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Voluntary sovereign debt exchanges

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ABSTRACT

We show that some recent sovereign debt restructurings were characterized by (i) the absence of missed debt payments prior to the restructurings, (ii) reductions in the government's debt burden, and (iii) increases in the market value of debt claims for holders of the restructured debt. Since both the government and its creditors are likely to benefit from such restructurings, we label these episodes as "voluntary" debt exchanges. We present a model in which voluntary debt exchanges can occur in equilibrium when the debt level takes values above the one that maximizes the market value of debt claims. In contrast to previous studies on debt overhang, in our model opportunities for voluntary exchanges arise because a debt reduction implies a decline of the sovereign default risk. This is observed in the absence of any effect of debt reductions on future output levels. Although voluntary exchanges are Pareto improving at the time of the restructuring, we show that eliminating the possibility of conducting voluntary exchanges may improve welfare from an ex ante perspective. Thus, our results highlight a cost of initiatives that facilitate debt restructurings.

1. Introduction

In sovereign debt restructurings the government swaps previously issued debt for new debt. This paper studies restructurings that are characterized by (i) the absence of missed debt payments prior to the restructurings, (ii) reductions in the government's debt burden, and (iii) increases in the market value of debt claims for holders of the restructured debt. Since both the government and its creditors are likely to benefit from such restructurings, we label these episodes as "voluntary" debt exchanges.[1]

We first show evidence that some recent sovereign debt restructurings fit within our definition of a voluntary debt exchange. We then show that a sovereign default model à la Eaton and Gersovitz (1981) can account for voluntary exchanges. Finally, we show that within our framework, and in spite of benefiting all parties involved at the time of the restructuring, voluntary debt exchanges are not always optimal for the government from an ex ante perspective.

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[1] Our definition of voluntary exchange should not be interpreted as indicating that the participation of all creditors in the exchange was strictly voluntary. First, it is difficult to determine the extent to which governments coerced bondholders into participating in a debt exchange (Enderlein et al., 2012; Zettelmeyer et al., 2012). Furthermore, even when a debt restructuring is beneficial for creditors as a group, individual creditors could benefit from free-riding on the participation of other creditors. For a discussion of collective action problems associated with debt restructuring see Wright (2011) and the references therein.

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Formally, we analyze a small open economy that receives a stochastic endowment stream of a single tradable good. The government is benevolent, issues long-term debt in international markets, and cannot commit to repay its debt. Sovereign debt is held by risk-neutral foreign investors. We extend this baseline model of sovereign default by allowing for voluntary debt exchanges. We assume that the cost of debt exchanges is stochastic and may be either low (zero) or high. The high cost is assumed to be high enough to prevent the government from launching a voluntary exchange. This assumption intends to capture difficulties in the implementation of voluntary exchanges. At the beginning of each period, the debt exchange cost is realized together with the endowment shock. When the cost is low, the government chooses whether to conduct a voluntary debt exchange. If the government conducts a voluntary exchange, the post-exchange debt level is endogenously determined as the outcome of a Nash bargaining game. If the government does not conduct a voluntary exchange, it decides whether to declare an outright default. An outright default is followed by a stochastic period of exclusion from capital markets and lower income levels while this exclusion lasts.

Why does debt relief lead to capital gains for the holders of restructured debt? In our model this occurs because lower debt levels reduce future default risk and thus increase the market value of the bond holders' debt portfolio. Fig. 1 shows the market value of debt as a function of the debt stock. If the state of the economy is represented by a point like *A*, a marginal decline of the debt level is more than compensated by an increase in bond prices, producing an increase in the market value of bond holders' debt portfolio. In this case, both the government and its creditors could benefit from a debt restructuring that reduces the debt stock (for example, to point *B*).

We show that our model can account for the frequency of voluntary debt exchanges suggested by the findings in Cruces and Trebesch (2013), while still accounting for other features of the data highlighted in the sovereign debt literature. Using a calibration for an archetypical emerging economy, opportunities for voluntary debt exchanges arise in 11% of the simulation periods. That is, in 11% of the simulation periods the initial debt level is higher than the debt level that maximizes the market value of debt claims. We show that those periods arise after sufficiently negative aggregate income shocks. However, an outright default does not need to be imminent in all those periods—i.e., the government would not necessarily default if the cost of conducting a voluntary exchange was high. The presence of voluntary exchanges in periods without an imminent default is only possible in our model because we allow the government to issue long-term debt.

Among voluntary debt exchanges that prevented an outright default in the period of the exchange, the average capital gain from holdings of the restructured debt is 141%. The sovereign enjoys an average debt reduction of 23% and an average welfare gain at the time of the exchange equivalent to a permanent consumption increase of 0.6%. Among voluntary debt exchanges conducted in periods in which the government would have chosen to stay current on its debt, the average capital gain is 6%, the average debt reduction is 7%, and the sovereign enjoys an average welfare gain equivalent to a permanent consumption increase of 0.2%.

In spite of the Pareto gains that result at the moment of a voluntary exchange, eliminating the possibility of conducting a voluntary exchange may improve welfare from an ex ante perspective. The anticipation of future voluntary exchanges leads lenders to offer lower prices for government bonds. This is costly for an impatient government that is eager to borrow. This finding highlights a cost of initiatives to facilitate sovereign debt restructurings.

1.1. Related literature

The possibility that a sovereign debtor and its creditors can jointly benefit from a debt reduction is discussed by Froot (1989), Krugman (1988a,b), and Sachs (1989), among others. They present "debt overhang" models with exogenous debt

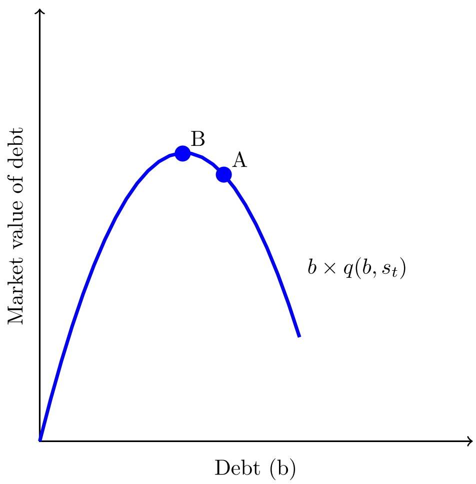

Fig. 1. Shape of the market value of the debt stock. The market value in period t is given by $b \times q(b, s_t)$, where b denotes the number of bonds outstanding, s_t denotes other state variables, and $q(b, s_t)$ denotes the bond price. Point A characterizes a state in which a debt reduction to a point like B produces capital gains from holdings of the restructured debt.

levels and show that debt relief generates Pareto gains when future debt obligations are high enough to lower aggregate investment. By increasing aggregate investment, a debt relief increases the resources available for creditors to confiscate in the case of a default.[2] In contrast, we present a model in which a debt relief has no effect on future available resources, but it weakens incentives to default, producing capital gains for debt holders. Our findings also complement their results by showing that (i) opportunities for Pareto improving debt reliefs arise endogenously in a calibrated model with empirically plausible implications, and (ii) governments may want to prevent debt reliefs from an ex ante perspective, even when these reliefs generate Pareto gains at the time of the debt restructuring.

Aguiar et al. (2009) study a model with endogenous debt levels in which a government that cannot commit to future actions may accumulate excessive debt claims, causing a debt overhang effect on investment. The optimal allocation under no commitment lies on the constrained efficient frontier, which rules out the possibility of mutually beneficial debt reductions once the sovereign is in a situation of debt overhang. In contrast, we focus on debt reliefs that produce Pareto gains at the time of the restructuring even though they may not be optimal from an ex ante perspective.

In the recent quantitative papers on sovereign default that followed Aguiar and Gopinath, (2006) and Arellano, (2008), the possibility of engaging in mutually beneficial debt relief is ruled out by assumption. We extend the baseline model by allowing for this possibility, which enables us to account for some of the heterogeneity among debt restructurings. We show that the model can account for voluntary debt exchanges while still accounting for other features of the data emphasized in the sovereign default literature.

The rest of the paper proceeds as follows. Section 2 argues that some recent sovereign debt restructurings fit within our definition of a voluntary debt exchange. Section 3 presents a four-period model in which voluntary debt exchanges arise in equilibrium and can be detrimental to ex ante welfare. Section 4 introduces the quantitative model. Section 5 discusses the calibration. Section 6 presents the results. Section 7 concludes.

2. Voluntary debt exchanges in the data

This section presents evidence that shows that some recent sovereign debt restructurings fit within our definition of a voluntary debt exchange. Recall that we define a voluntary debt exchange as a debt restructuring such that (i) the government does not miss any debt payments prior to the restructuring, (ii) there is a decline in the government's debt burden, and (iii) bond holders enjoy capital gains from participating in the restructuring.

A significant fraction of sovereign debt restructurings was not preceded by missed debt payments. For instance, Cruces and Trebesch (2013) study 180 sovereign debt restructurings between 1970 and 2010 and show that in 77 of these 180 restructurings the government did not miss debt payments prior to the restructuring.

Most sovereign debt restructurings are characterized by debt reductions. Measuring the implied debt reductions requires assumptions that allow for the comparison of debt instruments with different characteristics. One measure of debt reduction is the "haircut." There are different haircut measures. Recently, the one proposed by Sturzenegger and Zettelmeyer (2005) has been widely accepted and used (see, for example, Cruces and Trebesch, 2013):

$$Haircut = 1 - \frac{\text{Present value of debt obtained in the restructuring}}{\text{Present value of debt surrendered in the restructuring}}, \tag{1}$$

where both present values are computed using the yields of the debt received at the moment of the restructuring. Using this measure, Cruces and Trebesch (2013) find that only six of the 180 restructurings they study present negative haircuts. This indicates that governments benefited from a debt reduction in the remaining 174 cases.[3]

We find that some recent sovereign debt restructurings that occurred in the absence of missed debt payment were characterized by a lower debt burden for the government and a higher market value of the debt claims of holders of restructured debt. We compute capital gains for holders of restructured debt using bond prices to determine the value of the debt portfolio of these bond holders, before and after the restructurings:

$$\text{Capital gain} = \frac{\sum_{i=J+1}^{J+I} q_i(T) \times B_i}{(1+r)^{T-t} \sum_{j=1}^{J} q_j(T-t) \times B_j} - 1, \tag{2}$$

where T denotes the period for which prices of bonds obtained in the exchange are available, $T-t$ denotes the period for which we compute the market value of the portfolio surrendered in the exchange, $q_i(t)$ denotes the unit price of bonds of type i in period t, and B_i denotes the number of outstanding bonds of type i. Eq. (2) refers to a case in which the debt portfolio $(B_1, \ldots, B_J)$ is exchanged for the debt portfolio $(B_{J+1}, \ldots, B_{I+J})$.

