

# Online Appendix for “Sovereign Defaults and Banking Crises”

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1 This Online Appendix presents the details of a number of analyses and robustness tests  
2 that are referred to in the main paper. Section A presents a sensitivity analysis to assess  
3 the robustness of the main quantitative results in the main paper. Section B discusses some  
4 simplifying assumptions and how relaxing them may affect the main results.

## 5 **A. Sensitivity Analysis**

6 In this section we vary the value of some key parameters in order to get an insight on  
7 how each of them affect the dynamics. Note that parameter values are changed one at a  
8 time (i.e. keeping the values of all other parameters unchanged). Table 1 summarizes the  
9 findings of this exercise. <sup>1</sup>

### 10 *A.1. Tightness of the working capital constraint*

11 Let us first consider how the model behaves with different values of  $\gamma$ . This parameter  
12 governs the tightness of the working capital constraint,  $\gamma \in (0, 1]$ . A high (low) value of  $\gamma$   
13 means that firms need to pay up-front a higher (lower) proportion of their wage bill; this  
14 means that private credit in the form of working capital loans is more (less) important for  
15 production.

16 Panel B of Table 1 shows that the model performs as expected: for lower values of  $\gamma$   
17 (cases in which private credit is not so important for production), default is not very costly.  
18 Consequently, the government is tempted to default too often. Creditors, understanding this,  
19 reduce lending in the government bonds market. Along those lines, values of  $\gamma \leq .30$  produce

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<sup>1</sup>While columns 1 to 5 have self-explanatory headings, columns 6 and 7 warrant a minor clarification: they report output drops and credit drops around defaults (measured as peak-to-through using the de-trended series), respectively.

20 a mean debt ratio of zero and an observed default rate is zero as well. On the other hand,  
21 high values of  $\gamma$  make defaults very costly. This raises the observed debt ratios and lowers  
22 the observed default rates. In these less frequent (i.e., rarer than the benchmark) defaults,  
23 the costs in terms of output and credit drops are considerable larger than in the benchmark  
24 calibration (precisely because higher exposure ratios bring higher output and credit drops  
25 during defaults). These dynamics imply a non-monotonic behavior of the default rate as the  
26 value of  $\gamma$  increases. This leads to (for example) having two scenarios with a zero default  
27 rate and with zero spreads that are very different: on the one hand, low enough values of  
28  $\gamma$ , for which there is no lending (default temptation is too high), and on the other hand,  
29 sufficiently large values of  $\gamma$ , for which there are large debt and exposure ratios (default costs  
30 are too large).

### 31 *A.2. Financial exclusion after defaults*

32 Next, let's consider how the model economy reacts to changes in the re-entry probability  
33 ( $\phi$ ). Panel C of Table 1 has the results for the sensitivity analysis regarding parameter  $\phi$ .

34 When the government can re-access credit markets immediately after a default ( $\phi = 1$ ),  
35 the overall costs of a default (exclusion from credit markets being among them) are reduced.  
36 A lower default cost renders repudiation more attractive, so we see that for  $\phi = 1$  default  
37 is more frequent. Consequently, the government has to pay higher spreads. If, on the other  
38 hand, the value of  $\phi$  decreases (making re-access to credit markets less frequent), then the  
39 exclusion cost of default is larger, default is chosen less frequently, and the government can  
40 obtain better debt prices (i.e., it can pay lower spreads).

41 Figure 1 shows how a credit crunch looks in the model. The benchmark calibration of  
42 the model features a collapse in the private sector credit (i.e., working capital loans to firms,  
43 in the model). The workings of a credit crunch are clear from both panels in Figure 1: as  
44 firms are in need of external financing, when loanable funds shrink, output shrinks along  
45 with them.

46 Figure 1 also shows the effect of exclusion from financial markets: if the government re-  
47 mains excluded, the private credit reduces (and remains low) and the output decline becomes  
48 more protracted. On the other hand, an immediate re-access to the credit market implies a

49 rapid recovery in both credit and output. <sup>2</sup>

### 50 A.3. Relative weights in the social welfare function

51 The model in the main article makes the (common) assumption that the planner only  
 52 cares about the households utility. However, we can study the dynamics of the model under  
 53 different social welfare functions. In particular, one could study the default incentives and  
 54 the transmission mechanism from defaults to banking crises when the planner cares about a  
 55 weighted average of all residents utilities: households and bankers.

