market debt, with Unrestricted ($\Gamma(m) = +\infty$) bilateral loans and $\theta = 0.5$, conditional on the default spell lasting exactly two years.