Debt dilution and sovereign default risk*

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Abstract

We measure the effects of debt dilution on sovereign default risk and show how these effects can be mitigated with debt contracts promising borrowing-contingent payments. First, we calibrate a baseline model à la Eaton and Gersovitz (1981) to match features of the data. In this model, bonds' values can be diluted. Second, we present a model in which sovereign bonds contain a covenant promising that after each time the government borrows it pays to the holder of each bond issued in previous periods the difference between the bond market price that would have been observed absent current-period borrowing and the observed market price. This covenant eliminates debt dilution by making the value of each bond independent from future borrowing decisions. We quantify the effects of dilution by comparing the simulations of the model with and without borrowing-contingent payments. We find that dilution accounts for 84% of the default risk in the baseline economy. Similar default risk reductions can be obtained with borrowing-contingent payments that depend only on the bond market price. Using borrowing-contingent payments is welfare enhancing because it reduces the frequency of default episodes.

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1 Introduction

This paper presents a measure of the effects of the debt dilution induced by governments' borrowing decisions and shows how these effects can be mitigated with debt contracts that specify borrowing-contingent payments. If governments could commit to not dilute the value of current bond issuances with future borrowing, this would allow them to sell bonds at a higher price. Sovereign debt dilution may become a problem when governments do not have the ability to make such commitment.

Participants in various credit markets have made efforts to mitigate the dilution problem, which is suggestive of the relevance assigned to this issue. Corporate debt contracts often include covenants intended to limit debt dilution (Asquith et al. (2005), Smith and Warner (1979) and Rodgers (1965) discuss corporate debt covenants). In some cases, debt claims differ in their seniority—if existing debt is senior to new issuances, this may mitigate the dilution problem. A seniority structure is common in corporate debt and collateralized loans to households. In contrast, sovereign bonds typically do not present differences in legal seniority but include a pari passu clause and a negative pledge clause that prohibits future issuances of collateralized debt.[1] These clauses are intended to avoid making new debt senior to previously issued debt, but do not make existing debt senior to debt that will be issued in the future.

The weaker protection against sovereign debt dilution may be due in part to the weak enforcement of sovereign debt claims.[2] Overall, it seems clear that existing sovereign debt contracts do not eliminate the risk of debt dilution.

The possibility of sovereign debt dilution has also received considerable attention in both academic and policy discussions. Several studies describe the benefits of eliminating debt dilution.

[1]Sturzenegger and Zettelmeyer (2006) explain how in Ecuador's 2000 sovereign debt restructuring, the government exchanged defaulted bonds for new bonds that included a clause specifying that if a default occurred within 10 years following the restructuring agreement, the government would extend new bonds to the holders of the restructured debt. Sturzenegger and Zettelmeyer (2006) argue that the "effect of this was to offer a (limited) protection of bond holders against the dilution of their claims by new debt holders in the event of default." However, the inclusion of such debt covenants is much more an exception than a rule. It has also been argued that loans from institutions such as the International Monetary Fund or the World Bank receive de facto seniority over loans from private agents (see, for example, Saravia (2010)).

[2]This weak enforcement has lead to several proposals to induce more orderly sovereign debt restructurings (see, for example, Bolton and Skeel (2005), Borensztein et al. (2004), G-10 (2002), IMF (2003), Krueger and Hagan (2005), and Paulus (2002)).

For instance, Bizer and DeMarzo (1992) show how dilution may lead to equilibria with higher debt levels and higher interest rates implied by higher default probabilities. It has also been argued that dilution may lead to excessive issuance of short-term debt (Kletzer (1984)), or of debt that is hard to restructure after a default (Bolton and Jeanne (2009)), which in turn could increase the likelihood and/or severity of sovereign debt crises. Bolton and Skeel (2005) argue for the importance of being able to grant seniority to debt issued while the country is negotiating with holders of debt in default, as observed in corporate bankruptcy procedures. Borensztein et al. (2004) suggest changes in national and international laws that may facilitate the introduction of debt contracts that provide some protection against debt dilution.[3] While these studies suggest that debt dilution may be an important source of inefficiencies in debt markets, they do not quantify the effects of dilution.

We do so using a default framework à la Eaton and Gersovitz (1981).[4] Formally, we analyze a small open economy that receives a stochastic endowment stream of a single tradable good. The government's objective is to maximize the expected utility of private agents. Each period, the government makes two decisions. First, it decides whether to default on previously issued debt. Second, it decides how much to borrow. The government can borrow by issuing non-contingent long-duration bonds, as in Hatchondo and Martinez (2009). The cost of defaulting is represented by an endowment loss that is incurred in the default period.

The most common modeling approach for the study of debt dilution is to focus on the effect of seniority clauses.[5] However, it is well known that seniority does not fully eliminate debt dilution if new borrowing increases the default probability (see, for example, Bizer and DeMarzo (1992)). Therefore, in general, one cannot measure accurately the effects of dilution by comparing equilibria with and without seniority. Furthermore, seniority clauses may not be

[3] Detragiache (1994), Eaton and Fernandez (1995), Niepelt (2008), Sachs and Cohen (1982), Tirole (2002), and UN (2004) also discuss inefficiencies raised by debt dilution.

[4] This framework has been used in many recent studies. See, for instance, Aguiar and Gopinath (2006), Alfaro and Kanczuk (2009) Arellano (2008), Boz (2011), Cuadra et al. (2010), D'Erasmo (2008), Durdu et al. (2010), Lizarazo (2005, 2006), and Yue (2010). These models share blueprints with the models used in studies of household bankruptcy—see, for example, Athreya et al. (2007), Chatterjee et al. (2007), Li and Sarte (2006), Livshits et al. (2008), and Sanchez (2010).

[5] For instance, Bi (2006) analyzes a model with one-quarter and two-quarter bonds and studies the effects of making earlier issuances senior to new issuances.

useful to eliminate sovereign debt dilution in reality. Weak enforcement of sovereign debt claims may constitute an obstacle to implementing a meaningful seniority structure.

A second approach for the study of debt dilution is to compare equilibria obtained with long-duration and with one-period bonds.[6] However, one-period bonds do not only eliminate dilution but also increase rollover risk. We show that, in general, one cannot measure accurately the effects of dilution by comparing equilibria with one-period and with long-duration bonds: Lower default probabilities with one-period bonds are not only the result of the elimination of debt dilution but also the result of the lower debt levels chosen by the borrower to mitigate rollover risk. Furthermore, while eliminating dilution with borrowing-contingent payments increases welfare, replacing long-duration bonds by one-period bonds in the presence of rollover risk decreases welfare.

We propose a new approach for the study of the effects of debt dilution. We modify the baseline model by assuming that sovereign bonds include a covenant specifying that after each time the government borrows it compensates existing bondholders by paying the difference between the bond market price that would have been observed absent current-period borrowing and the observed market price. With this borrowing-contingent payment, bonds' values become independent from future borrowing and thus, there is no dilution caused by borrowing decisions. We measure the effects of dilution by comparing simulations of the baseline model (with dilution) with the ones of the modified model (without dilution). We impose discipline to our quantitative exercise by calibrating the baseline model to match data from an economy facing default risk (Argentina before its 2001 default).

We find that, if the sovereign eliminates debt dilution, the number of defaults per 100 years decreases from 5.6 (with dilution) to 0.9 (without dilution). That is, dilution accounts for 84% of the default risk in the simulations of the baseline model. Reducing the default frequency is beneficial for welfare because defaulting is ex-ante inefficient. Thus, our exercise is indicative of

[6]Intertemporal debt dilution only appears with long-duration bonds. With one-period bonds, when the government decides its current issuance level, the outstanding debt level is zero (either because the government honored its debt obligations at the beginning of the period or because it defaulted on them). Thus, the government cannot dilute the value of debt issued in previous periods. Chatterjee and Eyigungor (forthcoming) and Hatchondo and Martinez (2009) show that in a sovereign default framework, equilibrium default risk is significantly higher with long-duration bonds than with one-period bonds.

the quantitative importance of dilution and supports the view that dilution should be a central issue in discussions of sovereign debt management and the international financial architecture (e.g., Borensztein et al. (2004)).