We are interested in the effect of debt reductions on the market value of the creditors' debt portfolio. Therefore, we compute the market value of the portfolio surrendered in the exchange at different pre-restructuring dates in an attempt to

[2] It should be noted that creditors have recently faced difficulties when intending to confiscate assets of a defaulting country (see, for instance, Panizza et al., 2009; Hatchondo and Martinez, 2011). In many countries (including the United States), there are legal procedures that creditors may follow once individuals or corporations renege on their debt. Creditors rely on the enforcement of domestic courts to get—partially—repaid. However, it is difficult for courts to enforce government payments.

[3] It should be noted that debt reductions are only a partial measure of the benefits accrued to sovereign debtors. Debt restructurings typically benefit debtor governments by smoothing or extending the maturity profile of debt payments.

Table 1
Capital gains from holdings of restructured debt in recent sovereign debt restructurings.

Features of the restructuring	Belize	Dom. Rep.	Greece	Pakistan	Ukraine	Uruguay
First date with all bond prices	2/21/07	5/18/05	3/9/12	n.a.	3/30/00	9/23/03
Extended deadline for participating	2/15/07	7/20/05	4/4/12	12/13/99	3/15/00	5/22/03
Original deadline for participating	1/26/07	5/11/05	3/8/12	12/13/99	3/15/00	5/15/03
Launch of the exchange	12/18/06	4/20/05	2/24/12	11/15/99	2/14/00	4/10/03
Formal exchange announcement	8/2/06	3/30/05	2/24/12	8/15/99	2/4/00	3/11/03
Informal exchange announcement (IA)	7/15/06	4/16/04	7/21/11	1/15/99	1/14/00	2/18/03
Capital gains with price of surrendered portfolio at						
First date with all bond prices	0.10	0.08	0.02	n.a	−0.01	0.01
Extended deadline for participating	0.11	0.11	−0.27	0.05	0.14	0.10
Original deadline for participating	0.10	0.08	0.03	0.05	0.14	0.12
Launch of the exchange	0.05	0.04	−0.01	0.04	0.18	0.30
Formal exchange announcement	−0.05	0.06	−0.01	0.00	0.24	0.28
Informal exchange announcement	−0.11	0.24	−0.59	0.07	0.48	0.22
One month before IA	−0.14	0.27	−0.60	0.22	0.49	0.04
Three months before IA	−0.19	0.10	−0.66	−0.02	0.39	0.34
Six months before IA	0.05	n.a.	−0.69	−0.38	−0.12	0.38
Acceptance rate	0.98	0.94	0.97	0.99	0.99	0.95
Haircut	0.24	0.05	0.65	0.15	0.18	0.10

control for the effect of the anticipation of the restructuring on this market value. If the success of the exchange is perfectly anticipated, at the time of the exchange, the value of the portfolio surrendered by lenders should be equal to the value of the portfolio they receive in the exchange.

Table 1 presents creditors' capital gains from holding restructured debt in six recent episodes.[4] The table reports capital gains up to six months before the exchange announcement. Details on the construction of debt portfolio values are presented in the Appendix.

Fig. 2 presents the market value of debt portfolios for the six exchanges in Table 1. Fig. 2 and Table 1 show that four of the debt exchanges under consideration produced capital gains when we considered the price of surrendered bonds at the moment of the informal announcement. These are the episodes in Dominican Republic, Pakistan, Ukraine, and Uruguay. Fig. 2 shows a common pattern in the value of the portfolio surrendered in the exchange for these four episodes: this value increases in the weeks preceding the exchange.

All episodes in Table 1 are characterized by debt reductions, as measured by positive haircuts (the table presents the haircuts computed by Cruces and Trebesch, 2013; Zettelmeyer et al., 2012). The restructurings in Belize and Greece had the highest haircuts among the six episodes we consider. This is consistent with the capital losses we measure for those episodes.

Notice that the capital gain measured as in Eq. (2) equals the opposite of the haircut measure (1) for the case in which present values are computed using the yield implicit in each bond price—and thus the present value of payments promised in a bond is given by its price. Thus, the key difference between the haircut and capital gain measures presented in Table 1 is in the yield used to compute the present value of debt surrendered in the restructuring: for the haircut measure, we use the yields implicit in the price of bonds obtained in the restructuring, and for the capital gain measure, we use the yields implicit in the price of bonds surrendered in the restructuring. This indicates that for the four restructurings that present capital gains—and therefore fit within our definition of voluntary exchanges—the implied yields in sovereign bond prices decreased after these restructurings. This could be due to a decline in country risk implied by each restructuring, which is consistent with our theory of voluntary debt exchanges.

3. A four-period example

This section presents a four-period model that allows us to illustrate how voluntary debt exchanges can occur in equilibrium, and how eliminating the possibility of voluntary exchanges may improve welfare from an ex ante perspective. This model features several stark assumptions that we relax when we study the quantitative model presented in Section 4.

[4] Secondary market bond prices make it easier to compute the market values of debt for bond debt restructurings than for bank debt restructurings. Cruces and Trebesch (2013) show that out of the 180 debt restructurings that took place between 1970 and 2010, 162 were bank debt restructurings and only 18 were bond debt restructurings. Out of the 18 bond debt restructurings, nine occurred without the government missing debt payments prior to the restructuring. These are the recent restructurings in Belize, Dominica, Dominican Republic, Grenada, Kenya, Moldova, Pakistan, Ukraine, and Uruguay. Table 1 presents data for the five of these restructurings for which we found bond price data, plus the 2012 Greece restructuring.

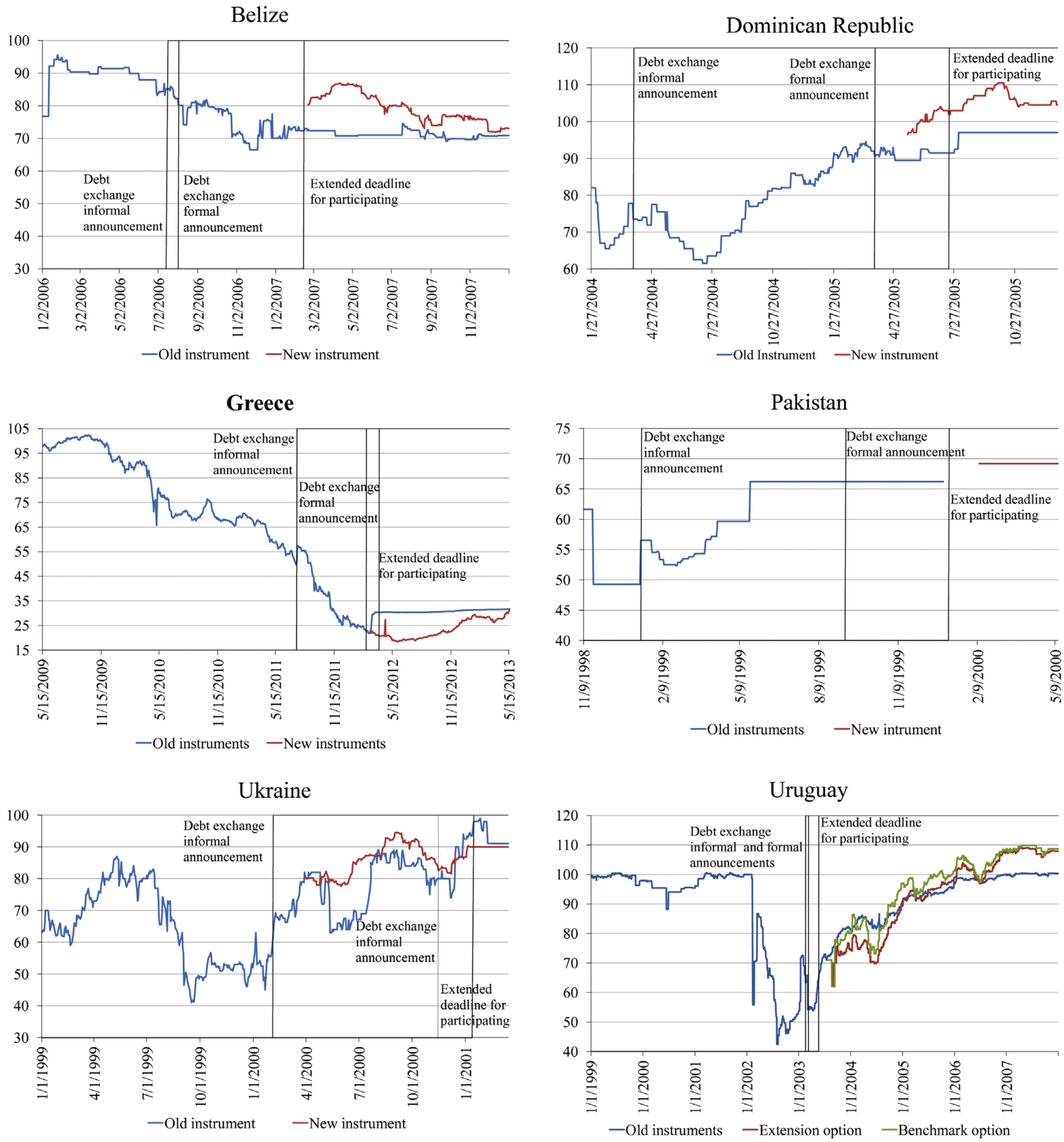

Fig. 2. Market value of pre-restructuring and post-restructuring debt portfolios. Bond prices are expressed as a percentage of the par value of pre-restructuring bonds.