Formally, the planner's optimization problem can now be written recursively as:

$$\mathcal{V}(b, k, z) = \max_{d \in \{0,1\}} \{(1-d)\mathcal{V}^{nd} + d\mathcal{V}^d\} \quad (1)$$

56 where  $\mathcal{V}^{nd}$  ( $\mathcal{V}^d$ ) is the value of repaying (defaulting). Given that there are two types of  
 57 residents (households and bankers), the overall objective function of the planner is a convex  
 58 combination of the value functions of the two types of residents. Then:

$$\mathcal{V}^i(b, z) = \theta V^i(b, k, z) + (1-\theta)W^i(b, k, z),$$

59 where  $i = \{nd, d\}$  and  $\theta \in [0, 1]$  is the weight assigned to the households' *happiness* in the  
 60 planner's objective function. The parameter  $\theta$  gives the model a certain flexibility. Letting  $\theta$   
 61 be equal to one collapses this specification to the benchmark calibration studied in the main  
 62 article. Moving  $\theta$  to zero implies that the planner will only care about bankers.

63 Therefore, the value of no-default is:

$$\mathcal{V}^{nd}(b, k, z) = \max_{\{c,x,n,k',b'\}} \{\theta V^{nd}(b, k, z) + (1-\theta)W^{nd}(b, k, z)\} \quad (2)$$

64 subject to:

$$V^{nd}(b, k, z) = U(c, n) + \beta \mathbb{E}V^{nd}(b', k', z') \quad (\text{hh's value function})$$

$$W^{nd}(b, k, z) = x + \delta \mathbb{E}W^{nd}(b', k', z') \quad (\text{banker's value function})$$

$$65 \quad g + b = \tau wn + b'q \quad (\text{gov't b.c.})$$

$$c + x + g + k' = zF(n) + A + s(k) \quad (\text{resources const.})$$

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<sup>2</sup> As in Mendoza and Yue (2012), the v-shaped recovery of output after a default event is driven by two forces: TFP and re-access to credit. TFP is mean-reverting and thus very likely to recover after defaults. Also, when the sovereign regains access to credit markets, then the output recovery is even faster.

$$\left. \begin{aligned}
x &= (A + s(k) + b)(1 + r) - k' - qb' \\
q &= \delta \mathbb{E} \{(1 - d')(1 + r')\} \\
1 &= s_k(k') \delta \mathbb{E} \{1 + r'\} \\
r &= \frac{znF_n}{A+s(k)+b} - \frac{1}{\gamma} \\
-\frac{U_n}{U_c} &= (1 - \tau)w \\
w &= \frac{zF_n}{(1+\gamma r)}
\end{aligned} \right\} \text{(comp. eq. conditions)}$$

The value of default is:

$$\mathcal{V}^d(k, z) = \max_{\{c, x, n, k'\}} \{\theta V^d(k, z) + (1 - \theta)W^d(k, z)\} \quad (3)$$

subject to:

$$V^d(k, z) = U(c, n) + \beta \mathbb{E} \{\phi V(0, k', z') + (1 - \phi)V^d(k', z')\} \quad (\text{hh's value function})$$

$$W^d(k, z) = x + \delta \mathbb{E} \{\phi W(0, k', z') + (1 - \phi)W^d(k', z')\} \quad (\text{banker's value function})$$

$$g = \tau wn \quad (\text{gov't b.c.})$$

$$c + x + g + k' = zF(n) + A + s(k) \quad (\text{resources const.})$$

$$\left. \begin{aligned}
x &= (A + s(k))(1 + r) - k' \\
1 &= s_k(k') \delta \mathbb{E} \{1 + r'\} \\
r &= \frac{znF_n}{A+s(k)} - \frac{1}{\gamma} \\
-\frac{U_n}{U_c} &= (1 - \tau)w \\
w &= \frac{zF_n}{(1+\gamma r)}
\end{aligned} \right\} \text{(comp. eq. conditions)}$$