Eliminating dilution allows the government to choose a low default risk. With dilution, default risk is high even if the government chooses very low debt levels that almost certainly would not trigger a default in the following period. This is the case because future governments would increase the debt level. As long as a government cannot control the choices of future governments, it cannot choose low default risk. Promising borrowing-contingent payments allow the government to moderate the borrowing levels that future governments will choose.

The government's ability to choose low default risk is reflected in its borrowing opportunities (i.e., the set of combinations of levels of debt and interest rates the government can choose from). Thus, eliminating dilution shifts the set of government's borrowing opportunities. The equilibrium combinations of debt and interest rate levels in the simulations without dilution are not part of the government's choice set with dilution. For no-dilution equilibrium debt levels, the equilibrium interest rate would be about 400 basis points higher in the economy with dilution. Borrowing-contingent payments weaken the government's incentives to issue debt and thus imply lower future issuance levels. For any debt level, the expectation of lower future issuance levels implies a lower default probability. This in turn allows the government to pay a lower interest rate.

The borrowing-contingent payments that eliminate dilution may be difficult to implement in practice. This is because determining these payments requires knowledge of the price at which bonds would have traded in the absence of current-period borrowing. While that price can be easily computed in our simulations, it may be difficult to determine in practice.

However, we show that most gains from eliminating dilution can be obtained with two simple borrowing-contingent payments that depend only on the sovereign bond market price. In one, the sovereign promises to pay a predetermined share of current borrowing revenues to the holder of each bond issued in previous periods. In the other, borrowing-contingent payments are a decreasing function of the bond market price. The benchmark default frequency is reduced 69% with the first scheme and 86% with the second scheme.

It should be emphasized that our findings are not based on the assumption that the government cannot default on borrowing-contingent payments in the same way it can default on other payments. Sovereign debt contracts often contain an acceleration clause and a cross-default clause (for example, see IMF (2002)). The first clause allows creditors to call the debt they hold in case the government defaults on a payment. The cross-default clause states that a default in any government obligation constitutes a default in the contract containing that clause. These clauses imply that in practice, when the government chooses to default on a payment it chooses to default on all its debt. The implementation of borrowing-contingent payments only requires that defaulting on borrowing-contingent payments would trigger acceleration and cross-default clauses and, therefore, a default on all government debt.

The borrowing-contingent payments studied in this paper resemble covenants commonly used in corporate debt contracts to transfer resources from debtors to creditors when credit quality deteriorates (for instance, because of an increase in indebtedness). For example, Asquith et al. (2005) document such "interest-increasing performance pricing" and find lower interest rates for contracts with this pricing.

Borrowing-contingent payments also resemble taxes used in previous studies for eliminating overborrowing by private debtors (see Bianchi (forthcoming) and the references therein). In these studies, borrowing by one agent increases other agents' cost of borrowing and the probability of a crisis. Taxing private borrowing reduces the frequency of crises. In this paper, the borrowing by future governments increases the current government's cost of borrowing and the default probability. Borrowing-contingent payments "tax" borrowing by future governments and thus reduce default risk.

The rest of the article proceeds as follows. Section 2 introduces the model. Section 3 discusses the calibration. Section 4 presents the results. Section 5 concludes and discusses possible extensions of our analysis.

2 The model

We first discuss the baseline model with debt dilution and later introduce borrowing-contingent payments that allows us to quantify the role of debt dilution.

2.1 The baseline environment

There is a single tradable good. The economy receives a stochastic endowment stream of this good y_t, where

$$\log(y_t) = (1-\rho)\,\mu + \rho \log(y_{t-1}) + \varepsilon_t,$$

with $|\rho| < 1$, and $\varepsilon_t \sim N\left(0, \sigma_\epsilon^2\right)$.

The government's objective is to maximize the present expected discounted value of future utility flows of the representative agent in the economy, namely

$$E\left[\sum_{t=0}^{\infty} \beta^t u\left(c_t\right)\right],$$

where E denotes the expectation operator, β denotes the subjective discount factor, and the utility function is assumed to display a constant coefficient of relative risk aversion denoted by γ. That is,

$$u\left(c\right) = \frac{c^{(1-\gamma)} - 1}{1-\gamma}.$$

As in Hatchondo and Martinez (2009), we assume that a bond issued in period t promises an infinite stream of coupons, which decreases at a constant rate δ. In particular, a bond issued in period t promises to pay one unit of the good in period $t+1$ and $(1-\delta)^{s-1}$ units in period $t+s$, with $s \geq 2$.

Each period, the government makes two decisions. First, it decides whether to default. Second, it chooses the number of bonds that it purchases or issues in the current period.

As in previous studies of sovereign default, the cost of defaulting is not a function of the size of the default. Thus, as in Arellano and Ramanarayanan (2010), Chatterjee and Eyigungor (forthcoming), and Hatchondo and Martinez (2009), when the government defaults, it does so on all current and future debt obligations. This is consistent with the behavior of defaulting governments in reality. As mentioned in the introduction, sovereign debt contracts often contain acceleration and cross-default clauses. These clauses imply that after a default event, future debt obligations become current.[7] Following previous studies, we also assume that the recovery rate for debt in default is zero.

Lenders are risk neutral and assign the value e^{-r} to payoffs received in the next period. Bonds are priced in a competitive market inhabited by a large number of identical lenders, which implies that bond prices are pinned down by a zero expected profit condition.

When the government defaults, it faces an income loss of $\phi(y)$ in the default period. In Section 4.3 we show that our findings are robust to assuming that the government is excluded from capital markets after a default episode.[8] Following Chatterjee and Eyigungor (forthcoming), we assume a quadratic loss function $\phi(y) = d_0 y + d_1 y^2$.

The government cannot commit to future default and borrowing decisions. Thus, one may interpret this environment as a game in which the government making the default and borrowing decisions in period t is a player who takes as given the default and borrowing strategies of other players (governments) who will decide after t. We focus on Markov Perfect Equilibrium. That is, we assume that in each period, the government's equilibrium default and borrowing strategies depend only on payoff-relevant state variables. As discussed by Krusell and Smith (2003), there may be multiple Markov perfect equilibria in infinite-horizon economies. In order to avoid this problem, we solve for the equilibrium of the finite-horizon version of our economy, and we increase the number of periods of the finite-horizon economy until value functions and bond prices for the first and second periods of this economy are sufficiently close. We then use the first-period equilibrium functions as the infinite-horizon-economy equilibrium functions.

[7]The type of acceleration clauses depend on the details of each bond contract and on the jurisdiction under which the bond was issued (see IMF (2002)). For instance, in some cases it is necessary that creditors holding a minimum percentage of the value of the bond issue request their debt to be accelerated for their future claims to become due and payable. In other cases, no such qualified majority is needed.

[8]Hatchondo et al. (2007) solve a baseline model of sovereign default with and without the exclusion punishment and show that eliminating this punishment only affects significantly the debt level generated by the model.