3.1. Environment

The economy lasts for four periods, denoted by $t \in \{1,2,3,4\}$. The government receives a sequence of endowments y_t. The endowment in periods 1 and 2 is equal to $\bar{y}$. The endowment realization in period 3 (y_3) is drawn from an uniform distribution with support $[0,1]$. The endowment realization in period 4 is drawn from an uniform distribution with support $[0,y_3]$.

In the first three periods, the government can issue sovereign bonds promising to pay one unit of the good in period 4. Thus, the maturity of bonds issued in periods 1 and 2 is higher than one period. Foreign competitive lenders are risk neutral and face an opportunity cost of funds equal to the real interest rate, which is assumed to be equal to zero. Each period, the government maximizes the expected utility of a risk-neutral representative agent, who discounts future payoffs with the factor $\beta < 1$.

In period 3, the government can conduct a voluntary debt exchange. If a voluntary exchange is conducted, the post-exchange debt is such that it maximizes the market value of debt claims. The cost of conducting a debt exchange for the government is zero with probability π or prohibitively large with probability $1-\pi$.

In period 4, the government can declare an outright default on its debt. If the government defaults, it loses a fraction ϕ of its period 4 income. Thus, consumption is given by

$$\begin{aligned}
c_1 &= \overline{y} + b_2 q_1(b_2),\\
c_2 &= \overline{y} + (b_3 - b_2) q_2(b_3),\\
c_3 &= y_3 + \hat{e}[b_4 - \hat{b}^E(b_3, y_3)] q_3(b_4, y_3) + (1-\hat{e})(b_4 - b_3) q_3(b_4, y_3),\\
c_4 &= y_4 - \hat{d}(b_4, y_4)\phi y_4 - [1 - \hat{d}(b_4, y_4)] b_4,
\end{aligned}$$

where $b_t \geq 0$ denotes the number of sovereign bonds at the beginning of period t, q_t denotes the bond price in period t, $\hat{e}$ is an indicator function that equals 1 (0) if the cost of an exchange is low (high), $\hat{b}^E$ denotes the government debt level after the exchange stage when the cost of an exchange is low, and $\hat{d}$ denotes the government's outright default decision that takes a value of 1 (0) when the government chooses to default (repay).

3.2. Results

We solve the model backward. Next, we describe the equilibrium functions for each period.

3.2.1. Period 4

The government maximizes period 4 consumption by defaulting if and only if $b_4 > \phi y_4$. Thus,

$$\hat{d}(b_4, y_4) = \begin{cases} 0 & \text{if } b_4 \leq \phi y_4 \\ 1 & \text{otherwise.} \end{cases}$$

3.2.2. Period 3

The period 4 default decision implies that the equilibrium bond price at the end of period 3 is given by

$$q_3(b_4, y_3) = \begin{cases} 1 - \dfrac{b_4}{\phi y_3} & \text{if } b_4 \leq \phi y_3 \\ 0 & \text{otherwise.} \end{cases}$$

Using this bond-price function, we can find the equilibrium period 3 borrowing rule. Note that the government is indifferent among all levels of bond issuance such that $b_4 \geq \phi y_3$—because for such issuance levels $q_3(b_4, y_3) = 0$ and $\hat{d}(b_4, y_4) = 1$. Let b denote the government's debt level at the time it chooses its period 3 borrowing (i.e., after the debt exchange stage). If $b \geq \phi y_3$, the equilibrium b_4 is undetermined—$q_3(b_4, y_3) = 0$ and $\hat{d}(b_4, y_4) = 1$ for all $b_4 \geq b$. We assume that in these situations, the equilibrium $b_4 = b$.

At the time the government chooses its period 3 borrowing (i.e., after the debt exchange stage), for any debt level b, the period 3 equilibrium borrowing rule is given by

$$\hat{b}_4(b, y_3) = \begin{cases} \dfrac{b + (1-\beta)\phi y_3}{2-\beta} & \text{if } b \leq \phi y_3 \\ b & \text{otherwise.} \end{cases}$$

Thus, the debt market value after the debt exchange stage is equal to

$$MV_3(b, y_3) = b q_3\left(\hat{b}_4(b, y_3), y_3\right) = \begin{cases} \dfrac{\phi y b - b^2}{\phi y (2-\beta)} & \text{if } b \leq \phi y_3 \\ 0 & \text{otherwise.} \end{cases}$$

The debt level that maximizes this market value is $\phi y_3/2$. Therefore, a debt exchange would only be conducted if $b_3 > \phi y_3/2$ (and thus the government benefits from a debt reduction). Consequently,

$$\hat{b}^E(b_3, y_3) = \min\{b_3, \phi y_3/2\}.$$

The analysis above implies that as long as it is possible that a voluntary debt exchange is not too expensive ($\pi > 0$), and the government chooses to start period 3 with a positive debt level ($b_3 > 0$), voluntary debt exchanges are part of the equilibrium of this model. In particular, the probability of observing a voluntary debt exchange is given by $\max\{2\pi b_3/\phi, 1\}$.

3.2.3. Period 2

The following proposition characterizes the period 2 bond price function (proofs are presented in the Appendix).

Proposition 1 (*Period 2 bond price function*). *The equilibrium period 2 bond price function q_2 is increasing with respect to the probability of a voluntary exchange π. If $\pi = 0$, $q_2(b_3) = 0$ for all $b_3 \geq \phi$. In contrast, if $\pi > 0$, $q_2(b_3) > 0$ for all $b_3 \geq 0$. Furthermore,* $\lim_{b_3 \to \infty} q_2(b_3) = 0$.

Proposition 1 shows that the possibility of a voluntary debt exchange lowers the period 2 financial cost of borrowing. That is because a voluntary exchange would increase the expected debt recovery rate (this is why a voluntary exchange would increase the debt market value).

We show next that, in contrast to what is observed in period 2, the borrowing cost may increase in period 1. In order to do so, we focus on the extreme case in which it is always free for the government to conduct a voluntary exchange in period 3 ($\pi = 1$; when we study the quantitative model we calibrate π to obtain a plausible frequency of voluntary debt exchanges). The next proposition states that for sufficiently large debt levels contracted in period 1, the government fully dilutes those claims in period 2.

Proposition 2 (*Full debt dilution*). *Suppose* $\pi = 1$. *There is a threshold* $\overline{b} \le \phi/2$ *such that,* $\hat{b}_3(b_2) = +\infty$ *for any* $b_2 \ge \overline{b}$.

In order to provide intuition for Proposition 2, consider the case in which the government enters period 2 with a debt level that, with probability 1, will be renegotiated and reduced down to $\phi y_3/2$ in period 3. In that case, the best strategy for an impatient government ($\beta < 1$) is to assign de facto seniority to bond holders who buy debt issued in period 2 (over those who bought debt in period 1). The government does so by issuing an infinitely large amount of debt in period 2. This ensures that the share of post-exchange bonds received by investors who purchased debt in period 1 is infinitesimal. With this strategy, the government increases consumption in period 2 by diluting the value of the claims of those who bought bonds in period 1.

3.2.4. Period 1

We next present a numerical example that illustrates how the possibility of a voluntary debt exchange strengthens the government's ability to dilute in period 2 the value of debt issued in period 1. Thus, the possibility of a voluntary debt exchange limits the amount of debt that the government can issue in period 1, and the government is better off without the possibility of a voluntary exchange. This example assumes that $\beta = 0.75$, $\phi = 0.5$, and $\underline{y} = 0$. We illustrate the gains from eliminating the possibility of voluntary debt exchanges by comparing the model economies with $\pi = 0$ and $\pi = 1$.

The left panel of Fig. 3 shows that the possibility of a voluntary debt exchange ($\pi = 1$) increases the period 1 financial cost of borrowing. In particular, with $\pi = 1$, there is a threshold $\overline{b} = 0.02$ such that for all $b_2 \ge \overline{b}$, the price lenders are willing to pay for sovereign bonds is zero. The threshold $\overline{b}$ is the one presented in Proposition 2 (Fig. 4). For all $b_2 \ge \overline{b}$, lenders expect that the government will choose to fully dilute the value of debt issued in period 1. Therefore, lenders are not willing to pay for sovereign bonds.

Furthermore, the price at which the government can issue bonds is also lower in the economy with voluntary exchanges for issuance levels below $\overline{b}$. This occurs because the level of period 2 debt dilution is also higher in the economy with voluntary exchanges for period 1 issuance levels below $\overline{b}$. Recall that the price at which the government can sell bonds in period 2 is higher in the economy with voluntary exchanges (Proposition 1). Therefore, the level of period 2 government issuances is higher in the economy with voluntary exchanges. Consequently, period 1 lenders expect the value of their debt holdings to be diluted more in the economy with voluntary exchanges and, therefore, are willing to pay less for bonds issued in period 1.

The right panel of Fig. 3 shows that the government is better off when it cannot conduct voluntary debt exchanges ($\pi = 0$). This is the case because the possibility of conducting voluntary debt exchanges worsens the government's borrowing opportunities and impairs the government's ability to increase consumption in period 1. In this numerical example, the government enters period 3 with $b_3 > 0$. This implies that voluntary debt exchanges are a feature of the equilibrium. Voluntary debt exchanges are observed for period 3 income realizations $y_3 \in [0, 2b_3/\phi)$.