Panel D of Table 1 presents the results for using different values in the relative weights of the planner's objective function (i.e., different values for the parameter  $\theta$ ). We can see that response of the default rate is non-monotonic. For high values of  $\theta$  (i.e., high relative weight to the households' utility) the default frequency is lower: the planner values the households utility more, these agents have concave utility functions and therefore dislike profoundly swings in consumption and leisure, and increases in distortionary taxes, hence it is in the planner's best interest to keep crisis events relatively infrequent. As the parameter  $\theta$  increases, the planner assigns less and less weight to the households utility and so crises are more frequent and spreads are higher. The case of  $\theta = 0$  where the planner only cares about the welfare of the bankers is an extreme one: since the bankers receive the entire hit of the defaults it is now optimal to never default.

## 80 B. Discussion of additional simplifying assumptions

81 The model described in the main article involved a series of simplifying assumptions that  
82 were made in order to isolate the effect that a sovereign default has on the banking and  
83 productive sectors of the economy. This subsection discusses ways to relax two of these  
84 assumptions and the implications of doing so. <sup>3</sup>

85 *Constant government spending.* In order to simplify the optimal fiscal policy planning, our  
86 model assumes a constant level of government expenditures,  $g$ . While this is a useful first ap-  
87 proximation, relaxing this assumption could improve the model’s quantitative performance.  
88 A commonly used alternative is to render  $g$  valuable by including it in the agents’ preferences.  
89 In this case,  $g$  becomes an extra fiscal policy instrument: the planner understands that a  
90 higher  $g$  implies either higher taxation or higher indebtedness, but also takes into account  
91 the agents’ preferences for  $g$ . Then, when the country defaults and consumption declines,  
92 the planner will find it optimal to decrease  $g$  as well in order to satisfy the intra-temporal  
93 optimality condition relating private and public consumption. Thus, if government spending  
94 were to be “endogenized” in this way, the model would be able to account for the observed  
95 pro-cyclicality of government spending (see Cuadra et al., 2010).

96 Another alternative is to follow the tradition of Lucas and Stokey (1983) and have  $g$   
97 follow an exogenously given stochastic process. Extending in this way the model presented  
98 in the main article, “good times” and “bad times” will now be indexed by the realizations of  
99 both the TFP process and the “expenditure” shock. We consider that, while enriching the  
100 environment, this second alternative does not add any new insights to our understanding of  
101 the dynamics of sovereign debt, bank lending, and defaults. <sup>4</sup>

102 *Total defaults.* The model economy in the main article is based on the assumption that  
103 sovereigns can either repay in full or default in full. This is an assumption shared by most of  
104 the papers in the quantitative literature on sovereign debt and default. In models à la Eaton

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<sup>3</sup>The main article already has a discussion of other (main) simplifying assumptions.

<sup>4</sup>The computational challenge of adding an “expenditure” shock, as in Lucas and Stokey (1983) (or Aiyagari et al., 2002), boils down to adding an extra exogenous state variable, which increases the state space but keeps the algorithm and solution method otherwise unchanged.

105 and Gersovitz (1981), this assumption is easily justified by making the cost of the default  
106 independent of its size: if a country is to suffer the costs of defaulting, it had better obtain all  
107 the possible gains thereof, which implies a full repudiation. In our environment, the cost of a  
108 default (i.e., the output decline) is not independent but actually a function of the amount of  
109 debt repudiated. The very nature of the model renders it a suitable laboratory for studying  
110 the extent to which sovereigns would like to conduct partial defaults, and also for analyzing  
111 the dynamics of such defaults.

112 Recent work by Arellano et al. (2013) has incorporated the option for sovereigns in models  
113 of this type to partially default on their debts. One advantage of our framework over Arellano  
114 et al. (2013)'s is that in our environment incentives to default on fractions of the debt arise  
115 endogenously rather than by assuming an ad hoc "cost-of-default" function that depends on  
116 the amount of defaulted debt. Studying the reasons why countries may partially default on  
117 their debts is nonetheless beyond the scope of this study.