2.2 Recursive formulation of the baseline framework

Let b denote the number of outstanding coupon claims at the beginning of the current period, and b' denote the number of outstanding coupon claims at the beginning of the next period. A negative value of b implies that the government was a net issuer of bonds in the past. Let d denote the current-period default decision. We assume that $d = 1$ if the government defaulted in the current period and $d = 0$ if it did not. The number of bonds issued by the government is given by $-[b' - (1-d)(1-\delta)b]$. Let V denote the government's value function at the beginning of a period, that is, before the default decision is made. Let $\tilde{V}$ denote its value function after the default decision has been made. Let F denote the conditional cumulative distribution function of next-period endowment y'. For any bond price function q the function V satisfies the following functional equation:

$$V(b, y) = \max_{d\epsilon\{0,1\}} \{d\tilde{V}(1, b, y) + (1-d)\tilde{V}(0, b, y)\}, \tag{1}$$

where

$$\tilde{V}(d, b, y) = \max_{b' \leq 0} \left\{ u(c) + \beta \int V(b', y') F(dy' \mid y) \right\}, \tag{2}$$

and

$$c = y - d\phi(y) + (1-d)b - q(b', y)[b' - (1-d)(1-\delta)b]. \tag{3}$$

The bond price is given by the following functional equation:

$$\begin{aligned} q(b', y) &= \int e^{-r}[1 - h(b', y')] F(dy' \mid y) \\ &\quad + (1-\delta) \int e^{-r}[1 - h(b', y')]\ q(g(h(b', y'), b', y'), y') F(dy' \mid y), \end{aligned} \tag{4}$$

where h and g denote the future default and borrowing rules that lenders expect the government to follow.[9] If the government defaults (does not default), $h = 1$ ($h = 0$). The function g

[9] The price of a bond issued at date t consists of the discounted sum of future payoffs. Formally,

$$q(b_{t+1}, y_t) = \sum_{i=1}^{\infty} (1-\delta)^{i-1} e^{-ir} \int ... \int [1 - h(b_{t+i}, y_{t+i})] F(dy_{t+i} \mid y_{t+i-1}) ... F(dy_{t+1} \mid y_t), \tag{5}$$

where b_{t+i} denotes the government's equilibrium asset position i periods ahead, which is a function of b_{t+1} and determines the number of coupons that will mature next period. The first term in the right-hand side of equation (4) equals the expected value of the next-period coupon payment promised in a bond. The second term in the right-hand side of equation (4) equals the expected value of all other future coupon payments, which is summarized by the expected price at which the bond could be sold in the next period.

Equations (1)-(4) illustrate that the government finds its optimal current default and borrowing decisions taking as given its future default and borrowing decision rules h and g. In equilibrium, the optimal default and borrowing rules that solve problems (1) and (2) must be equal to h and g for all possible values of the state variables.

Definition 1 *A Markov Perfect Equilibrium is characterized by*

1. a set of value functions $\tilde{V}$ and V,
2. a default rule h and a borrowing rule g,
3. a bond price function q,

such that:

(a) given h and g, V and $\tilde{V}$ satisfy equations (1) and (2) when the government can trade bonds at q;

(b) given h and g, the bond price function q is given by equation (4); and

the sequence of income realizations $y_{t+1}, ..., y_{t+i-1}$ (we omit these arguments to simplify the notation).
Equation (5) can be decomposed as

$$q(b_{t+1}, y_t) = e^{-r} \int [1 - h(b_{t+1}, y_{t+1})]\, F(dy_{t+1} \mid y_t) +$$

$$(1-\delta)\, e^{-r} \int \left[\sum_{i=1}^{\infty} (1-\delta)^{i-1} e^{-ir} \int ... \int [1 - h(b_{t+1+i}, y_{t+1+i})]\, F(dy_{t+1+i} \mid y_{t+i}) ... F(dy_{t+2} \mid y_{t+1}) \right] F(dy_{t+1} \mid y_t)$$

$$= e^{-r} \int [1 - h(b_{t+1}, y_{t+1})]\, F(dy_{t+1} \mid y_t) + (1-\delta)\, e^{-r} \int q(b_{t+2}, y_{t+1}) F(dy_{t+1} \mid y_t).$$

(c) the default rule h and borrowing rule g solve the dynamic programming problem defined by equations (1) and (2) when the government can trade bonds at q.

2.3 A framework without debt dilution

In this section, we propose a modification to the model presented in Section 2.1 that will allow us to study an economy without debt dilution and, in turn, to measure the effects of debt dilution. We eliminate debt dilution—caused by borrowing decisions—by introducing a borrowing-contingent debt covenant. The covenant specifies that if the sovereign borrows, it has to pay to the holder of each previously-issued bond the difference between the counterfactual bond price one would have observed absent new borrowing and the observed bond market price.[10] This covenant eliminates debt dilution by making the value of each bond independent from future borrowing decisions.

We assume that a default on borrowing-contingent payments triggers acceleration and cross-default clauses that make all the government's debt obligations become current: if the government selectively defaults on borrowing-contingent payments, it has to cancel all current and future debt obligations, discounting future debt obligations at the risk-free rate. Consequently, selectively defaulting on borrowing-contingent payments cannot be better than buying back all government debt. Therefore, the next subsection presents the recursive formulation of the framework with borrowing-contingent payments without giving the government the option to selectively default on borrowing-contingent payments (but giving the government the option to buy back its debt).

2.4 Recursive formulation of the framework without debt dilution

As before, let q denote the price function of sovereign bonds. Let $\tilde{b} \equiv (1-\delta)b < 0$ denote the interim number of next-period coupon obligations. As in Section 2.1, when the government

[10]To be precise, these payment obligations do not only depend on the borrowing decision (how many bonds are issued in the current period). They also depend on the current income realization and the debt level, given that the counterfactual bond price one would have observed absent new borrowing requires knowledge of the income and debt level, and of the bond price function. While this price is easy to compute in our simulations, it would be difficult to determine in reality. In Section 4.4 we show that most benefits from eliminating dilution can be obtained with borrowing-contingent payments that do not depend on this counterfactual price.

wants to buy back its bonds, it does so at the secondary-market price. If the government issues $\tilde{b}-b'>0$ bonds, borrowing-contingent payments are given by $-\tilde{b}[q(\tilde{b},y)-q(b',y)]$. Suppose the bond price is higher when the debt level is lower because the default probability is increasing with respect to the debt level (as is always the case for the parameterizations we study). The equilibrium bond price is then given by

$$\begin{aligned} q(b',y) \;\; = \;\; & \int e^{-r}\left[1-h\left(b',y'\right)\right] F\left(dy' \mid y\right) \\ & + (1-\delta) \int e^{-r}\left[1-h\left(b',y'\right)\right] \max\left\{0, q(\tilde{b}',y') - q(g(h(b',y'),b',y'),y')\right\} F\left(dy' \mid y\right) \\ & + (1-\delta) \int e^{-r}\left[1-h\left(b',y'\right)\right] q(g(h(b',y'),b',y'),y') F\left(dy' \mid y\right). \end{aligned} \tag{6}$$

The first term of the right-hand side of equation (6) represents the expected value of the next-period coupon payment. The second term represents the expected value of borrowing-contingent payments. The third term represents the expected value of a bond at the end of next period—after the lender received the coupon and borrowing-contingent payments. Note that, because of the borrowing-contingent payments, the future value of a lender's investment may be affected by the income shock, a debt buyback, and a default, but not by new borrowing. Thus, there is no debt dilution in this framework.

The government's budget constraint reads

$$c = y - d\phi\left(y\right) + (1-d)b + q(b',y)(\tilde{b}-b') + \tilde{b}\max\{0, q(\tilde{b},y)-q(b',y)\}. \tag{7}$$

The last term of the right-hand side of equation (7) represents the government's borrowing-contingent payment. After replacing equations (3) and (4) by equations (6) and (7) in the dynamic programming problem described in Section 2.2, we obtain the problem without debt dilution.