In summary, we have presented a numerical example in which voluntary debt exchanges occur in equilibrium. At the time of the debt exchange, these exchanges produce Pareto gains by reducing the government's debt burden while

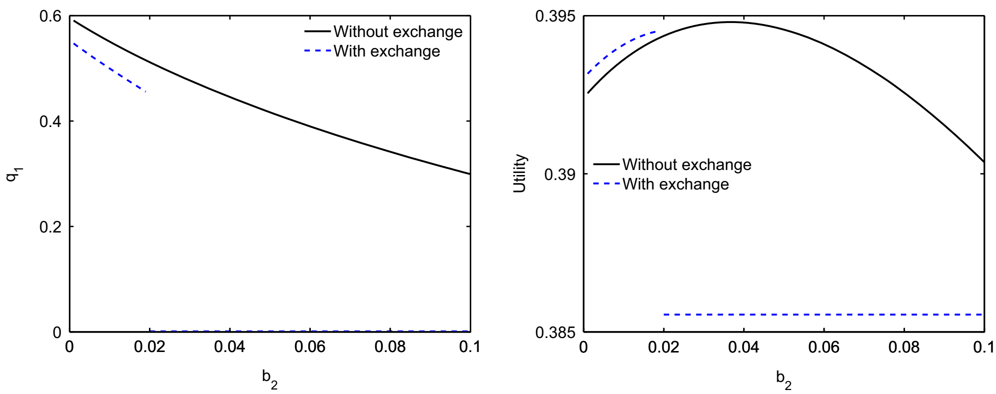

Fig. 3. Equilibrium with and without voluntary debt exchanges ($\pi = 1$ and $\pi = 0$, respectively). The left panel presents the price at which the government can sell bonds in period 1. The right panel presents the government's period 1 objective function.

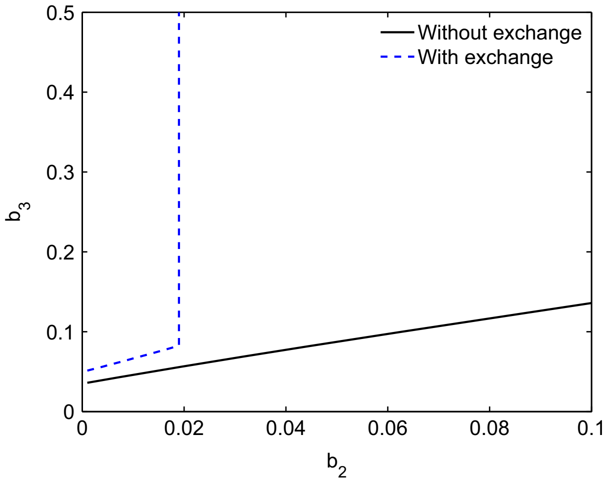

Fig. 4. Equilibrium borrowing decision in period 2 as a function of initial debt.

increasing the market value of debt claims. Nevertheless, eliminating the possibility of conducting voluntary exchanges would enable the government to achieve a more front-loaded consumption profile and thus, would increase its welfare. We next show that ex ante welfare is also reduced by the possibility of conducting voluntary debt exchanges in a quantitative model with empirically plausible parameter values.

4. Model

There is a single tradable good. The economy receives a stochastic endowment stream of this good y_t, where

$$\log(y_t) = (1-\rho)\mu + \rho\log(y_{t-1}) + \varepsilon_t,$$

with $|\rho| < 1$ and $\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$.

The government's objective is to maximize the present expected discounted value of future utility flows of the representative agent in the economy, namely

$$\mathbb{E}_t \sum_{j=t}^{\infty} \beta^{j-t} u(c_j),$$

where $\mathbb{E}$ denotes the expectation operator, β denotes the subjective discount factor, and the utility function is assumed to display a constant coefficient of relative risk aversion denoted by γ. That is,

$$u(c) = \frac{c^{(1-\gamma)} - 1}{1-\gamma}.$$

As in Hatchondo and Martinez (2009), we assume that a bond issued in period t consists of a perpetuity with geometrically declining coupon obligations. In particular, a bond issued in period t promises to pay one unit of the good in period $t+1$ and $(1-\delta)^{s-1}$ units in period $t+s$, with $s \geq 2$.

The government cannot commit to repay its debt. We refer to events in which the government reneges on its debt as outright defaults. As in previous studies, the cost of an outright default does not depend on the size of the default. Thus, when the government defaults, it does so on all current and future debt obligations.[5] Following previous studies, we also assume that the recovery rate for outright defaults is zero.

An outright default triggers exclusion from credit markets for a stochastic number of periods. Income is given by $y - \phi(y)$ in every period in which the government is excluded from credit markets. As in Arellano (2008), we assume that it is proportionally more costly to default in good times. This is a property of the endogenous default cost derived by Mendoza and Yue (2012). As in Chatterjee and Eyigungor (2012), we assume a quadratic loss function for income during a default episode $\phi(y) = \max\{d_0 y + d_1 y^2, 0\}$. They show that this function allows the equilibrium default model to match the behavior of the spread in the data.

Lenders are risk neutral and discount future payoffs at the rate r. Bonds are priced in a competitive market inhabited by a large number of identical lenders, which implies that bond prices are pinned down by a zero expected profit condition.

[5] This is consistent with the behavior of defaulting governments in reality. Sovereign debt contracts often contain acceleration and cross-default clauses. These clauses imply that after a default event, future debt obligations become current (IMF, 2002).

Our departure from the baseline setup is to allow for voluntary debt exchanges. We present a stylized model of voluntary exchanges. We intend to capture the difficulties in implementing a voluntary restructuring by assuming that the cost of debt exchanges is i.i.d. and may take a low (zero) or a high value. The high cost is assumed to be sufficiently elevated to make it optimal for the government to not launch a voluntary exchange when the cost is high.[6]

When the cost of a voluntary exchange is zero, the government chooses whether to conduct an exchange. If the government conducts a voluntary exchange, it reduces its debt level. We assume that the post-exchange debt level is endogenously determined in a Nash bargaining game in which the government and bond holders negotiate over the debt level.[7]

The government cannot commit to future outright default, exchange, and borrowing decisions. Thus, one may interpret this environment as a game in which the government making the exchange, default, and borrowing decisions in period t is a player who takes as given the exchange, default, and borrowing strategies of other players (governments) who will decide after t. We focus on Markov Perfect Equilibrium. That is, we assume that in each period, the government's equilibrium exchange, default, and borrowing strategies depend only on payoff-relevant state variables. Krusell and Smith (2003) discuss the possibility of multiple Markov perfect equilibria in infinite-horizon economies. In order to avoid this problem, we solve for the equilibrium of the finite-horizon version of our economy. That is, we increase the number of periods of the finite-horizon economy until value functions and bond prices for the first and second periods of this economy are sufficiently close. We then use the first-period equilibrium policy functions as an approximation of the stationary equilibrium policy functions.

4.1. Recursive formulation

Let b denote the number of outstanding coupon claims at the beginning of the current period. Let $\theta \in \{L, H\}$ denote the exchange-cost shock, where L (H) denotes the low (high) value of the shock. Let V denote the value function of a government that is not currently in default. This function satisfies the following functional equations:

$$V(b, y, L) = \max\{V^E(b, y), V^P(b, y), V^D(y)\}, \tag{3}$$

and

$$V(b, y, H) = \max\{V^P(b, y), V^D(y)\}, \tag{4}$$

where V^E denotes the continuation value when the government launches a debt exchange, V^P denotes the continuation value when the government pays its debt obligations (and does not launch an exchange), and V^D denotes the continuation value when the government declares an outright default.

The value function of paying current debt obligations and not launching an exchange is represented by the following functional equation:

$$V^P(b, y) = \max_{b' \geq 0, c} \{u(c) + \beta \mathbb{E}_{(y', \theta')|y} V(b', y', \theta')\}, \tag{5}$$

subject to

$$c = y - b + q(b', y)[b' - (1-\delta)b],$$

where $b' - (1-\delta)b$ represents current debt issuances, and q denotes the price of a bond at the end of a period. Let $\hat{b}^P$ denote the government's borrowing rule when it stays current on its debt obligations.

The value function when the government declares an outright default satisfies the following functional equation:

$$V^D(y) = u(y - \phi(y)) + \beta \mathbb{E}_{(y', \theta')|y}[(1-\psi)V^D(y') + \psi V(0, y', \theta')], \tag{6}$$

where ψ denotes the probability of regaining access to capital markets. Let $\hat{d}$ denote the government's default strategy. The function $\hat{d}$ takes a value of 1 when the government declares an outright default and takes a value of 0 when it stays current on its debt.

Finally, the value function when the government launches a debt exchange satisfies

$$V^E(b, y) = V^P(\hat{b}^E(b, y), y), \tag{7}$$

where $\hat{b}^E(b, y)$ denotes the post-exchange debt level. That is, in a restructuring period the government services the debt agreed on the restructuring and then it issues (or buys back) debt.

[6] Voluntary exchanges may be prevented because of collective action problems (Wright, 2011), which are sometimes mitigated by the inclusion of collective action clauses, minimum participation thresholds, and exit consents in debt contracts (IMF, 2013). We interpret a low exchange cost shock as representing circumstances in which collective action problems can be overcome. We calibrate the probability of such shock targeting the frequency of voluntary exchanges. Since voluntary exchanges in our model are only conducted when there is no cost for the sovereign, our results may provide an upper bound of the benefits derived from debt exchanges.

[7] We rule out the possibility that bond holders can extract side payments from the government in order to accept a restructuring proposal.