118 **References for Online Appendix**

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Table 1: Sensitivity Analysis.

	Moments (in %)					
	Default rate	$E\{b/y\}$	$E\{R_s\}$	Exposure	$y \downarrow$	Credit $\downarrow$
<b>Panel A.</b>						
Data	2.5	11.32	7.44	26.5	13.67	40.11
Benchmark calibration	2.6	11.54	7.39	26.8	7.16	8.00
<b>Panel B.</b> Working capital constraint (benchmark: $\gamma = .52$ )						
$\gamma = 0.3$	0	0	0	0	n.a.	n.a.
$\gamma = 0.45$	19.24	2.11	45.30	6.48	3.77	4.08
$\gamma = 0.65$	0.31	43.74	3.91	56.89	15.50	24.83
$\gamma = 1$	0	50.13	0	61.10	n.a.	n.a.
<b>Panel C.</b> Reentry Probability (benchmark: $\phi = .5$ )						
$\phi = 0.10$	1.22	26.40	5.84	45.56	7.52	8.73
$\phi = 0.25$	2.39	17.17	7.36	35.42	7.27	9.46
$\phi = 0.75$	4.04	12.81	8.93	29.67	9.05	14.09
$\phi = 1$	7.97	15.90	14.80	36.05	11.88	25.71
<b>Panel D.</b> HH weight in social welfare function (benchmark: $\theta = 1$ )						
$\theta = 0$	0	21.57	0	30.55	n.a.	n.a.
$\theta = 0.25$	8.28	17.23	13.06	36.03	8.49	18.42
$\theta = 0.50$	3.75	11.01	7.30	25.66	7.49	9.43
$\theta = 0.75$	3.12	10.24	6.80	24.17	7.03	8.38



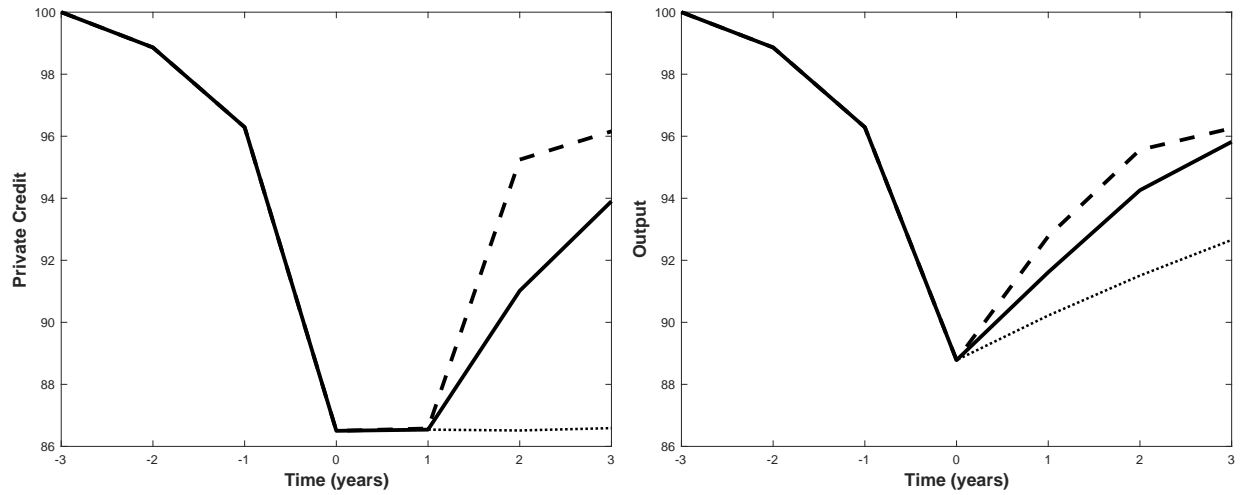


Figure 1: Private Credit, Output, and Financial Exclusion. The left panel corresponds to Private Credit. The right panel corresponds to Output. Both series are normalized so that  $T - 3 = 100$ . The solid line (—) is for the model average, the dashed line (---) is for the case of immediate re-access, and the dotted line (⋯) is for the no re-access case.