3 Calibration

Table 1 presents the calibration. We assume that the representative agent in the sovereign economy has a coefficient of relative risk aversion of 2, which is within the range of accepted

Borrower's risk aversion	γ	2
Interest rate	r	1%
Output autocorrelation coefficient	ρ	0.9
Standard deviation of innovations	σ_ϵ	2.7%
Mean log output	μ	$(-1/2)\sigma_\epsilon^2$
Duration	δ	0.0341
Discount factor	β	0.969
Default cost	d_0	-0.69
Default cost	d_1	1.01
Risk premium	α	4

Table 1: Parameter values.

values in studies of business cycles. A period in the model refers to a quarter. The risk-free interest rate is set equal to 1%. The parameter values that govern the endowment process are chosen so as to mimic the behavior of GDP in Argentina from the fourth quarter of 1993 to the third quarter of 2001, as in Hatchondo et al. (2009). The parameterization of the output process is similar to the parameterization used in other studies that consider a longer sample period (see, for instance, Aguiar and Gopinath (2006)).

With $\delta = 3.41\%$, bonds have an average duration of 4.19 years in the simulations of the baseline model.[11] Cruces et al. (2002) report that the average duration of Argentinean bonds included in the EMBI index was 4.13 years in 2000. This duration is not significantly different from what is observed in other emerging economies. Using a sample of 27 emerging economies, Cruces et al. (2002) find an average duration of 4.77 years, with a standard deviation of 1.52.

We calibrate the discount factor and the output cost (two parameter values) to target three

[11] We use the Macaulay definition of duration, which with the coupon structure in this paper is given by

$$D = \frac{1 + r^*}{\delta + r^*},$$

where r^* denotes the constant per-period yield delivered by the bond.

moments: a mean spread—i.e., the difference between the sovereign bond yield and the risk-free interest rate—of 7.4, a standard deviation of the spread of 2.5, and a mean debt level of 28% of the mean quarterly output in the pre-default samples of our simulations (the exact definition of these samples is presented in Section 4.1).[12] The targets for the spread distribution are taken from the spread behavior in Argentina before its 2001 default (see Table 2). Regarding the debt level, for the period we study, Chatterjee and Eyigungor (forthcoming) target a mean level of unsecured sovereign debt of 70% of quarterly output. Since our model is a model of external debt and Sturzenegger and Zettelmeyer (2006) estimate that 60% of the debt Argentina defaulted on was held by residents, we choose to target a mean debt level that is roughly 40% of the value targeted by Chatterjee and Eyigungor (forthcoming). In Section 4.3 we show that our findings are robust to targeting a higher debt level.

4 Results

As in Hatchondo et al. (2010), we solve the models numerically using value function iteration and interpolation.[13] First, we show that debt dilution accounts for most of the default risk in the benchmark economy. Second, we present the welfare gains from eliminating dilution. Third, we discuss the robustness of our measurement of the effects of debt dilution. Fourth, we show that most gains from eliminating dilution can be obtained with simpler borrowing-contingent payment schemes that may be easier to implement. Fifth, we compare long-duration debt with borrowing-contingent payments to one-period debt. Finally, we compare the allocation without dilution with the allocation that the government could attain if it could trade a full range of one-period state contingent bonds.

[12]The discount factor value we obtain is relatively low but higher than the ones assumed in previous studies (for instance, Aguiar and Gopinath (2006) assume $\beta = 0.8$). Low discount factors may be a result of political polarization in emerging economies (see Amador (2003) and Cuadra and Sapriza (2008)).

[13]We use linear interpolation for endowment levels and spline interpolation for asset positions. The algorithm finds two value functions, $\tilde{V}(1,\cdot,\cdot)$ and $\tilde{V}(0,\cdot,\cdot)$. Convergence in the equilibrium price function q is also assured.

4.1 Dilution and default risk

This subsection measures the effects of debt dilution. In order to do so, it presents simulation results from the models with and without debt dilution. Table 2 reports moments in the data and in our simulations.[14] As in previous studies, we report results computed for pre-default simulation samples. The exception is the default frequency, which we compute using all simulation periods. We simulate the model for a number of periods that allows us to extract 500 samples of 32 consecutive periods before a default. We focus on samples of 32 periods because we compare the artificial data generated by the model with Argentine data from the fourth quarter of 1993 to the third quarter of 2001.[15] In order to facilitate the comparison of simulation results with the data, we only consider simulation sample paths in which the last default was declared at least two periods before the beginning of each sample.

The moments reported in Table 2 are chosen so as to illustrate the ability of the model to replicate distinctive business cycle properties of emerging economies. These economies feature a high, volatile, and countercyclical interest rate, and high consumption volatility. To compute the quarterly interest rate spread we first calculate the yield that makes the present value of future payments promised in a bond—which for the no-dilution case includes borrowing-contingent payments—equal to the bond price. Then, we calculate the quarterly spread as the difference between this yield and the risk-free rate. The annualized spread (R_s) is four times the quarterly

[14] The data for output and consumption were obtained from the Argentine Finance Ministry. The spread before the first quarter of 1998 is taken from Neumeyer and Perri (2005), and from the EMBI Global after that.

[15] The qualitative features of this data are also observed in other sample periods and in other emerging markets (see, for example, Aguiar and Gopinath (2007), Alvarez et al. (2011), Boz et al. (2011), Neumeyer and Perri (2005), and Uribe and Yue (2006)). The only exception is that in the data we consider, the volatility of consumption is slightly lower than the volatility of income, while emerging market economies tend to display a higher volatility of consumption relative to income.

	Data	Dilution	No dilution
Defaults per 100 years		5.59	0.91
Mean debt market value		0.21	0.20
Mean debt face value	0.28	0.30	0.21
$E(R_s)$	7.44	7.21	0.95
$\sigma\left(R_s\right)$	2.51	2.54	0.68
$\sigma(\tilde{y})$	3.17	2.97	3.28
$\sigma(\tilde{c})/\sigma(\tilde{y})$	0.94	1.03	1.10
$\rho\left(\tilde{c}, \tilde{y}\right)$	0.97	1.00	0.99
$\rho\left(R_s, \tilde{y}\right)$	-0.65	-0.81	-0.63

Table 2: Business cycle statistics. The second column is computed using data from Argentina from 1993 to 2001. Other columns report the mean of the value of each moment in 500 simulation samples. Each sample consists of 32 periods before a default episode.

spread.[16]

In Table 2, the logarithm of income and consumption are denoted by $\tilde{y}$ and $\tilde{c}$, respectively. The standard deviation of x is denoted by $\sigma\left(x\right)$ and is reported in percentage terms. The coefficient of correlation between x and z is denoted by $\rho\left(x, z\right)$. Moments are computed using detrended series. Trends are computed using the Hodrick-Prescott filter with a smoothing parameter of $1,600$. Table 2 also reports the mean debt market value (computed as the mean b divided by

[16]In the economy without dilution, let

$$\mathcal{C}(b, y) = max\left\{q(b(1-\delta)(1-h(b,y)), y) - q(g(h(b,y), b, y), y), 0\right\}$$

denote the per-bond borrowing-contingent payment when the initial state is given by the vector (b, y). Let

$$q^{DF}(b', y; i) = e^{-i}\int\left[1 + \mathcal{C}(b', y') + (1-\delta)q^{DF}(g(h(b', y'), b', y'), y'; i)\right]F(dy' \mid y)$$

denote the price of a default-free bond that pays the coupon and $\mathcal{C}$ every period, where i denotes the constant rate at which future payments are discounted. In state (b, y), the yield of a defaultable bond is defined as the rate i^* that satisfies

$$q^{DF}(g(h(b,y), b, y), y, i^*) = q(g(h(b,y), b, y), y).$$

The annualized interest rate spread is therefore defined as $R_s = 4(i^* - r)$.