The post-exchange debt level is endogenously determined in a Nash bargaining game in which bond holders' bargaining power is constant over time and denoted by α, namely

$$\hat{b}^E(b,y) = \underset{b^E}{\operatorname{argmax}}\{[MV(b^E,y) - MV(b,y)]^{\alpha}[V^P(b^E,y) - \max\{V^P(b,y), V^D(b,y)\}]^{1-\alpha}\}$$
$$\text{s.t.} \quad MV(b^E,y) - MV(b,y) \geq 0, \quad \text{and}$$
$$V^P(b^E,y) - \max\{V^P(b,y), V^D(b,y)\} \geq 0,$$

where $MV(b,y)$ denotes the market value of debt claims outstanding at the beginning of the period. If it is optimal for the government to default when it starts with a state vector (b, y), the market value is zero. If it is optimal to repay, the market value equals the sum of current coupon payments and the value at which bond holders can sell their bond portfolio. Formally,

$$MV(b,y) = b[1-\hat{d}(b,y)][1+(1-\delta)q(\hat{b}^P(b,y),y)]. \tag{8}$$

The surplus that each party obtains in the restructuring consists of the difference between the continuation value if the restructuring takes place and the continuation value without restructuring. If a restructuring does not take place, the market value of bond holders' debt claims is $MV(b,y)$, and the continuation value for the government is $\max\{V^P(b,y), V^D(b,y)\}$. If a restructuring that brings the debt level to b^E takes place, the market value of bond holders' debt claims is $MV(b^E,y)$ and the continuation value for the government is $V^P(b^E,y)$.

Let $\hat{e}$ denote the government's exchange strategy. The function $\hat{e}$ takes a value of 1 (0) when the government chooses to (to not) conduct a debt exchange. Recall that we assume that the high cost of an exchange is such that $\hat{e}(b,y,H) = 0$ for all (b,y).

The assumption that bond holders price bonds in competitive markets implies that

$$q(b',y)(1+r) = \mathbb{E}_{(y',\theta')|y}\left\{\frac{\hat{e}(b',y',\theta')\hat{b}^E(b,y)}{b'}\left[1+(1-\delta)q\left(\hat{b}^P\left(\hat{b}^E(b',y'),y'\right),y'\right)\right]\right.$$
$$\left. + [1-\hat{e}(b',y',\theta')][1-\hat{d}(b',y',\theta')][1+(1-\delta)q(\hat{b}^P(b',y'),y')]\right\}, \tag{9}$$

where $\hat{b}^E(b,y)/b' \leq 1$ denotes the number of bonds received in the exchange per bond surrendered. Notice that the model equivalent to the definition of haircut presented in Section 1 is given by $1-\hat{b}^E(b,y)/b'$.

4.2. Equilibrium definition

A *Markov perfect equilibrium* is characterized by

1. rules for voluntary exchanges $\hat{e}$, default $\hat{d}$, and borrowing $\hat{b}^P$,
2. and a bond price function q,

such that

i. given a bond price function q, the policy functions $\hat{d}$, $\hat{e}$, and $\hat{b}^P$ solve the Bellman equations (3)–(7),
ii. given policy rules $\hat{d}$, $\hat{e}$, and $\hat{b}^P$, the bond price function q satisfies (9).

5. Calibration

Table 2 presents the benchmark values given to the parameters in the model. A period in the model refers to a quarter. The coefficient of relative risk aversion is set equal to 2, the risk-free interest rate is set equal to 1%, and the discount factor is set equal to 0.975. These are standard values in quantitative business cycle and sovereign default studies. The average duration of sovereign default events is three years ($\psi^d = 0.083$), in line with the duration estimated by Dias and Richmond (2007).

We use Mexico as reference for the calibration. Mexico is an archetypical emerging economy that is commonly used as reference in previous works that study these economies (see, for example, Aguiar and Gopinath, 2007; Neumeyer and Perri, 2005; Uribe and Yue, 2006). The parameter values that govern the income process are estimated using GDP data from the first quarter of 1980 to the fourth quarter of 2011.

We set $\delta = 3.3\%$. With this value, bonds have an average duration of five years in the simulations, which is roughly the average debt duration observed in Mexico according to Cruces et al. (2002).[8]

[8] We use the Macaulay definition of duration that, with the coupon structure in this paper, is given by $D = (1+r^*)/(\delta + r^*)$, where r^* denotes the constant per-period yield delivered by the bond. Using a sample of 27 emerging economies, Cruces et al. (2002) find an average duration of foreign sovereign debt in emerging economies—in 2000—of 4.77 years, with a standard deviation of 1.52.

Table 2
Benchmark parameter values.

Risk aversion	γ	2
Risk-free rate	r	0.01
Discount factor	β	0.975
Probability of re-entry after default	ψ	0.083
Probability of low exchange cost	π	0.02
Income autocorrelation coefficient	ρ	0.94
Standard deviation of innovations	σ_ε	1.5%
Mean log income	μ	$(-1/2)\sigma_\varepsilon^2$
Debt duration	δ	0.033
Income cost of defaulting	d_0	-1.01683
Income cost of defaulting	d_1	1.18961
Creditors bargaining power	α	1

We allow creditors to have full bargaining power during the debt exchange bargaining game. In terms of Fig. 1, this means that if the government conducts a debt exchange in a period characterized by point A, it lowers its debt level to the one depicted by point B. It is often argued that friendly restructurings maximize creditors' recovery rate subject to bringing the debt to a "sustainable" level. This seems consistent with our baseline of $\alpha = 1$. In Section 6.6 we present comparative statics with respect to α.

We calibrate the values of d_0, d_1, and π (the probability of a low exchange cost) targeting the average levels of spread and debt between 1995 (due to data availability) and 2011, and a share of voluntary debt exchanges to default episodes of 30%. Debt restructurings that fit within our definition of voluntary exchange are likely to be considered sovereign defaults. For instance, this is often the case for the six restructurings described in Section 2 (for example, Cruces and Trebesch, 2013 describe these episodes as defaults). Credit-rating agencies consider a "technical" default an episode in which the sovereign makes a restructuring offer that contains terms less favorable than the original debt, leaving considerable ambiguity on how to determine whether terms offered to investors were less favorable. Usually, the haircut definition presented in Section 2 is used as a measure of investor losses.[9] This interpretation of the haircut would be consistent with considering all voluntary exchanges as sovereign defaults, as we do for interpreting simulation results. Cruces and Trebesch (2013) report that 43% of the default episodes they study occur before the government defaults on debt payments. As we show in Section 2, not all of these episodes result in capital gains for investors. Thus, 43% is an upper bound for the share of voluntary debt exchanges to default episodes.

6. Results

We first show that the model simulations reproduce features of the data. We then discuss the opportunities for voluntary debt exchanges in the simulations and the role of long-term debt in generating these opportunities, the gains at the moment of a voluntary exchange, and the ex ante gains from avoiding voluntary debt exchanges. Finally, we present comparative statics with respect to creditors' bargaining power in determining the post-exchange debt level.

The recursive problem is solved using value function iteration. The approximated value and bond price functions correspond to the ones in the first period of a finite-horizon economy with a number of periods large enough to make the maximum deviation between the value and bond price functions in the first and second period small enough. We solve for the optimal debt level in each state by searching over a grid of debt levels and then using the best point on that grid as an initial guess in a nonlinear optimization routine. The value functions V^E, V^P, and V^D, and the bond price function q are approximated using linear interpolation over y and cubic spline interpolation over debt levels.[10] We use 50 grid points for both debt and income. Expectations are calculated using 75 quadrature points for the income shock.

In order to compute the sovereign spread implicit in a bond price, we first compute the yield i an investor would earn if it holds the bond to maturity (forever in the case of our bonds) and no default is ever declared. This yield satisfies

$$q_t = \frac{1}{(1+i)} + \sum_{j=1}^{\infty} \frac{(1-\delta)^j}{(1+i)^{j+1}}.$$

The sovereign spread is the difference between the yield i and the risk-free rate r. We report the annualized spread

$$r_t^s = \left(\frac{1+i}{1+r}\right)^4 - 1.$$

Debt levels in the simulations are calculated as the present value of future payment obligations discounted at the risk-free rate, i.e., $b_t(1+r)(\delta+r)^{-1}$, where $\delta = 1$ for one-period bonds. We report debt levels as a percentage of annualized income.

[9] In fact, the objective of Sturzenegger and Zettelmeyer (2005) when they introduce this definition is to measure investors' losses.

[10] Hatchondo et al. (2010) discuss the advantages of using interpolation and solving for the equilibrium of a finite-horizon economy.

We generate 500 samples of 500 periods each. We extract the last 120 periods of samples in which the last default was observed at least 20 periods prior to the beginning of those 120 periods. With the exception of the default rates, moments in the simulations correspond to the averages over those samples. The frequencies of outright defaults and debt restructurings are computed using all simulation data. Table 3 presents the calibration targets and the corresponding moments in the simulations. The table shows that the simulations approximate the targets reasonably well.

6.1. *Nontargeted simulation moments*

Table 4 reports nontargeted simulation moments. The table shows that allowing for voluntary exchanges does not compromise the default model's ability to account for key features of business cycle statistics in emerging economies (Aguiar and Gopinath, 2006, 2007; Neumeyer and Perri, 2005; Uribe and Yue, 2006 document those features). Our model is able to account for a volatile and countercyclical spread that leads to more borrowing in good times than in bad times, as reflected by a volatile and countercyclical trade balance.

6.2. *Voluntary debt exchanges*

Opportunities for voluntary debt exchanges arise on average in 11 out of 100 simulation periods. That is, in 11% of the simulation periods the initial debt level b is higher than the debt level that maximizes the market value of debt (denoted by $\overline{b}(y)$). There are two reasons why this may happen. First, b may be higher than $\overline{b}(y)$ because of a sufficiently negative income shock. Even when the government chooses $\hat{b}^P(b,y) < \overline{b}(y)$, it may still be that next period $\hat{b}^P(b,y) > \overline{b}(y')$. The left panel of Fig. 5 depicts the market value of debt claims for two income levels. The figure shows that declines in income shift the market value curve toward the origin and reduce the debt level that maximizes *MV*. This occurs because, as is standard in models of sovereign default that generate countercyclical spreads, lower income levels imply higher default probabilities. This is illustrated in the right panel of Fig. 5, which shows that lower income levels are associated with lower bond prices (that reflect higher default probabilities).

Furthermore, even when the government starts the period with $b < \overline{b}(y)$, it may choose to end a period with a debt level higher than the one that maximizes the debt market value. Fig. 6 presents an example of these situations in the simulations. We find however that such examples are only observed in 3% of the simulation periods. Furthermore, as illustrated in Fig. 6, b' is typically very close to b in these periods.