$\delta + r^*$, where r^* is the mean equilibrium interest rate) and the mean debt face value (computed as the mean b divided by $\delta + r$).

Table 2 shows that the baseline model with dilution matches the data reasonably well. As in the data, in the simulations of the baseline model consumption and income are highly correlated, and the consumption volatility is higher than the income volatility. The model also approximates reasonably well the moments used as targets (the mean debt level, and the mean and standard deviation of the spread). Estimating the default probability in the data is difficult. Using a sample of 68 countries between 1970 and 2010, Cruces and Trebesch (2011) find a frequency of 6.6 defaults every 100 years. Arellano (2008) targets a frequency of 3 defaults per 100 years because that is the number of defaults observed in Argentina during the last 100 years. The default frequency in our benchmark simulation is between those numbers. In Section 4.3, we show that, when the risk aversion of lenders is calibrated to generate a yearly default frequency of three defaults per 100 years, the effects of debt dilution are similar to the ones reported using our baseline calibration.

What are the quantitative effects of debt dilution? Table 2 shows that debt dilution accounts for 84% of the default risk in the simulations of the baseline model. The number of defaults per 100 years decreases from 5.59 in the baseline to 0.91 in the model without debt dilution. Debt dilution also accounts for 87% of the spread paid by the sovereign. The standard deviation of the spread decreases from 2.54 with debt dilution to 0.68 without debt dilution. The mean face value of outstanding bonds declines 32%. Most of this decline is explained by the lower interest rate in the simulations of the model without debt dilution: The mean market value of outstanding bonds decreases only by 9%.

In order to shed light on how eliminating debt dilution affects the government's optimal decisions it is illustrative to consider its first-order conditions. We use $f_j(x_1, ..., x_n)$ to denote the first-order derivative of the function f with respect to the argument x_j. The first-order condition in the benchmark economy (with dilution) is given by

$$u_1(c)q^{Dil}(b', y) = \beta \int V_1^{Dil}(b', y')\, F(dy' \mid y) - u_1(c)q_1^{Dil}(b', y)[b' - (1-d)\tilde{b}], \tag{8}$$

where the superindex Dil denotes variables in the economy with dilution. The left-hand side of equation (8) represents the marginal benefit of borrowing. By issuing one extra bond today, the government can increase current consumption by $q^{Dil}(b', y)$ units. The right-hand side of equation (8) represents the marginal cost of borrowing. The first term in the right-hand side represents the future cost of borrowing: by borrowing more, the government decreases expected future consumption. The second term in the right-hand side represents the current cost of borrowing: by borrowing more, the government decreases the issuance price of every bond it issues in the current period, which in turn decreases current consumption.

Assuming that the zero-profit bond price is decreasing in the debt level (as we find it is the case for the parameterizations we study) and the government chooses to borrow ($b' < \tilde{b}$), the first-order condition in the economy without dilution is given by

$$u_1(c)q^{\text{No dil}}(b', y) = \beta \int V_1^{\text{No dil}}(b', y')\, F(dy' \mid y) - u_1(c)q_1^{\text{No dil}}(b', y)b', \tag{9}$$

where the superindex No dil is used to denote variables in the economy with no dilution.

The comparison of equations (8) and (9) shows how our modification to the baseline model affects the tradeoffs faced by the government when it issues debt. In equation (8), the current cost of borrowing depends on the new issuances: The government only internalizes as a cost the decline in the value of bonds issued in the current period. It does not internalize as a cost the decrease in the value of the debt issued in previous periods. In contrast, equation (9) shows that with our modification to the baseline model, the current cost of borrowing depends on the entire debt stock at the end of the period. That is, the government chooses its issuance level internalizing the dilution in the value of debt issued in previous periods.

Equation (9) illustrates the tradeoffs faced by the government for a given bond price. But the change in the government's tradeoffs also affect the bond price schedule the government faces when issuing debt. We illustrate that in Figure 1. The figure presents the spread demanded by lenders as a function of the face value of next-period debt. The figure also presents the combination of spread levels and next-period debt chosen by the government when its initial debt level is the average level in the simulations of each case considered in the graph.

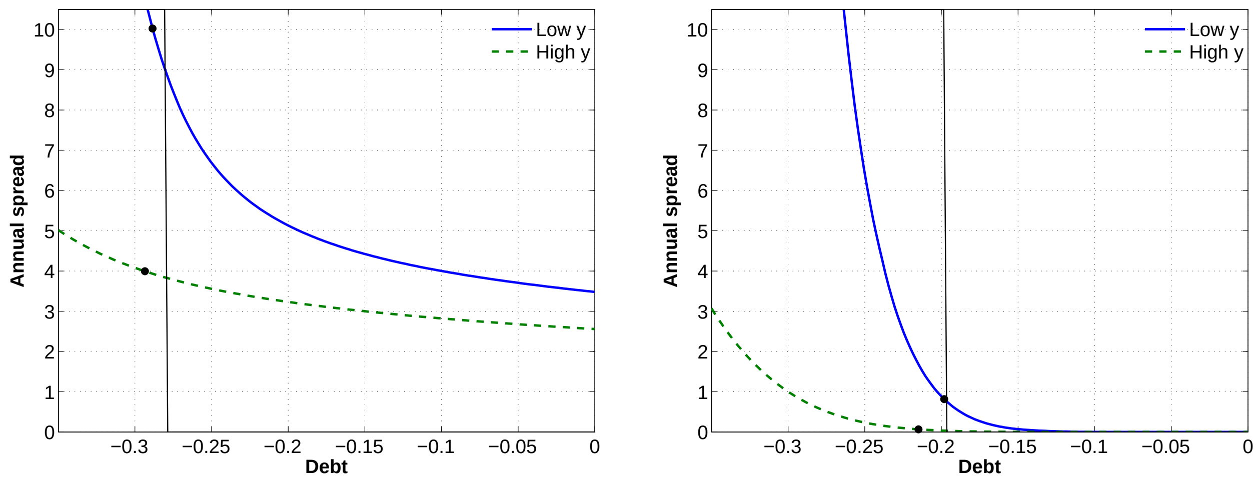

Figure 1: Menu of combinations of spreads and next-period debt levels $(\frac{b'}{\delta+r})$ from which the government can choose. The left panel corresponds to the baseline case. The right panel corresponds to the case without debt dilution. In each of these two cases, solid dots illustrate the optimal decision of a government that inherits a debt level equal to the average debt observed in our simulations for that case. Vertical lines mark the government's debt level before its issuance decision. The low (high) value of y corresponds to an endowment realization that is one standard deviation below (above) the unconditional mean.

Figure 1 shows that a shift in the government's choice set plays an important role in accounting for the reduction in spreads implied by the elimination of debt dilution: Even for the same debt levels, spread levels are higher in the benchmark than in the no-dilution model. For the equilibrium debt levels without dilution, equilibrium spread levels would be about 400 basis points higher in the economy with dilution. In the model without dilution, borrowing-contingent payments weaken the governments incentives to issue debt and thus imply lower future issuance levels. For any debt level, the expectation of lower future issuance levels implies a lower default probability. This in turn allows the government to pay a lower interest rate.

Figure 1 also helps us understand why consumption volatility is higher in the economy without dilution. As illustrated in Figure 1, when income is low, issuance levels tend to be lower in the economy without dilution than in the benchmark with dilution (in the figure, issuance levels are represented by the horizontal distance between the dark dots and the vertical solid line).[17]

[17]Notice that, for the same debt level, the spread curves in Figure 1 are steeper when income is lower. This implies that in the economy without dilution, for the same issuance level, borrowing-contingent payments would

Thus, the government is more effective in mitigating the effects of low income realizations on consumption in the benchmark with dilution.