Fig. 7 presents the equilibrium outright default and voluntary exchange policies. Fig. 7 shows that the government prefers to participate in a debt exchange only when aggregate income takes a sufficiently low value (gray and black areas). The income threshold at which the government is indifferent between participating and not participating in a debt exchange is higher than the income threshold at which it is indifferent between an outright default and repaying current debt obligations. This also means that all outright defaults in the simulations would have been avoided with a low cost of voluntary exchange. In addition, Fig. 7 shows that for some state realizations the government would choose to conduct a voluntary exchange even if the impossibility of an exchange would not have triggered an outright default (gray area). This is consistent with the presence of opportunities for capital gains at debt levels that would not trigger an outright default, as illustrated in the left panel of Fig. 5.

Table 3
Calibration targets and simulation moments. Moments for the simulations correspond to the mean value of each moment in 500 simulation samples.

Moment	Targets	Simulations
Mean debt-to-GDI	44	42
Mean r_s	3.40	3.35
Voluntary debt exchanges/defaults	0.30	0.27

Table 4
Nontargeted simulation moments. The standard deviation of x is denoted by $\sigma(x)$. The coefficient of correlation between x and z is denoted by $corr(x,z)$. Moments for the simulations correspond to the mean value of each moment in 500 simulation samples.

Moment	Mexico	Simulations
$\sigma(r_s)$	1.6	1.4
$corr(r_s, y)$	−0.5	−0.7
$\sigma(c)/\sigma(y)$	1.3	1.3
$Corr(tb, y)$	−0.7	−0.8
$Corr(r_s, tb)$	0.8	0.9

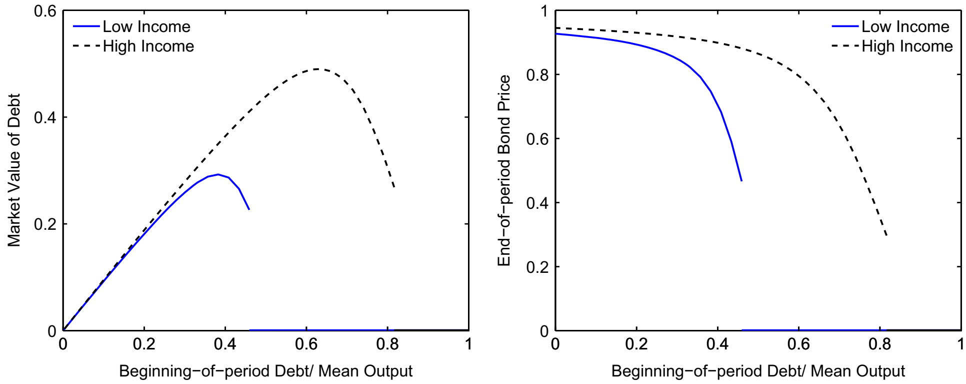

Fig. 5. Market value of the debt stock and bond prices. The left panel plots $MV(b,y)$. The right panel plots the end-of-period bond price when the cost of an exchange is high: $[1-\hat{d}(b,y,H)]q(\hat{b}^{P}(b,y),y)$. The figure refers to income levels one standard deviation below and above the unconditional mean as low income and high income.

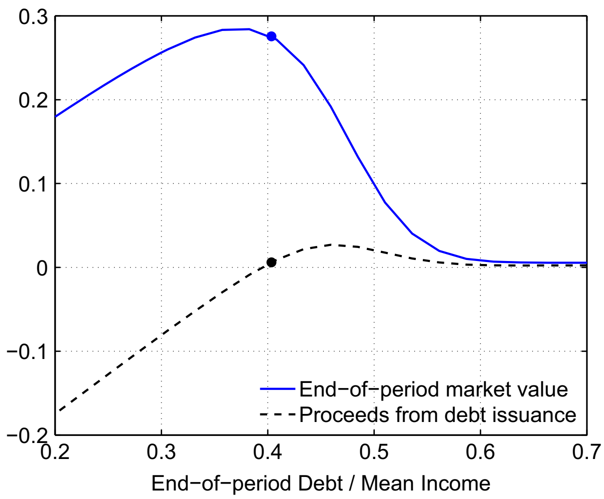

Fig. 6. Debt market value and proceeds from debt issuances. The end-of-period market value of debt claims is defined as $q(b',y)b'$. The proceeds of debt issuances consist of $q(b',y)[b'-(1-\delta)b]$. The graph assumes that the state is characterized by an initial debt level of 41% of mean income and that the current income realization is one standard deviation below the mean. The solid dots represent the optimal decision of a sovereign that did not exchange debt in the period.

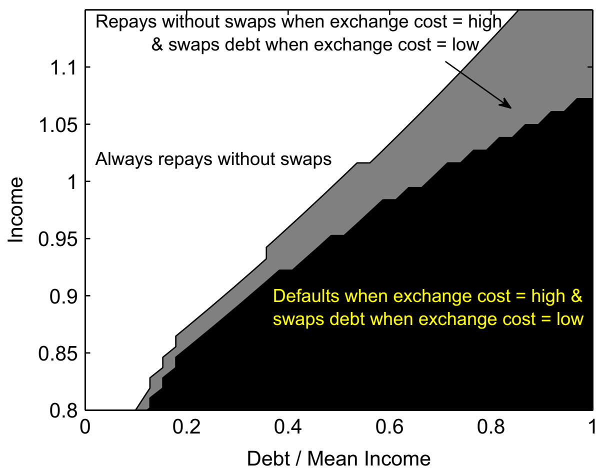

Fig. 7. Equilibrium voluntary exchange and outright default policies. For each debt level, the figure presents the maximum income level for which the government would choose to declare an outright default (if the cost of a debt exchange is high) or participate in a debt exchange (if the cost of a debt exchange is low).

Table 5
Simulations in the benchmark model and with one-period debt.

Moment	Benchmark	One-period bonds
Mean debt-to-GDP	42.4	32.3
Mean r_s	3.35	0.14
Outright defaults per 100 years	2.21	0.14
Voluntary debt exchanges per 100 years	0.81	0.003

6.3. The role of long-term debt

Table 5 presents the benchmark simulations (when the government issues only long-term debt) and simulations of the model with only one-period debt ($\delta=1$ while maintaining the remaining parameter values presented in Table 2). When the government only issues one-period debt, the spread and default rates are significantly lower than in the baseline calibration (Hatchondo and Martinez, 2009). In addition, the share of default episodes that can be described as voluntary debt exchanges drops from 27% in our benchmark to 2% with one-period debt.[11] This is the case because in the one-period-bond version of the model, all voluntary exchanges prevent an outright default. Notice that with one-period bonds, the market value of the debt stock at the beginning of a period is given by $MV(b,y)=[1-\hat{d}(b,y)]b$. Therefore, a debt reduction can only increase this market value (making a voluntary exchange possible) when the debt reduction prevents an outright default. Consequently, with one-period bonds, the share of defaults due to voluntary exchanges is π (2%). This share is higher with long-term bonds as voluntary exchanges occur when an outright default would not have occurred. As illustrated in Figs. 5 and 6, at the beginning of a period, the debt level may be higher than the one that maximizes the debt market value (generating the possibility of a voluntary exchange) even when this debt level would not lead the government to outright default in the current period. In fact, only 5% of voluntary exchanges in our benchmark simulations prevent an outright default in the period of the exchange.

Furthermore, the government would never choose a debt level higher than the one that maximizes the debt market value in the one-period-bond version of the model. Such a choice is only optimal when the issuance proceeds are different from the market value (as in our baseline calibration with long-term debt). This is not the case with one-period bonds.[12] Choosing a debt level higher than the one that maximizes the debt market value implies choosing a debt level such that the effect of additional borrowing on the market value is negative. If the government repays its debt and does not launch a voluntary exchange, the first order condition for debt accumulation is given by:

$$\frac{d(q(b',y)b')}{db'}=-\frac{\beta}{u'(c)}\frac{d(\mathbb{E}_{(y',\theta')|y}V(b',y',\theta'))}{db'}+(1-\delta)b\frac{d(q(b',y'))}{db'}, \tag{10}$$

assuming the value and bond price functions are differentiable.

The left hand side of Eq. (10) represents the marginal effect of the last bond issued in the period on the market value of debt claims outstanding at the end of the period. In the model simulations, the first term of the right-hand side is always positive (the government is worse off when it starts a period with a higher debt level) while the second term is negative (bond prices are lower when the government chooses a higher debt level). Thus, the government only chooses debt levels higher than the one that maximizes the market value when that second term is low enough. This is never the case in the one-period-bond version of the model ($\delta=1$), where the second term is equal to zero.

6.4. Gains at the moment of a voluntary debt exchange

Table 6 reports the average capital gain, haircut, and welfare gain from voluntary exchanges. The table shows that, consistent with our definition, these exchanges produce gains for lenders (as indicated by positive capital gains) and the government (as indicated by positive haircuts and welfare gains). We measure welfare gains as the constant proportional change in consumption that would leave domestic consumers indifferent between not having the option to conduct a voluntary exchange ($\theta=H$) and having this option ($\theta=L$):

$$\left(\frac{V(b,y,L)}{V(b,y,H)}\right)^{1/(1-\gamma)}-1.$$

6.5. Ex ante gains from eliminating the possibility of voluntary debt exchanges

Fig. 8 presents the proportional consumption compensation that leaves domestic residents indifferent between living in an economy without debt exchanges ($\pi=0$) and moving to an economy with debt exchanges ($\pi=0.02$). A positive number

[11] Recall that, as discussed in the calibration section, we count both outright defaults and voluntary debt exchanges as default episodes.

[12] When the government issues debt, it is concerned about the market value of the debt claims that it is issuing (i.e., the issuance proceeds) and not about the overall value of the debt stock (Hatchondo et al., 2012b). With one-period debt, debt issued in the current period constitutes the total debt stock.