4.2 Welfare gains from eliminating dilution

We next show that it is welfare enhancing to implement the borrowing-contingent payments that eliminate debt dilution. Eliminating dilution reduces the frequency of defaults, and with that it reduces the deadweight losses caused by defaults. We measure welfare gains as the constant proportional change in consumption that would leave a consumer indifferent between continuing living in the benchmark economy with dilution and moving to an economy without dilution. The welfare gain of moving from the benchmark economy to the economy without dilution is given by

$$\left(\frac{V^{\text{No dil}}(b,y)}{V^{\text{Dil}}(b,y)}\right)^{\left(\frac{1}{1-\gamma}\right)} - 1.$$

Figure 2 presents welfare gains from implementing the borrowing-contingent payments that eliminate dilution. The figure considers two initial debt levels: zero, and the mean debt level in the simulations of the economy with dilution. The figure shows that for both cases there are positive welfare gains from eliminating dilution.

In order to eliminate dilution in an economy with positive debt levels, the government promises borrowing-contingent payments to holders of existing debt. This is costly for the government and explains why in Figure 2 welfare gains from eliminating dilution are lower when there is initial debt (except for lower income levels for which the government chooses to default).[18]

Figure 2 also presents welfare gains from the case in which the government captures existing bondholders' capital gains from the introduction of borrowing-contingent payments that eliminate debt dilution. We assume the government captures these gains through a debt ex-

be larger when income is lower.

[18]Welfare gains are larger in a default period (when the government writes off all debt liabilities) than when the government enters the period without debt. After a default, the government wants to smooth out the income cost of defaulting. Thus, in a default period, the government has stronger incentives to borrow than when it enters the period without debt. This explains why the improvement in borrowing terms implied by the elimination of dilution is more valuable in default periods than when the government enters the period without debt.

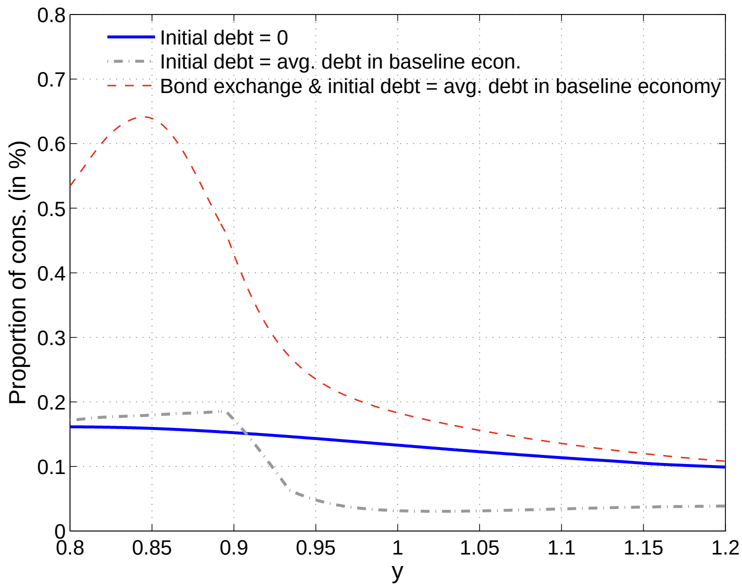

Figure 2: Consumption compensation (in percentage terms) that makes domestic agents indifferent between living in an economy with or without dilution. The figure was constructed assuming that the initial debt level is equal to zero or to the mean debt level in the simulations of the economy with dilution. A positive number means that agents prefer the economy without dilution.

change: The government makes a take-it-or-leave-it debt buyback offer with the promise that these borrowing-contingent payments will be implemented only if this offer is accepted. Thus, the government offers bondholders to buy back previously issued bonds at the price that would have been observed if borrowing-contingent payments were never implemented. That price is lower than the no-dilution price at which the government would be able to issue debt after implementing borrowing-contingent payments. By assuming that the government makes a take-it-or-leave-it offer, we focus on the extreme case in which it reaps all capital gains.[19] The case in which borrowing-contingent payments are introduced without a debt exchange constitutes the other extreme case in which bondholders enjoy all these gains.

It should be mentioned that one may want to take our measure of the welfare gain with a grain of salt. In particular, one could argue that our measure is too low. Since there is no production in our setup, we cannot capture productivity gains from reducing the level and

[19]It is also assumed that there are no output costs triggered by that debt exchange.

volatility of interest rates.[20] Several studies find evidence of a significant effect of interest rates on productivity (through the allocation of factors of production), and of a significant role of interest rate fluctuations in the amplification of shocks (see, for example, Mendoza and Yue (forthcoming), Neumeyer and Perri (2005), and Uribe and Yue (2006)). Furthermore, the welfare gain is increasing in the level of debt, and we chose to calibrate our model to a low debt level to resemble the level of sovereign defaultable debt held by foreigners. In the next subsection we present a calibration with a higher debt level and show that eliminating dilution results in a larger welfare gain.

4.3 Robustness

In this subsection, we show that our finding of a strong effect of debt dilution on sovereign default risk is robust to changes in the benchmark economy we study. We focus on three changes to our benchmark. First, we modify the cost of defaulting to allow for a higher debt stock. Second, we assume that lenders are risk averse. Third, we introduce sudden-stop shocks into the model (while we still assume risk-averse lenders). For all cases, we show that eliminating dilution leads to a significant reduction of the default frequency and increases welfare.

In order to present a case with higher debt levels, we assume a larger cost of defaulting. A defaulting economy is banned from international capital markets for a stochastic number of periods and it loses a fraction of its output in every period it remains excluded. The probability of reentry to capital markets is constant over time. This is the same cost of defaulting considered in most previous studies of sovereign default (e.g., Aguiar and Gopinath (2006) and Arellano (2008)). We assume that the probability of reentry equals 0.282, which is the value used by Arellano (2008). The only parameters that change compared to the benchmark parameterization are the ones that determine the output loss after a default. We set $d_0 = -0.7043$ and $d_1 = 0.9236$. This calibration implies a debt level of 78 percent of quarterly output, a value slightly higher than the one targeted by Chatterjee and Eyigungor (forthcoming).

[20]The development of a sovereign default framework that accommodates effects of interest rates on factors allocation is the subject of ongoing research (see, for example, Mendoza and Yue (forthcoming) and Sosa Padilla (2010)).

We introduce risk premium as Arellano and Ramanarayanan (2010). We assume that the price of sovereign bonds satisfies a no arbitrage condition with stochastic discount factor $M(y', y) = e^{-r-\alpha\varepsilon' - 0.5\alpha^2\sigma_\epsilon^2}$. Several studies document that the risk premium is an important component of sovereign spreads and that a significant fraction of the spread volatility in the data is accounted for by the volatility in the risk premium (see, for example, Borri and Verdelhan (2009), Broner et al. (2007), Longstaff et al. (2011), and González-Rozada and Levy Yeyati (2008)).

The discount factor $M(y', y)$ is a special case of the discrete-time version of the Vasicek one-factor model of the term structure (see Vasicek (1977) and Backus et al. (1998)). With our formulation, the risk premium is determined by the income shock in the borrowing economy. The advantage of our formulation is that it avoids introducing additional state variables to the model. However, it may be more natural to assume that the lenders' valuation of future payments is not perfectly correlated with the sovereign's income.