Table 6
Gains from a voluntary debt exchange. Capital gains for exchanges that prevent an outright default are computed using the market value of the debt stock the quarter before the exchange.

Feature of the restructuring	Exchanges that prevent an outright default	Other exchanges
Average capital gain (%)	141	6
Average haircut (%)	23	7
Average welfare gain (%)	0.6	0.2

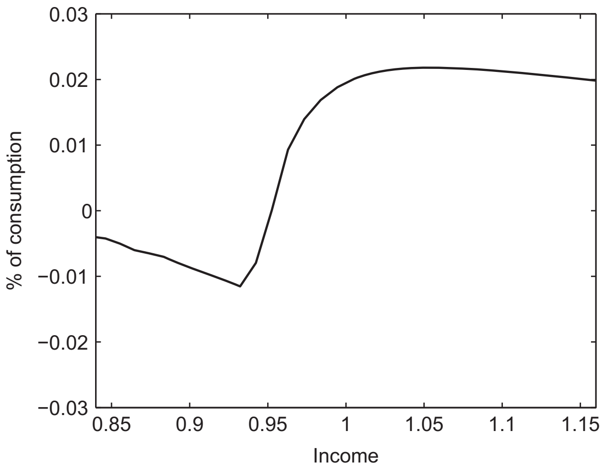

Fig. 8. Welfare gains derived from eliminating the possibility of voluntary exchanges. Welfare gains are computed for a debt level equal to the mean debt level in the simulations of the benchmark ($\pi=0.02$), and before current exchange cost realization.

means that domestic residents are better off without debt exchanges. The welfare gain is evaluated at a debt level equal to the mean debt observed in the simulations of the benchmark economy ($\pi=0.02$), and before the government learns the current exchange cost. Formally, the welfare gain is computed as:

$$\left(\frac{\pi_0 V(b,y,L;\pi_0)+(1-\pi_0)V(b,y,H;\pi_0)}{\pi_1 V(b,y,L;\pi_1)+(1-\pi_1)V(b,y,H;\pi_1)}\right)^{1/(1-\gamma)}-1,$$

for $\pi_0=0$ and $\pi_1=0.02$.

Fig. 8 shows that eliminating the possibility of conducting a voluntary exchange may improve welfare for sufficiently high income levels. This is so even though voluntary exchanges are Pareto improving at the time of the restructuring (Table 6). For high income levels, it is not optimal for the government to conduct a voluntary exchange in the current period, even when the exchange cost is low. Furthermore, it is unlikely that the government will conduct a voluntary exchange in the near future.

On the contrary, for sufficiently low income levels, the possibility of conducting voluntary exchanges improves welfare. This occurs because the government is likely to benefit from a voluntary exchange when income and the cost of an exchange are low. This is illustrated in Fig. 9, which shows that a voluntary exchange would allow the government to increase consumption when income is low.

The average welfare gain from eliminating the possibility of conducting voluntary exchanges is 0.015%. Gains from eliminating the possibility of conducting voluntary exchanges are consistent with governments' reluctance to issue easier to restructure debt. Since our benchmark probability of a low exchange cost is only 2%, welfare gains from lowering this probability to zero are modest.

Fig. 9 also shows that for high income levels, consumption is (slightly) lower in the economy with voluntary exchanges. This happens because, as in the four-period model, the possibility of conducting voluntary debt exchanges shrinks the set of borrowing opportunities available to the government (see Fig. 3). Fig. 10 illustrates how the spread compensation demanded by investors is higher in the economy with debt exchanges than in the one without debt exchanges.

We next show that (as in the four-period model) welfare gains from eliminating the possibility of voluntary debt exchanges arise because of the effects of this possibility on the government's borrowing choices. Table 7 reports that, in

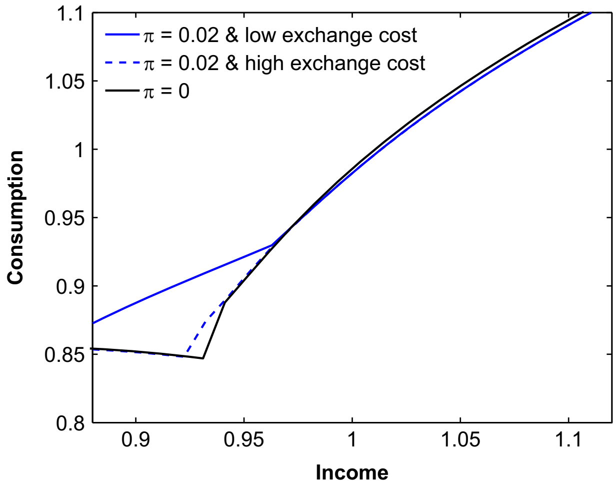

Fig. 9. Consumption in the economies with and without voluntary exchanges ($\pi = 0.02$ and $\pi = 0$). The initial debt level is equal to the mean debt observed in the simulations of the benchmark economy ($\pi = 0.02$).

equilibrium, the government borrows (slightly) more in the benchmark economy with voluntary debt exchanges. This is consistent with higher spreads and a higher frequency of outright defaults in this economy.

To further illustrate the effect of changes in the government's borrowing choices on the welfare gains from eliminating the possibility of voluntary exchanges, we solved a counterfactual economy in which voluntary exchanges are possible ($\pi = 0.02$) but the government is forced to follow the borrowing rule corresponding to the economy without voluntary exchanges. In the counterfactual economy, the government chooses optimally whether to pay its debt, conduct a voluntary exchange, or outright default. Furthermore, bond prices satisfy the lenders' expected zero profit condition.

The left panel of Fig. 11 presents welfare gains of moving from the counterfactual economy ($\pi = 0.02$ and the government's optimal borrowing rule in the economy with $\pi = 0$) to an economy without debt exchanges ($\pi = 0$). The figure shows negative welfare gains for all income levels. This contrasts with the positive welfare gains from eliminating the possibility of voluntary exchanges presented in Fig. 8, indicating that the gains in Fig. 8 are due to the changes in the government's borrowing choices that we do not allow when constructing Fig. 11.

Welfare is higher in the counterfactual economy than in the economy without exchanges because the set of borrowing opportunities available to the government is larger in the counterfactual economy. The right panel of Fig. 11 shows that the spread compensation demanded by investors is lower in the counterfactual economy than in the economy without exchanges. This contrasts with the negative effect of voluntary exchanges on the government's borrowing set presented in Fig. 10, indicating that this negative effect is due to the changes in the government's borrowing choices that we do not allow in the counterfactual economy.

6.6. *Comparative statics with respect to creditors' bargaining power*

Table 8 shows that, as one would expect, reducing creditor's bargaining power (up to 0.5) increases the average haircut in voluntary exchanges, increases the frequency of voluntary exchanges, and, consequently, increases the welfare gain from eliminating the possibility of conducting these exchanges. The table also shows that changes in bargaining power do not affect significantly other implications of the model, which may not be surprising since voluntary exchanges are infrequent.

Fig. 12 illustrates the haircut after a voluntary debt exchange as a function of income for two economies: the benchmark ($\alpha = 1$) and an economy with $\alpha = 0.7$. The figure shows that when creditors do not have full bargaining power, the post-exchange debt level presents a discontinuity. This discontinuity arises because, as illustrated in Fig. 5, the market value presents a discontinuity, dropping to zero in states in which the government would choose an outright default if the cost of conducting a voluntary exchange is high. A lower market value of debt enables the government to increase the debt relief that it can extract from creditors.[13]

[13] The discontinuity in Fig. 12 also implies a discontinuity in the value function V^E. This creates problems when interpolation methods are used to compute expectations. We solve this problem approximating two auxiliary functions for the post-exchange debt level (and thus two auxiliary functions for the value of conducting an exchange): one function assumes that the government does not default in the current period in any state (and follows the optimal strategies in the future), and the other function assumes that the government always defaults in the current period if there is no voluntary exchange (and follows the optimal strategies in the future). We then use the first (second) function to interpolate for states in which the government repays (defaults).

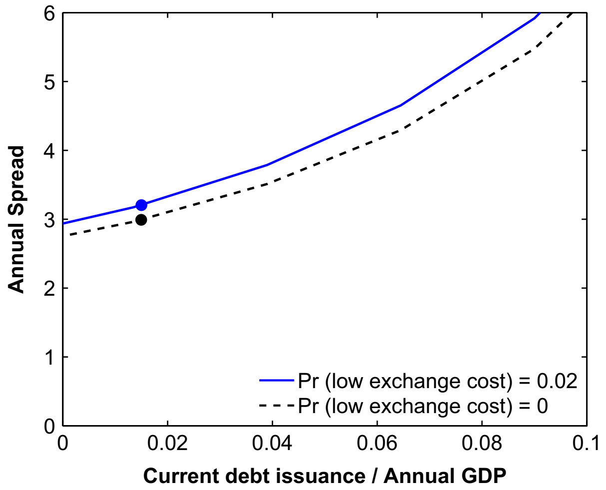

Fig. 10. Schedule of borrowing and spreads available to the government in the economies with and without voluntary exchanges ($\pi = 0.02$ and $\pi = 0$). The figure assumes that the current income is equal to the mean income. The solid dots represent optimal choices when the current debt level equals the mean debt in the benchmark simulations ($\pi = 0.02$).

Table 7
Simulations when the government cannot conduct voluntary exchanges.