Our third robustness exercise shows that our measurement of the quantitative effect of dilution on default risk is robust to assuming that there is a shock to the cost of borrowing that is not perfectly correlated with the sovereign's income. To this end, we introduce sudden-stop shocks. These shocks have received considerable attention in the international macroeconomics literature (see Durdu et al. (2009) and the references therein) but have been mostly ignored in the quantitative sovereign default literature (Chatterjee and Eyigungor (forthcoming) is a notable exception). We add a shock s such that when $s = 1$ ($s = 0$) the government can (cannot) issue debt—the government can always buy back previously issued debt. We denote by π_1 (π_0) the probability of $s_{t+1} = s_t$ conditional on $s_t = 1$ ($s_t = 0$). Thus, the value functions with sudden stops are given by

$$V(b, y, s) = \max_{d\epsilon\{0,1\}} \{d\tilde{V}(1, b, y, s) + (1-d)\tilde{V}(0, b, y, s)\},$$

$$\tilde{V}(d, b, y, 1) = \max_{b' \leq 0} \left\{ u(c) + \beta \int [\pi_1 V(b', y', 1) + (1-\pi_1) V(b', y', 0)]\, F(dy' \mid y) \right\},$$

and

$$\tilde{V}(d,b,y,0) = \max_{b(1-\delta)\leq b'\leq 0} \left\{ u\left(c\right) + \beta \int \left[\pi_0 V(b',y',0) + (1-\pi_0) V(b',y',1)\right] F\left(dy' \mid y\right) \right\}.$$

We calibrate π_1 and π_0 assuming that there is a 5.5% unconditional probability of observing $s = 0$ and sudden-stop periods last for one year on average (using annual data for a sample of developing countries from 1980 to 2003, Eichengreen et al. (2008) estimate a 5.5% probability of observing a sudden stop).

Table 3 shows that there is a strong effect of debt dilution on sovereign default risk in the three economies studied in this subsection. The effects of dilution on other variables are also very similar across the different exercises. The exception is the debt level. When the government is excluded from capital markets after a default, the market value of debt is higher in the economy without dilution. The better borrowing terms that the government receives in the economy without dilution increases the value of having access to debt markets and, therefore, it makes defaulting more costly in that case. Eliminating dilution is also beneficial for welfare in all the cases studies.

4.4 Alternative borrowing-contingent payments

Implementing the borrowing-contingent payments that eliminate dilution would require knowledge of the fundamentals that determine bond prices (income and debt in the model) and the mapping from fundamentals onto bond prices. In this section, we study the effects of two simpler debt covenants.

We first study the case in which borrowing-contingent payments are a predetermined fixed share of borrowing revenues per unit of outstanding debt. We search for the optimal share of borrowing revenues the government should promise to existing creditors. We find that this share is such that on average the government pays to holders of debt issued in previous periods 36% of its issuance revenues.[21]

[21] It should be mentioned that promising higher borrowing-contingent payments is not necessarily costly for the government because it allows the government to sell bonds at a higher price.

	Higher debt		Risk-averse lenders		Sudden Stops	
	Dilution	No dilution	Dilution	No dilution	Dilution	No dilution
Defaults per 100 years	4.52	1.02	3.10	0.42	5.07	0.86
Mean debt market value	0.55	0.56	0.20	0.18	0.23	0.19
Mean debt face value	0.78	0.59	0.28	0.19	0.32	0.20
$E(R_s)$	7.12	1.11	7.04	0.99	7.64	0.52
$\sigma(R_s)$	2.72	0.75	2.19	0.71	3.05	0.65
$\sigma(y)$	2.98	3.22	2.03	3.36	3.07	3.25
$\sigma(c)/\sigma(y)$	1.08	1.26	1.04	1.08	1.03	1.11
$\rho(c, y)$	0.99	0.98	1.00	0.99	0.99	0.99
$\rho(R_s, y)$	-0.79	-0.65	-0.82	-0.63	-0.80	-0.62
Welfare gain (% of cons.)		0.33		0.13		0.12

Table 3: Robustness exercises. Welfare gains correspond to the average gain for the case of zero debt.

Table 4 presents simulation results for the economy with the fixed-share borrowing-contingent payments. The table shows these payments allow the government to achieve 83% of the decline in the default frequency and 69% of the ex-ante welfare gain it achieves with the borrowing-contingent payments that eliminate dilution.

The main difference between fixed-share borrowing-contingent payments and the borrowing-contingent payments that eliminate dilution is that the former are an increasing function of the post-issuance bond price $q(b', y)$ while the latter are a decreasing function of $q(b', y)$. Next, we study the effects of introducing borrowing-contingent payments that are a decreasing function of the post-issuance bond price (but do not depend on the bond price that would have been observed in the absence of borrowing). In particular, we assume that the covenant specifies that if the government issues debt in the current period, it has to pay $A - q(b', y)$ per bond issued in previous periods. We search for the optimal value of A and find that this value is 1.75% higher than the price of a risk-free bond without borrowing-contingent payments. Table 4 shows that this optimal borrowing-contingent function would allow the government to achieve the same

	Dilution	No dilution	Fixed share	Linear
Defaults per 100 years	5.59	0.91	1.71	0.81
Mean debt market value	0.21	0.20	0.17	0.19
Mean debt face value	0.30	0.21	0.20	0.20
$E(R_s)$	7.21	0.95	2.01	0.79
$\sigma(R_s)$	2.54	0.68	1.31	0.66
$\sigma(y)$	2.97	3.27	3.05	3.28
$\sigma(c)/\sigma(y)$	1.03	1.10	0.96	1.10
$\rho(c, y)$	1.00	0.99	0.99	0.99
$\rho(R_s, y)$	-0.81	-0.63	-0.64	-0.61
Welfare gain (% of cons.)		0.13	0.09	0.13

Table 4: Simulation results for different borrowing-contingent payments. Welfare gains correspond to the average gain for the case of zero debt.

welfare gain as with the borrowing-contingent payments that eliminate dilution, with a slightly lower default frequency. In summary, our findings indicate that simple borrowing-contingent payments could also reduce default risk significantly and be welfare enhancing.

4.5 Borrowing-contingent payments vs. one-period bonds

In this subsection, we show that in general one cannot measure the effects of debt dilution by comparing equilibria with long-duration and one-period bonds. With one-period bonds, when the government issues debt, the outstanding debt level is zero (either because the government honored its debt at the beginning of the period or because it defaulted on it) and, thus, the government cannot dilute the value of debt issued in previous periods. But with one-period bonds the government has to pay back its entire debt stock in every period, which increases its exposure to rollover risk.

Table 5 presents simulation results for two models without dilution: Our model with the borrowing-contingent payments that eliminate dilution, and a one-period bond model (i.e., a model with $\delta = 1$). The table presents results for both the baseline and the sudden-stop versions of the model. Table 5 shows that simulation results obtained with one-period bonds differ from those obtained with borrowing-contingent payments. Because one-period bonds may increase the government's exposure to rollover risk, differences in results are wider for the sudden-stop model, for which rollover risk is more significant. With sudden stops, lower default probabilities with one-period bonds are not only the result of the elimination of debt dilution but also the result of the lower debt levels chosen by the borrower to mitigate rollover risk.

	Baseline		Sudden stops	
	Borrowing-cont.	One-period	Borrowing-cont.	One-period
Defaults per 100 years	0.91	0.34	0.86	0.03
Mean debt (market value)	0.20	0.19	0.19	0.08
Mean debt (face value)	0.21	0.20	0.20	0.08
$E(R_s)$	0.95	0.44	0.52	0.15
$\sigma(R_s)$	0.68	0.45	0.65	0.27
$\sigma(y)$	3.28	3.10	3.25	3.22
$\sigma(c)/\sigma(y)$	1.10	1.23	1.11	1.28
$\rho(c, y)$	0.99	0.97	0.99	0.80
$\rho(R_s, y)$	-0.63	-0.73	-0.62	-0.41
Welfare gain (% of cons.)	0.13	0.13	0.12	-0.10

Table 5: Business cycle statistics without debt dilution.

In terms of welfare, when the government is subject to sudden-stop shocks and rollover risk is significant, replacing long-duration bonds by one-period bonds results in welfare losses. The ex-ante welfare loss from issuing one-period bonds instead of long-duration bonds is equivalent to a permanent consumption decline of 0.1%. In contrast, the ex-ante welfare gain from introducing borrowing-contingent payments is equivalent to a permanent consumption increase of 0.12%.

4.6 Borrowing-contingent payments vs. one-period state-contingent claims

Equation (7) makes clear that the borrowing-contingent payments needed to eliminate dilution are state-contingent. In this subsection, we explore how those borrowing-contingent payments perform against the best possible case with state-contingent one-period claims. It would be best for the government to transfer resources across periods using contracts with payoffs that are conditional on the past history of state realizations. For tractability reasons, in this subsection we consider a market structure in which the government can issue one-period Arrow-Debreu securities that pay off conditional on the next-period domestic income realization. The government is subject to the same limited liability constraint that is present in the benchmark economy.[22]

We assume that the government chooses how much it promises to pay next period for each realization of next-period income y' (payments can be negative). Let $b'(y')$ denote such government's promise. The government is subject to the same cost of defaulting that is present in the benchmark economy. Without loss of generality, we assume that the government only promises payments $b'(y')$ for which it would not choose to default. Lenders would not pay for a government's promise $b'(y')$ that the government would default on.

Let $\tilde{W}(d, b, y)$ denote the value function of a government that has chosen the default decision d after starting the period with debt b and income y. For any b_0 and b_1, $\tilde{W}(1, b_0, y) = \tilde{W}(1, b_1, y)$. Thus, since $\tilde{W}(0, b, y)$ is decreasing in b, for any income level y, there exists a debt level $\underline{b}(y')$ such that the government defaults if and only if its debt level is higher than $-\underline{b}(y')$. For any y, $\underline{b}(y)$ satisfies $\tilde{W}(0, \underline{b}(y), y) = \tilde{W}(1, \underline{b}(y), y)$, where

$$
\begin{aligned}
\tilde{W}(d, b, y) \quad = \quad & \max_{b'(y')} \left\{ u\left(c\right) + \beta \int W(b', y') F\left(dy' \mid y\right) \right\} \\
& s.t. \quad c = y - d\phi\left(y\right) + (1-d)b - e^{-r} \int b'(y') F\left(dy' \mid y\right) \\
& \qquad b'(y') \geq \underline{b}(y') \text{ for all } y',
\end{aligned}
$$

[22]Issuing long-term debt allows the government to bring resources forward from future periods, not only from the subsequent period. That is not an option in the economy with one-period Arrow-Debreu securities. Clearly, this limitation is not an issue in the absence of the limited liability constraint. But it can dampen the welfare gain from issuing Arrow-Debreu securities in the presence of such constraint.

and

$$W(b,y) = \max_{d\epsilon\{0,1\}} \{d\tilde{W}(1,b,y) + (1-d)\tilde{W}(0,b,y)\}.$$

Table 6 summarizes simulation results in the benchmark, and in the economies with the borrowing-contingent payments that eliminate dilution, and with state-contingent claims. Since there are no defaults with state-contingent claims, we cannot use the same criterion for selecting samples that we used for economies with defaults. In order to facilitate the comparison of simulations across economies, for the economy with state-contingent claims we use the same 500 samples of 32 periods used to compute the simulations in the benchmark economy (income shocks are the same, though the initial debt levels may differ).

	Benchmark	No dilution	State-contingent claims
Defaults per 100 years	5.59	0.91	0
Mean debt market value	0.21	0.20	0.30
Mean debt face value	0.30	0.21	0.30
$\sigma(y)$	2.97	3.28	2.97
$\sigma(c)/\sigma(y)$	1.03	1.10	0.67
$\rho(c,y)$	1.00	0.99	0.87
Welfare gain (% of cons.)		0.13	0.53

Table 6: Business cycle statistics in the benchmark economy and in the economies with the borrowing-contingent payments that eliminate dilution and with one-period state-contingent bonds.

Table 6 shows that the average ex-ante welfare gain from moving to an economy with state-contingent claims amounts to a permanent increase in consumption of 0.53%. There are three sources of welfare gains from using state-contingent claims. First, issuing state-contingent claims allows the government to avoid defaults. Second, when borrowing, the government is able to obtain more resources with state-contingent claims (as reflected in the higher market value of its debt). Third, the consumption process is more disentangled from the income process in the economy with state-contingent claims (as reflected in the lower consumption volatility).

Welfare gains from moving to an economy with borrowing-contingent payments are 25%

of those from introducing state-contingent claims. In contrast with state-contingent claims, borrowing-contingent payments do not eliminate defaults completely, reduce slightly the market value of the government's debt, and increase consumption volatility.

Our stylized framework is likely to overstate the advantages of introducing state-contingent claims over the introduction of borrowing-contingent payments. First, identifying the state is much more difficult in the real world than in our stylized model. In our model income shocks are the only source of uncertainty. However, Tomz and Wright (2007) argue that other determinants of the sovereigns' willingness to repay besides aggregate income play an important role in accounting for sovereign defaults. Identifying the shocks that affect default risk, measuring these shocks, and writing debt contracts with payments contingent on these shocks may be difficult. Second, writing debt claims contingent on income may suffer from verifiability and moral hazard issues that are not present in our stylized model. A government could manipulate the GDP calculation and final GDP data are available with a significant lag. It has also been argued that debt claims contingent on GDP may introduce moral hazard problems by weakening the government's incentives to implement growth-promoting policies, which improves the likelihood of repayment.[23]

5 Conclusions

We solved a baseline sovereign default framework à la Eaton and Gersovitz (1981) assuming that sovereign bonds contain a debt covenant promising that after each time the government borrows, it pays to the holder of each bond issued in previous periods the difference between the bond market price that would have been observed absent current-period borrowing and the observed market price. This covenant eliminates debt dilution—caused by borrowing decisions—by making the value of each bond independent from future borrowing decisions. We measured the effects of debt dilution by comparing the simulations of this model with the ones of the baseline model without borrowing-contingent payments. We found that even without commitment to

[23] See, for instance, Krugman (1988). These issues could be addressed by indexing debt contracts to variables that the government cannot control such as commodity prices or trading partners' growth rates (see for instance Caballero (2002)).

future repayment policies and without optimally designed contingent claims, if the sovereign eliminates debt dilution, the default probability decreases 84%. We also showed that most gains from eliminating dilution can be obtained with borrowing-contingent payments that depend only on the bond market price.

Our findings indicate that governments could benefit from committing to lower future borrowing levels, which governments could achieve through fiscal rules. Eliminating debt dilution should be an important motivation for the implementation of fiscal rules that could reduce significantly the risk of debt crises and the mean and volatility of interest rates (see Hatchondo et al. (2011)). Fiscal crises are occurring in countries with fiscal rules in part because of the weak enforcement of these rules. The borrowing-contingent covenants studied in this paper could enhance the enforcement of fiscal rules by providing incentives for lower future borrowing levels. Implementing these covenants would necessitate the same strict accounting norms necessary for the successful implementation of fiscal rules.

As in Chatterjee and Eyigungor (forthcoming) and Hatchondo and Martinez (2009), we assume that the government cannot choose the duration of its debt. Relaxing this assumption could enhance our understanding of the effects of debt dilution. It has been argued that the dilution problem may lead to excessive issuances of short-term debt (e.g., Kletzer (1984)). However, allowing the government to choose the duration of its debt would increase the computation cost significantly.[24]

[24] If one allows the government to choose a different duration of sovereign bonds each period, one would have to keep track of how many bonds the government has issued of each possible duration to determine government's liabilities (Arellano and Ramanarayanan (2010) study a model in which the government can choose to issue bonds with two possible durations). The computation cost of including additional state variables may be significant (see Hatchondo et al. (2010)).

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