Moment	Benchmark	No exchanges
Mean debt-to-GDP	42.5	42.3
Mean r_s	3.4	3.1
$\sigma(r_s)$	1.4	1.2
$Corr(r_s, y)$	−0.7	−0.3
Outright defaults per 100 years	2.2	2.1
Voluntary debt exchanges per 100 years	0.8	0

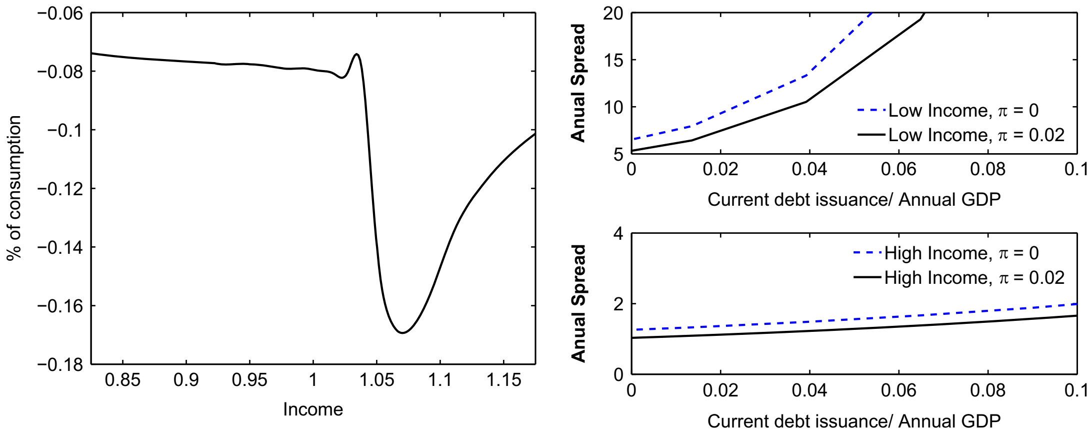

Fig. 11. Welfare gains from eliminating the possibility of voluntary exchanges and spread schedule for the counterfactual economy ($\pi = 0.02$ and the government's optimal borrowing rule in the economy with $\pi = 0$). Gains are computed for the mean debt level in the simulations of the benchmark, before the realization of the exchange cost.

7. Conclusions

This paper first documents that some recent sovereign debt restructurings were not preceded by missed debt payments and resulted in both sovereign debt reductions and capital gains for the holders of restructured debt. The paper then

Table 8
Simulations in economies with different creditors' bargaining power.

Moment	$\alpha = 1$	$\alpha = 0.7$	$\alpha = 0.5$
Mean debt-to-GDP	42.5	42.5	42.5
Mean r_s	3.4	3.5	3.6
Outright defaults per 100 years	2.2	2.3	2.3
Voluntary debt exchanges per 100 years	0.8	0.9	0.9
Average haircut (in %)	8	11	13
Welfare gain from eliminating VDE (% of cons.)	0.015	0.027	0.033

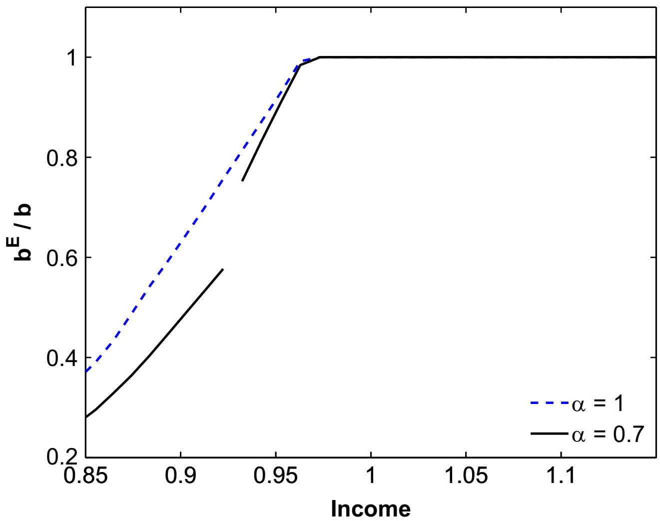

Fig. 12. Voluntary debt exchange recovery rate as a function of income and creditors' bargaining power for a debt level equal to the mean debt observed in the simulations of the benchmark economy.

proposes a model that accounts for such restructurings. In contrast with standard debt overhang arguments, our model delivers Pareto gains from debt restructurings even when debt reductions have no effect on future output levels. These gains arise only because of the decline of sovereign risk implied by the debt reduction. In our model, voluntary exchanges are possible after negative income shocks but do not require an outright default to be imminent. Most voluntary exchanges in the model simulations happen in periods in which the government would not choose to declare an outright default. The paper also shows that in spite of the Pareto improvement at the time of voluntary exchanges, the government may prefer an environment in which conducting these exchanges is more difficult.

Overall, the paper presents an attempt to enrich the understanding of gains from renegotiation between creditors and debtors, but it also highlights a time inconsistency problem in the government's decision of undertaking renegotiations.[14] The discussion of these issues would certainly benefit from a more thorough analysis of the possibility of Pareto gains in past sovereign debt restructuring experiences, and richer theories that, for instance, endogenize renegotiations both after outright defaults and in voluntary debt exchanges.[15]

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[14] Hatchondo et al. (2012a,b) highlight a time inconsistency problem in the government's borrowing decision. Hatchondo and Martinez (2013) highlight a time inconsistency problem in the government's debt maturity choice.

[15] Arguably, mechanisms that facilitate voluntary debt exchanges may also facilitate the restructurings that follow after an outright default. We abstract from that possibility. Benjamin and Wright (2008) and Yue (2010) model debt renegotiation but only after an outright default.

Appendix A. Supplementary material

Supplementary data associated with this article can be found in the online version at http://dx.doi.org/10.1016/j.jmoneco.2013.11.002.

References

Aguiar, M., Amador, M., Gopinath, G., 2009. Investment cycles and sovereign debt overhang. Rev. Econ. Stud. 76 (1), 1–31.
Aguiar, M., Gopinath, G., 2006. Defaultable debt, interest rates and the current account. J. Int. Econ. 69, 64–83.
Aguiar, M., Gopinath, G., 2007. Emerging markets business cycles: the cycle is the trend. J. Political Econ. 115 (1), 69–102.
Arellano, C., 2008. Default risk and income fluctuations in emerging economies. Am. Econ. Rev. 98 (3), 690–712.
Benjamin, D., Wright, M.L.J., 2008. Recovery Before Redemption? A Theory of Delays in Sovereign Debt Renegotiations. Manuscript.
Chatterjee, S., Eyigungor, B, 2012. Maturity, indebtedness and default risk. Am. Econ. Rev. 102 (6), 2674–2699.
Cruces, J.J., Trebesch, C., 2013. Sovereign defaults: the price of haircuts. Am. Econ. J.: Macroecon. 5 (3), 85–117.
Cruces, J.J., Buscaglia, M., Alonso, J., 2002. The Term Structure of Country Risk and Valuation in Emerging Markets. Manuscript. Universidad Nacional de La Plata.
Dias, D.A., Richmond, C., 2007. Duration of Capital Market Exclusion: An Empirical Investigation. Working Paper, UCLA.
Eaton, J., Gersovitz, M., 1981. Debt with potential repudiation: theoretical and empirical analysis. Rev. Econ. Stud. 48, 289–309.
Enderlein, H., Trebesch, C., von Daniels, L., 2012. Sovereign debt disputes: a database on government coerciveness during debt crises. J. Int. Money Finance 31, 250–266.
Froot, K.A., 1989. Buybacks, exit bonds, and the optimality of debt and liquidity relief. Int. Econ. Rev. 30 (1), 49–70.
Hatchondo, J., Martinez, L., 2009. Long-duration bonds and sovereign defaults. J. Int. Econ. 79 (1), 117–125.
Hatchondo, J., Martinez, L., Roch, F., 2012a. Fiscal Rules and the Sovereign Default Premium.
Hatchondo, J.C., Martinez, L., 2011. Legal protection to foreign investors. Econ. Q. 97 (2), 175–187.
Hatchondo, J.C., Martinez, L., 2013. Sudden stops, time inconsistency, and the duration of sovereign debt. Int. Econ. J. 27 (2), 217–228.
Hatchondo, J.C., Martinez, L., Sapriza, H., 2010. Quantitative properties of sovereign default models: solution methods matter. Rev. Econ. Dyn. 13 (4), 919–933.
Hatchondo, J.C., Martinez, L., Sosa-Padilla, C., 2012b. Debt Dilution and Sovereign Default Risk. Federal Reserve Bank of Richmond Working Paper No. 10.08R.
IMF, 2002. The Design and Effectiveness of Collective Action Clauses.
IMF, 2013. Sovereign Debt Restructuring Recent Developments and Implications for the Funds Legal and Policy Framework. International Monetary Fund.
Krugman, P., 1988a. Financing vs. forgiving a debt overhang. J. Dev. Econ. 29 (3), 253–268.
Krugman, P., 1988b. Market-Based Debt-Reduction Schemes. NBER Working Paper 2587.
Krusell, P., Smith, A., 2003. Consumption-savings decisions with quasi-geometric discounting. Econometrica 71 (1), 365–375.
Mendoza, E., Yue, V., 2012. A general equilibrium model of sovereign default and business cycles. Q. J. Econ. 127 (2), 889–946.
Neumeyer, P., Perri, F., 2005. Business cycles in emerging economies: the role of interest rates. J. Monetary Econ. 52, 345–380.
Panizza, U., Sturzenegger, F., Zettelmeyer, J., 2009. The economics and law of sovereign debt and default. J. Econ. Lit. 47 (3), 653–699.
Sachs, J., 1989. The Debt Overhang of Developing Countries. Basil Blackwell, Oxford.
Sturzenegger, F., Zettelmeyer, J., 2005. Haircuts: Estimating Investor Losses in Sovereign Debt Restructurings, 1998–2005. IMF Working Paper 05-137.
Uribe, M., Yue, V., 2006. Country spreads and emerging countries: who drives whom? J. Int. Econ. 69, 6–36.
Wright, M.L.J., 2011. Restructuring sovereign debts with private sector creditors: theory and practice. In: Primo Braga, Carlos A., Vincelette, Gallina A. (Eds.), Sovereign Debt and the Financial Crisis: Will This Time Be Different?, World Bank, Washington, DC, pp. 295–315.
Yue, V., 2010. Sovereign default and debt renegotiation. J. Int. Econ. 80, 176–187.
Zettelmeyer, J., Trebesch, C., Gulati, M., 2012. The Greek Debt Exchange: An Autopsy. Mimeo.